The Impact of Partial Measurement Invariance on Between-group Comparisons of Latent Means for a Second-Order Factor

by

Yixing Liu

A Thesis Presented in Partial Fulfillment
of the Requirements for the Degree
Master of Arts

Approved April 2016 by the
Graduate Supervisory Committee

Marilyn Thompson, Chair
Samuel Green
Roy Levy

ARIZONA STATE UNIVERSITY

May 2016

ABSTRACT

A simulation study was conducted to explore the influence of partial loading invariance and partial intercept invariance on the latent mean comparison of the second-order factor within a higher-order confirmatory factor analysis (CFA) model. Noninvariant loadings or intercepts were generated to be at one of the two levels or both levels for a second-order CFA model. The numbers and directions of differences in noninvariant loadings or intercepts were also manipulated, along with total sample size and effect size of the second-order factor mean difference. Data were analyzed using correct and incorrect specifications of noninvariant loadings and intercepts. Results summarized across the 5,000 replications in each condition included Type I error rates and powers for the chi-square difference test and the Wald test of the second-order factor mean difference, estimation bias and efficiency for this latent mean difference, and means of the standardized root mean square residual (SRMR) and the root mean square error of approximation (RMSEA).

When the model was correctly specified, no obvious estimation bias was observed; when the model was misspecified by constraining noninvariant loadings or intercepts to be equal, the latent mean difference was overestimated if the direction of the difference in loadings or intercepts of was consistent with the direction of the latent mean difference, and vice versa. Increasing the number of noninvariant loadings or intercepts resulted in larger estimation bias if these noninvariant loadings or intercepts were constrained to be equal. Power to detect the latent mean difference was influenced by estimation bias and the estimated variance of the difference in the second-order factor mean, in addition to sample size and effect size. Constraining more parameters to be equal between groups—even when

unequal in the population—led to a decrease in the variance of the estimated latent mean difference, which increased power somewhat. Finally, RMSEA was very sensitive for detecting misspecification due to improper equality constraints in all conditions in the current scenario, including the nonzero latent mean difference, but SRMR did not increase as expected when noninvariant parameters were constrained.

TABLE OF CONTENTS

Page

LIST OF TABLES vii

LIST OF FIGURES viii

CHAPTER

1 INTRODUCTION 1

Overview…. 1

First- and Second-Order CFA Models 3

First-Order CFA Models. 3

Second-Order CFA Models. 7

Measurement Invariance and Factorial Invariance 10

Simulation Studies Involving Methods for Testing Latent Mean Differences 20

The Purpose of the Current Study 23

2 METHODS 27

Overview…. 27

Representative Model 27

Research Conditions 31

Data Generation Conditions 32

Sample Size 32

Size of Latent Mean Difference for the Second-Order Factor. 32

Direction of Differences in Noninvariant Second-Order Factor Loadings when All First-Order Factor Loadings Were Invariant. 33

CHAPTER Page

Numbers and Directions of Differences in Noninvariant First-Order Factor Loadings when All Second-Order Factor Loadings Were Invariant. 35

The Combination of Noninvariance in Both First-Order and Second-Order Factor Loadings. 36

Direction of Differences in Noninvariant Intercepts of First-Order Factors when All Indicator Intercepts Were Invariant. 36

Numbers and Directions of Differences in Noninvariant Indicator Intercepts when All of the Intercepts of First-Order Factors Were Invariant. 36

The Combination of Noninvariance in Intercepts for Both First-Order Factors and Indicators. 38

Data Analysis Conditions. 38

Equality Constraints on Noninvariant Loadings or Intercepts. 39

Equality Constraints on Invariant Loadings or Intercepts. 39

Baseline Conditions. 39

Data Generation and Data Analysis Procedures 40

Outcomes of Interest 40

Type I Error Rate 41

Power 41

Estimation Bias. 41

Estimated Variance 42

CHAPTER Page

Coverage Rates of 95% Confidence Interval. 42

Standardized Root Mean Square Residual (SRMR) 42

Root Mean Square Error of Approximation (RMSEA). 42

3 RESULTS 44

Impact of Partial Loading Invariance on Estimation Bias for the Second-Order Factor Mean Difference 44

Impact of Partial Intercept Invariance on Estimation Bias for the Second-Order Factor Mean Difference 48

The Factors that Influence the Estimated Variance of the Latent Mean Difference of the Second-Order Factor 52

Type I Error Rates 52

Power…… 53

Coverage Rates of 95 % Confidence Interval 57

RMSEA……..... 57

SRMR…….. 59

4 DISCUSSION 63

Overview…..... 63

The Relationship between Noninvariance and Estimation Bias of the Second-order Factor 64

The General Influence of Misspecification of Noninvariant Factor Loadings and Intercepts on Estimation Bias for the Latent Mean Difference 65

CHAPTER Page

The Influence of Misspecification of Noninvariant Factor Loadings and Intercepts at Different Levels on Estimation Bias for the Latent Mean Difference of the Second-Order Factor 66

Mathematical Functions underlying the Influence of Misspecification of Noninvariant Loadings and Intercepts on Estimation Bias for the Latent Mean Difference for the Second-Order Factor.....67

Power (Type I Error Rates) 70

RMSEA and SRMR.....71

Recommendations.....74

Limitations.. 76

Significance and Conclusions.....77

REFERENCES 80

APPENDIX

A DETAILED RESULTS OF THE SIMULATION STUDY.....83

B THE COMPARISON BETWEEN THE RELATIVE BIAS AND ITS ROUGH ESTIMATE.....164

LIST OF TABLES

Table Page

1. Simulation Studies on Latent Mean Comparisons Using SMM 21

2. Manipulated Factors for Type of Partial Invariance in Loadings 34

3. Manipulated Factors for Type of Partial Invariance in Intercepts.....37

LIST OF FIGURES

Figure Page

1. Standard First-Order CFA Model 4
2. Standard First-Order CFA Model with Mean Structure....6
3. Second-Order CFA Model 8
4. Second-Order CFA Model with Mean Structure....10
5. Research Design 26
6. Representative Model....28
7. The Relationship between the Estimation Bias and Power 55

Chapter 1: Introduction

Overview

When conducting experimental or quasi-experimental studies, researchers are frequently interested in differences between population means on one or more variables that may be observed or latent. For the case in which the focus is between-group comparisons of latent means, structural equation modeling (SEM) approaches such as structured means modeling (SMM; Sörbom, 1974) and multiple-indicator multiple-cause modeling (MIMIC; Jöreskog & Goldberger, 1975) may be applied (Thompson & Green, 2013). Prior to conducting tests of differences in latent means, however, researchers must ensure that the specified measurement model is appropriate for both groups (Byrne, Shavelson, & Muthén, 1989; Meredith, 1993; Sörbom, 1974; Thompson & Green, 2013). Measurement invariance holds if a measuring device works in the same way across varied conditions (i.e., different populations, different time points) that are irrelevant to the attribute being measured (Millsap, 2011).

Within the framework of the common factor model, measurement invariance is evaluated in terms of factorial invariance, which focuses on the extent to which the factor structure is invariant across multiple populations or multiple occasions (Millsap, 2011). Factorial invariance is frequently characterized and examined based on sets of parameters that are progressively constrained to be equal between populations. For a factor analytic model, these include invariance of the model configuration, factor loadings, intercepts for measures, and residual variances for measures. Although invariance of all of the above-named sets of parameters is an ideal condition for conducting comparisons of latent means, it is quite difficult to achieve in reality. Byrne et al. (1989) provided a detailed strategy for

conducting latent mean comparisons under partial factorial invariance, a condition in which some, but not all, of the parameters with respect to the factor pattern and/or the magnitudes of factor loadings are invariant across groups.

Some researchers (e.g., Hancock, Lawrence, & Nevitt, 2000; Yang, 2008) have used simulation methods to examine empirically the robustness of latent mean comparisons to partial factorial invariance under different conditions, such as the magnitude of noninvariance, model size, effect size of latent mean differences, and sample size. Most of these simulation studies have focused on first-order confirmatory factor analysis (CFA) models; to the best of my knowledge, only one simulation study has examined latent mean comparisons for the general factor in second-order CFA models (Molenaar, Dolan, & Wicherts, 2009). In the research of Molenaar et al. (2009), the simulation conditions were informed by previous studies of gender difference in the general, second-order factor in a CFA model of Wechsler tests. Manipulated conditions were: 1) effect size of the latent mean difference of the second-order factor, 2) the number of noninvariant intercepts for the indicators, 3) the number of noninvariant intercepts for the first-order factors (first-order residual mean differences), and 4) the strength of the correlations among the first-order factors.

Little research has been conducted that focused on the impact of measurement noninvariance on latent mean comparisons for second-order CFA models. In particular, no systematic study has explored the impact of partial loading invariance or partial intercept invariance on between-group differences in estimation for higher-order factors in second-order CFA models. Addressing this need, the main purpose of the current thesis is to evaluate the robustness of latent mean comparisons for the higher-order factor to partial

loading invariance and partial intercept invariance under varied research conditions in second-order CFA models.

In this chapter, I first provide an introduction to first-order and second-order CFA models, factorial invariance, and latent mean comparisons. I then review the simulation literature on the influence of partial factorial invariance on latent mean comparisons. This review informs the design of the proposed simulation study presented in the methods chapter. It is expected that the results of this study will support recommendations for researchers who conduct latent mean comparisons within a second-order CFA framework under the common problems of partial loading invariance or partial intercept invariance at the first and/or second levels.

First- and Second-Order CFA Models

Confirmatory factor analysis (CFA) is based on the common factor model in which each observed measure is a linear function of one or more common factors and one unique factor. The purpose of CFA is to examine whether the specified model in which one or more latent factors are hypothesized to explain variation and covariation among a set of observed measures fit the observed data. The set of observed measures can be considered as the indicators for the corresponding latent factors. The variance of each indicator comprises two sources: common variance and unique variance. Common variance refers to the variance accounted for by the factor(s) and unique variance refers to the combination of systematic impact specific to the indicator and random error variance (Brown, 2006).

First-Order CFA Models

Figure 1 depicts an example of a first-order CFA model. In Figure 1, observed measures X1 – X8 are contained in the *X* vector (8×1); two correlated latent factors, $\xi 1$

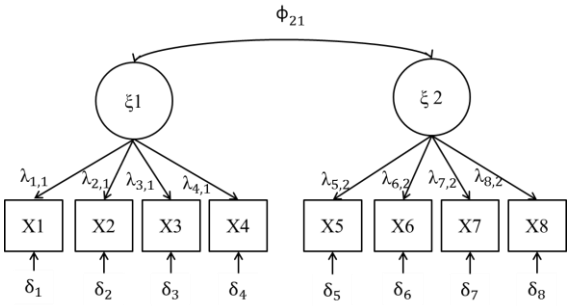


Figure 1 Standard First-Order CFA Model

and ξ2, underlying the set of observed measures are included in **ξ** vector (2 × 1); loadings $\lambda_{1,1}$, $\lambda_{2,1}$, $\lambda_{3,1}$ and $\lambda_{4,1}$ relate ξ1 to its indicators (X1, X2, X3 and X4) and loadings $\lambda_{5,2}$, $\lambda_{6,2}$, $\lambda_{7,2}$ and $\lambda_{8,2}$ relate ξ2 to its indicators (X5, X6, X7 and X8) in the $\mathbf{\Lambda_x}$ matrix (8 × 2); and residuals δ1, δ2, δ3, δ4, δ5, δ6, δ7, and δ8 are in the **δ** vector (8 × 1). Following notation from Bollen (1989), the equation relating factors to measures for a first-order CFA model is:

$$X = \mathbf{\Lambda_x}\,\boldsymbol{\xi} + \boldsymbol{\delta} \qquad (1)$$

The parameters to be estimated for a standard CFA model (i.e., no error covariance) are unknown elements in $\mathbf{\Lambda_x}$, variances for unknown elements in **δ,** and unknown elements in **Φ**, where **Φ** is the variance and covariance matrix for latent factors with dimensions 2 × 2 for this example. The purpose of CFA is to obtain a set of parameter estimates that minimize the difference between the model implied covariance matrix, **Σ(θ)**, and the sample covariance matrix, **S**. **Σ(θ)** is a function of model parameters and is obtained via the following formula:

$$\mathbf{\Sigma(\theta)} = \mathbf{\Lambda_x}\,\mathbf{\Phi}\,\mathbf{\Lambda'_x} + \mathbf{\Theta_\delta} \qquad (2)$$

where $\boldsymbol{\Lambda'}_{\mathbf{x}}$ is the transpose matrix of $\boldsymbol{\Lambda}_{\mathbf{x}}$ and $\boldsymbol{\Theta}_{\boldsymbol{\delta}}$ is the covariance matrix of residuals where the elements on the main diagonal are variances for the elements in $\boldsymbol{\delta}$ and the off-diagonal elements are error covariances (in a standard CFA model, all error covariances are 0s). In order to identify the model, each factor must be scaled either by fixing the value of one of the factor loadings or the factor variance, typically to 1.0.

When the mean structure is included in addition to the variance and covariance structure, indicator intercepts (ν_x) and latent means (κ) must be considered. In order to incorporate these additional parameters, as illustrated by Thompson and Green (2013), a unit predictor will be included in the model, which may be denoted using a triangle as shown in Figure 2. The equations for the first-order CFA model with mean structure are:

$$\boldsymbol{\xi} = \boldsymbol{\kappa}*1 + \boldsymbol{\zeta}$$

$$X = \boldsymbol{\nu}_{\mathbf{x}}*1 + \boldsymbol{\Lambda}_{\mathbf{x}}\boldsymbol{\xi} + \boldsymbol{\delta} \quad (3)$$

where $\boldsymbol{\kappa}$ is the vector containing intercepts for latent factors, $\boldsymbol{\zeta}$ is the vector containing disturbances for latent factors, and $\boldsymbol{\nu}_{\mathbf{x}}$ is the vector containing indicator intercepts. To better understand that the intercept for a given factor is actually the latent mean, the mean of $\xi 1$ is expressed using the following equation:

$$E(\xi 1) = E(\kappa_1*1 + \zeta_1) = E(\kappa_1*1) + E(\zeta_1) = \kappa_1 + E(\zeta_1) \quad (4)$$

where $E(\cdot)$ represents the function of expected value. Given that the expected value of ζ_1 is assumed to be zero, the expected value of $\xi 1$ is κ_1.

For a single group CFA model, the model cannot be identified if both the latent means and indicator intercepts are freely estimated. Generally, the latent means are set to zero (Thompson & Green, 2013). Another purpose for doing so is to scale the latent factors.

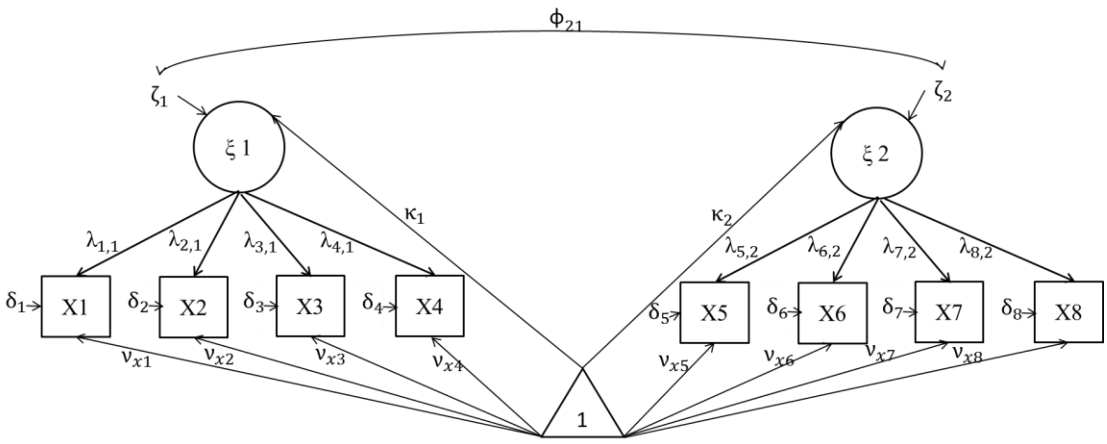


Figure 2 Standard First-Order CFA Model with Mean Structure

The residual **δ** encompasses both systematic error and random error. The expected value of the random part of **δ** is zero, and the mean of the systematic part can be absorbed into $\boldsymbol{\nu}_x$ (Millsap, 2011). Thus, the intercept of an indicator is actually a combination of the origin of a measure and the systematic impact specific to the measure. The model-implied mean vector can be written as:

$$\mathrm{E}(X) = \boldsymbol{\nu}_x + \boldsymbol{\Lambda}_x \boldsymbol{\kappa} \tag{5}$$

Because elements in **κ** are set to zero for the purposes of model identification and scaling the latent factors in a single group CFA model, the model-implied mean vector is equal to the vector containing indicator intercepts. Considering the example shown in Figure 2, eight means are included in the data input when including the mean structure in addition to the variance-covariance structure, and eight additional parameters (i.e., the intercepts for indicators) need to be estimated. Therefore, the mean structure part of the model is just-identified. The mean vector can be perfectly reproduced by indicator intercepts and this

part of model provides no additional information beyond the variance-covariance structure part of the model when fitting the model to data (Thompson & Green, 2013).

Second-Order CFA Models

Second-order CFA models have been used to model relationships among measures and factors in a variety of substantive areas. For example, in studies of intelligence, it is often hypothesized that a factor of general intelligence underlies a set of specific abilities such as verbal ability, spatial ability, and perceptual speed, and each of the specific abilities relates to a set of observed measures (Molenaar, Dolan, & Wicherts, 2009). Second-order CFA models can be applied when the correlations among the first-order factors are substantial and in accordance with the hypothesis that one or more second-order factors can account for the variation and covariation among the first-order factors (Chen, West, & Sousa, 2006). As noted by Chen, Sousa, and West (2005), second-order CFA models can offer several advantages over first-order CFA models. First, second-order CFA models improve the interpretability of relationships among first-order factors. Taking the example of intelligence tests, second-order CFA models can be used to examine whether or not the covariance among a set of specific ability constructs can be accounted for by general intelligence; this cannot be assessed using first-order CFA models with correlated factors. Second, the second-order CFA model is more parsimonious than the corresponding correlated first-order factor model as long as the number of first-order factors for a given second-order factor is larger than three. Third, the explained variance in each indicator can be partitioned into variance due to the second-order factor and variance due to the first-order factor. An example of the second-order CFA model is shown in Figure 3. Following

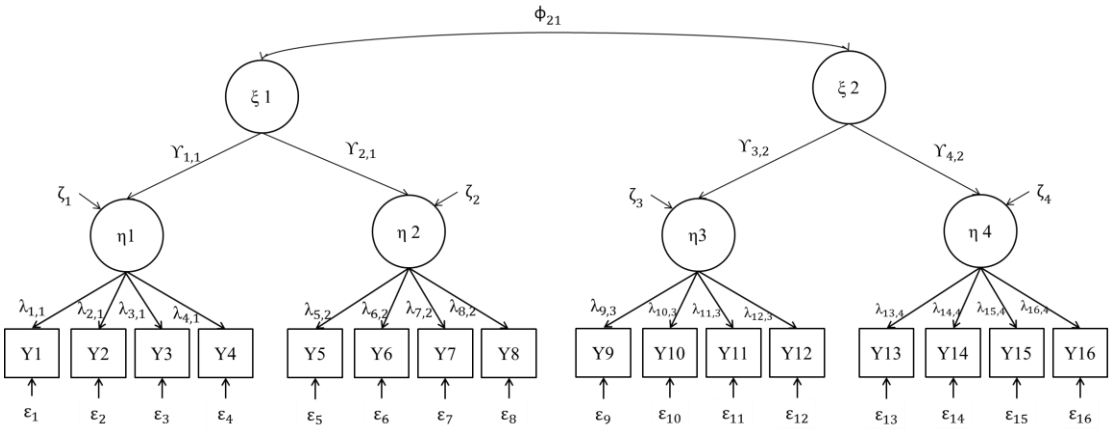


Figure 3 Second-Order CFA Model

notation used in Bollen (1989), the equations for the dependent variables in a second-order CFA model are:

$$\boldsymbol{\eta} = \boldsymbol{\Gamma}\boldsymbol{\xi} + \boldsymbol{\zeta}$$

$$Y = \boldsymbol{\Lambda}_{\mathbf{y}}\boldsymbol{\eta} + \boldsymbol{\varepsilon} \quad (6)$$

where $\boldsymbol{\eta}$ is an $n_{\text{first-order}}$ (the number of first-order factors) $\times 1$ vector containing first-order factors, $\boldsymbol{\Gamma}$ is an $n_{\text{first-order}} \times n_{\text{second-order}}$ (the number of second-order factors) matrix containing second-order factor loadings, $\boldsymbol{\xi}$ is an $n_{\text{second-order}} \times 1$ vector containing second-order factors, $\boldsymbol{\zeta}$ is an $n_{\text{first-order}} \times 1$ vector containing disturbances for first-order factors, Y is a p (the number of input indicators) $\times 1$ vector containing observed measures, $\boldsymbol{\Lambda}_{\mathbf{y}}$ is a $p \times n_{\text{first-order}}$ matrix containing first-order factor loadings, and $\boldsymbol{\varepsilon}$ is a $p \times 1$ vector containing residuals for indicators.

The parameters to be estimated are unknown elements in $\boldsymbol{\Gamma}$, $\boldsymbol{\Phi}$ (the variance-covariance matrix for second-order factors), $\boldsymbol{\psi}$ (the variance-covariance matrix for disturbances for first-order factors), $\boldsymbol{\Lambda}_{\mathbf{y}}$, and $\boldsymbol{\Theta}_{\boldsymbol{\varepsilon}}$ (the variance-covariance matrix for

residuals for indicators). The model-implied variance-covariance matrix as a function of model parameters is:

$$\Sigma = \boldsymbol{\Lambda_y \Gamma\, \Phi \Gamma' \Lambda_y' + \Lambda_y \psi\, \Lambda_y' + \Theta_\varepsilon} \,. \tag{7}$$

The strategies and criterions of identification for the second-order portion of the model are the same as those for first-order CFA models.

When mean structure is included in addition to the covariance structure for second-order CFA models, the new parameters include indicator intercepts (ν_y), intercepts for first-order factors (α), and second-order factor means (κ). An example of the second-order CFA model with mean structure is given in Figure 4, and the equations are:

$$\boldsymbol{\xi} = \boldsymbol{\kappa}^*1 + \zeta_\xi$$

$$\boldsymbol{\eta} = \boldsymbol{\alpha}^*1 + \boldsymbol{\Gamma\xi} + \boldsymbol{\zeta}$$

$$Y = \boldsymbol{\nu_y}^*1 + \boldsymbol{\Lambda_y \eta} + \boldsymbol{\varepsilon} \tag{8}$$

where $\boldsymbol{\kappa}$ is the vector containing intercepts for second-order factors, ζ_ξ is the vector containing disturbances for second-order factors, $\boldsymbol{\alpha}$ is the vector containing intercepts for first-order factors, and $\boldsymbol{\nu_y}$ is the vector containing indicator intercepts. Akin to first-order CFA models, the intercepts for second-order factors in second-order CFA models are their latent means. Given that the expected value for every element in $\boldsymbol{\zeta}$ is assumed to be 0, the equation for expected values of elements in $\boldsymbol{\eta}$ is:

$$\mathrm{E}\,(\boldsymbol{\eta}) = \boldsymbol{\alpha} + \boldsymbol{\Gamma\kappa} \tag{9}$$

And because the expected value of the random part for each element in $\boldsymbol{\varepsilon}$ is 0 and the expected value of the systematic part is absorbed into $\boldsymbol{\nu_y}$, the following model-implied mean structure can be obtained:

$$\mathrm{E}\,(Y) = \boldsymbol{\nu_y} + \boldsymbol{\Lambda_y}\,\mathrm{E}\,(\boldsymbol{\eta}) = \boldsymbol{\nu_y} + \boldsymbol{\Lambda_y}\,(\boldsymbol{\alpha} + \boldsymbol{\Gamma\kappa}) = \boldsymbol{\nu_y} + \boldsymbol{\Lambda_y\alpha} + \boldsymbol{\Lambda_y\Gamma\kappa} \tag{10}$$

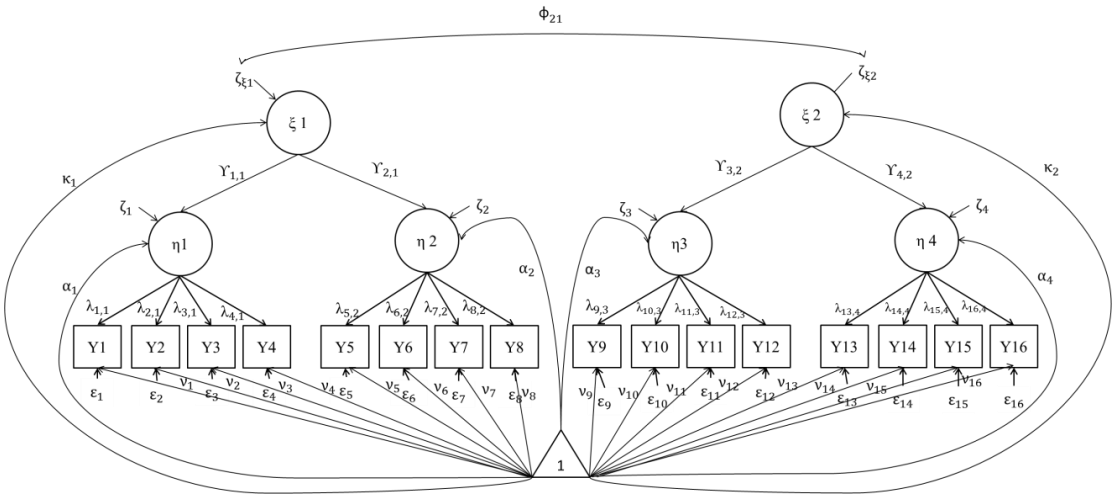


Figure 4 Second-Order CFA Model with Mean Structure

which indicates that the model implied mean vector in a second-order CFA model is a function of first-order factor loadings, second-order factor loadings, latent means for second-order factors, intercepts for first-order factors, and indicator intercepts. For the single group case, in order to identify the model and scale the latent variables, latent means for second-order factors and intercepts for first-order factors are constrained to zero and the model-implied mean vector can be perfectly reproduced by indicator intercepts. As is the case for first-order CFA models, inclusion of the mean structure is unnecessary for single-group, second-order CFA models.

Measurement Invariance and Factorial Invariance

Measurement invariance refers to the notion that some properties of a measure should be invariant across different conditions which are not the focus of the measure (Millsap, 2007, 2011). In order to define measurement invariance, assume X represents a vector of observed variables, W represents a vector of latent variables underlying X, G is

used as the indicator for group membership, and P(•) denotes the probability distribution in the population. The necessary and sufficient condition for measurement invariance is

$$P(X \mid W, G) = P(X \mid W) \tag{11}$$

This general definition of measurement invariance indicates that the conditional probability of observed scores X on latent variable W is independent of the group membership G (Mellenbergh, 1989; Meredith & Millsap, 1992; Millsap, 2007).

Questions of measurement invariance can be investigated in multiple frameworks, including the common factor model, item response theory models, and the latent class model. The focus here is restricted to the framework of CFA models. When measurement invariance is evaluated in the framework of common factor model, it is regarded as factorial invariance, which refers to the extent to which the factor structure underlying the observed measures is invariant across different conditions (e.g., populations, occasions, etc.; Millsap, 2007, 2011).

When applying CFA models in multiple-groups scenarios, a superscript (or subscript) must be added to the CFA model equations and formulas representing variance-covariance matrices and mean vectors based on model parameters (shown in Equations 2 - 10) to denote the group membership. As suggested by Millsap (2011), data analysts seeking to evaluate factorial invariance should test initially whether the variance-covariance matrices and mean vectors are the same across populations. If the null hypothesis is rejected which implies the differences in variance-covariance matrices, mean vectors, or both across the populations, factorial invariance is then examined using a series of nested models. Vandenberg and Lance (2000) described two types of invariance: measurement invariance and structural invariance. Measurement invariance concerns relationships between latent

factors and observed measures, whereas structural invariance concerns latent variables themselves. Measurement invariance should be established prior to evaluating structural invariance. In most applications (e.g., Schmitt & Kuljanin, 2008; Vandenberg & Lance, 2000), the least constrained model with respect to measurement invariance is the one used to test configural invariance in which equivalent factor structures are hypothesized. For this model, the elements in the loading matrix are not required to be equal across populations, but the pattern of zero and non-zero elements in the loading matrix should be the same. Mean structure is not relevant in testing configural invariance. Vandenberg and Lance pointed out (2000) that configural invariance is necessary to make subsequent analysis meaningful. However, Byrne et al. (1989) proceeded to assess factorial invariance and comparisons of latent means based on partial configural invariance in which unique cross-loadings or error covariances existed for each of the two groups. As suggested by Millsap (2011), independent cluster loading structure, in which one observed measure loads on one and only one latent factor, is not necessary for evaluating factorial invariance, but in the current thesis, I focus only on models with independent cluster loading structure.

Evaluation of measurement invariance proceeds by sequentially testing equality constraints on non-zero elements in the loading matrix ($\mathbf{\Lambda_x}$), intercept vector ($\mathbf{\nu_x}$), and the variance-covariance matrix of residual errors ($\mathbf{\Theta_\delta}$). If all freely estimated elements in a given matrix are invariant across populations, the model is considered to be fully invariant between these populations. Full invariance is an ideal condition for subsequent analysis, but may be hard to achieve in reality. In recent years, empirical examinations of partial invariance have become much more frequently observed in papers published after 2000, as identified in an electronic search of studies using the term “measurement invariance”

(Schmitt & Kuljanin, 2008). Partial invariance refers to conditions in which some but not all of the elements in a given matrix ($\mathbf{\Lambda_x}$, $\mathbf{\nu_x}$, or $\mathbf{\Theta_\delta}$) are invariant (Millsap, 2010). If models are found to be configurally invariant, the next step is to test metric invariance, which requires invariant factor loadings across the multiple groups, and can be expressed as:

$$\mathbf{\Lambda_{xg} = \Lambda_{xg'}} \quad (12)$$

in which $\mathbf{\Lambda_x}$ represents the loading matrix and the subscripts g and g' denote group membership. Loading invariance implies that for a one unit increase in the latent factor, the change in the indicator is equivalent across multiple groups. If loading invariance is satisfied, it is considered that the latent trait scores are on the same scale, which is useful for estimation of between-group differences on the latent traits (Reise, Widaman, & Pugh, 1993). When testing loading invariance across two groups, the most constrained model may be parameterized such that the factor variances in one group are constrained to a fixed value, the factor variances in the other group are freely estimated, and all loadings are constrained to be equal across the two groups (Thompson & Green, 2013; Yoon & Millsap, 2007). There are two advantages of such identification strategies. First, selection of an arbitrary reference indicator is avoided. Second, it is not necessary to assume the factor variances to be invariant across groups. If the model fit of the most constrained model used to test loading invariance significantly decreases in comparison with the configurally-invariant baseline model, it can be concluded that complete loading invariance is not achieved in terms of metric invariance. Loading constraints may then be relaxed based on modification indices one by one until such relaxation does not yield a significant difference in the model fit. In order to identify the model, at least one of the loadings for each factor should be equal across the two groups in addition to the constraints imposed on the factor

variances in one of the groups, and the model with such constraints is equivalent to the configurally invariant baseline model. Thus, if complete loading invariance is not achieved, at least two loadings for each factor should be equal across groups to obtain partial loading invariance.

If mean structure is of interest, scalar invariance is tested at the next step, which requires invariant intercepts for measures across the multiple groups, and the corresponding expression is:

$$\boldsymbol{\nu}_{xg} = \boldsymbol{\nu}_{xg'} \tag{13}$$

where $\boldsymbol{\nu}_x$ denotes the intercept vector and the subscript g and g' denote group membership. The intercept reflects the origin of a scale (Byrne & Stewart, 2006). As mentioned above, the systematic impact specific to an indicator is also absorbed into its intercept, therefore, different levels of such influence (i.e., method effects) on an indicator exerted in different groups will also lead to noninvariance in the intercept (Millsap, 2011). For a given observed measure, there is no need to test intercept invariance if a measure has already been deemed to have noninvariant loadings; intercepts for measures with noninvaraint factor loadings should be freely estimated (Thompson & Green, 2013). When testing intercept invariance, the most constrained model evaluated imposes equality constraints on intercepts of all measures having invariant factor loadings. If this model has significantly poorer fit than the model with complete or partial loading invariance, the intercept constraints are freed one by one until such relaxation does not improve the model fit significantly (Thompson & Green, 2013). The identification strategy for the variance and covariance structure of this model is the same as that used to test loading invariance. With respect to the mean structure, in order to scale latent factors, the means of factors in one

group are constrained to a fixed value (i.e., zero) and the means of factors in the other group are left unconstrained.

As reviewed by Vandenberg and Lance (2000), approximately half of the studies on factorial invariance included tests of invariance of the unique variances, which requires

$$\boldsymbol{\Theta}_{\delta g} = \boldsymbol{\Theta}_{\delta g'} \quad (14)$$

where $\boldsymbol{\Theta}_{\delta}$ denotes the variance-covariance matrix for residuals and the subscripts g and g' denote the group membership. Generally, the focus is on the unique variances (the main diagonal of $\boldsymbol{\Theta}_{\delta}$). If unique variances are invariant across groups in addition to complete invariance of structure, loadings, and intercepts, strict invariance is achieved (Meredith, 1993). Vandenberg and Lance (2000) found that most researchers tested unique variance invariance to examine indicator reliability and they pointed out that invariant factor variances are also required for the reliability invariance test because the reliability is defined as the proportion of variance in a measure due to true score variance. Generally, invariance of unique variances is not necessary for testing latent mean differences so, accordingly, is not required in the present study.

Although not required for testing of differences in latent means in first-order CFA models, structural invariance may also be examined. Structural invariance involves between-group equality constraints on elements in the variance-covariance matrix of latent factors ($\boldsymbol{\Phi}$) and the vector containing latent means ($\boldsymbol{\kappa}$). Failure to reject the null hypothesis that factor variances (the elements in the main diagonal of $\boldsymbol{\Phi}$) are invariant reflects the invariance of variability in the latent construct continuum across populations, and the test of invariant covariances among factors (the elements in off-diagonal of $\boldsymbol{\Phi}$) aims to examine the equality of correlations among factors (Vandenberg & Lance, 2000).

Before testing the invariance of latent means, complete or partial measurement invariance must be established. In particular, configural invariance, metric invariance (i.e., invariance of factor loadings), and scalar invariance (i.e., invariance of loadings and intercepts for measures) are assessed. In practice, inconsistencies have been observed with respect to the order of constraints used to test factorial invariance (Vandenberg & Lance, 2000). In most cases, forward approaches are used in which the least constrained model (i.e., the model with configural invariance) is specified first and a series of invariant constraints are imposed sequentially. Backward approaches (Horn & McArdle, 1992), in which analysis starts with the most constrained model (i.e., the model with configural invariance, metric invariance and scalar invariance) and then the constraints are gradually relaxed, can be also applied. Dimitrov (2010) presented two reasons that support forward approaches over backward approaches. First, some aspects of invariance are the prerequisite for tests of other aspects of invariance. For example, the intercept invariance is tested for measures having invariant loadings. Second, when the model fit is poor, it is difficult to figure out which aspect of invariance is violated.

If invariance of all parameters evaluate at a step is not achieved, noninvariant parameters should be identified; partial invariance results when between-group constraints on noninvariant parameters are relaxed. An alternative approach to identifying noninvaraint parameters has also been examined, in which all parameters within a set are allowed to be free (within identification limits) and then between-group constraints are sequentially imposed on parameters until further constraints produce significant decrements in fit (e.g., Lo & Thompson, 2009).

Researchers do not have consistent opinions about the extent to which partial measurement invariance can be allowed when latent mean differences are to be tested. Steenkamp and Baumgartner (1998) suggested using reference indicators to identify the model, and found that at least one indicator besides the reference indicator should have invariant factor loadings and intercepts to make latent mean comparisons meaningful. Thompson and Green (2013) pointed out that it is risky to estimate latent mean differences based on a single invariant measure. Supposing that for a given factor only one indicator is chosen to set equality constraints for estimating the latent mean difference, the model fit will be identical regardless of which indicator is chosen because these models are actually equivalent models. However, the latent mean difference estimate will change as the chosen indicator changes because the latent mean difference is estimated to reproduce the sample means for the chosen indicator. Reise et al. (1993) suggested that the majority of measures should have invariant loadings to avoid arbitrariness of latent mean comparisons if complete invariance was not achieved. Vandenberg and Lance (2000) also recommended a conservative manner to deal with noninvariance when testing latent mean differences in consideration of the possible threat of capitalization on chance in post-hoc, data-driven detection of noninvariance. They suggested that loading invariance constraints could be relaxed for only a minority of indicators based on strong theoretical grounds, and they also mentioned the importance of cross-validation.

With respect to the problem of post-hoc detection of noninvariance, Yoon and Millsap (2007) illustrated that indeterminacy in determining noninvariant measures exists if the noninvariant loadings in one group are proportional to those in the other group. Another concern about such data-driven detection of noninvariance is the variety of criteria

applied to inform which equality constraints should be relaxed. Generally, modification indexes (MIs) are employed to assist in determining whether a measure is invariant. In addition, expected parameter changes (Steenkamp & Baumgartner, 1998) and changes in comparative fit indices, such as Gamma hat and McDonald's Noncentrality Index (Cheung & Rensvold, 2002), have also been recommended to aid in assessing measurement invariance. Yoon and Millsap (2007) detected noninvariant loadings based on MI sequentially using a simulation study; their results indicated that the quality of detection depends on the number of noninvariant measures, sample size, and size of noninvariance. They found the perfect recovery rate (i.e., percentage of simulated samples in which all of noninvariant variables were detected) ranged from 0 to 83% across all of the research conditions, which suggests a fair degree of uncertainty exists in specifying the correct invariance constraints on loadings or intercepts. Lo and Thompson (2009) found similar results using chi-square difference tests to detect partial invariance in loadings; other fit indices did not perform as well for locating noninvariant loading parameters.

For second-order CFA models, when testing factorial invariance, invariance of additional types of parameters must be considered: first-order factor loadings (elements in $\mathbf{\Lambda_y}$), second-order factor loadings (elements in $\mathbf{\Gamma}$), intercepts of measured variables (elements in $\boldsymbol{\nu_y}$), intercepts of first-order factors (elements in $\boldsymbol{\alpha}$), disturbances of first-order factors (elements in $\boldsymbol{\psi}$), and residuals of measured variables (elements in $\mathbf{\Theta_\varepsilon}$). For the second-order part of the model, two methods may be applied in order to identify the covariance structure. As mentioned previously, to avoid selecting a reference first-order factor arbitrarily, it is necessary to fix the variances of the second-order factors in one group to a constant (i.e., a unit) and constrain at least one of the loadings for each second-order

factor to be invariant across groups. Alternatively, a reference first-order factor for each second-order factor can be selected and their loadings related to the second-order factor need to be fixed to a constant (i.e., a unit). For the identification of the covariance structure of the first-order part of the model, given that the variances of first-order factors cannot be directly estimated, generally a reference indicator for each first-order factor should be selected with its first-order factor loading fixed to a constant (e.g., set equal to 1.0).

If the latent mean difference of the second-order factor is of interest, the following method may be used for identification of the mean structure. To scale second-order factors, the latent means of second-order factors in one group (Group 1) are constrained to be zero, and the latent means of second-order factors in the other group (Group 2) are left unconstrained because they reflect latent mean differences that must be estimated. To scale the first-order factors, the intercepts for first-order factors in Group 1 are constrained equal to zero, and whether or not the intercepts for first-order factors in Group 2 need to be constrained to zero depends on the invariance tests for these intercepts. If an intercept for a given first-order factor is found to be noninvariant across groups, it will be freely estimated in Group 2 and it can be considered to reflect the mean differences in first-order factors that cannot be accounted for by the second-order factor. Given that the invariance of intercepts is only assumed for indicators having invariant loadings, in order to identify a second-order CFA model with mean structure, for each second-order factor, at least one first-order factor related to it should have both an invariant second-order factor loading and intercept, and for this first-order factor, at least one measure related to it should have an invariant first-order factor loading and indicator intercept. Additionally, for the first-order factor that does not have an invariant intercept, at least one measure related to it should

have an invariant indicator intercept. Before testing the latent mean difference of the second-order factor, it is necessary to test the invariance of first-order factor loadings, second-order factor loadings, indicator intercepts, and intercepts of first-order factors sequentially (Byrne & Stewart, 2006; Chen et al., 2005).

In previous studies (Byrne & Stewart, 2006; Chen et al., 2005), researchers examined the latent mean difference for the second-order factor using real data based on complete invariance of loadings and intercepts. To the best of my knowledge, no studies based on a real data set involved latent mean comparisons between groups for second-order factors under partial loading invariance or partial intercept invariance. As mentioned previously, Molenaar et al. (2009) explored the impact of partial intercept invariance using a simulation study, and the results will be discussed in the next section.

Simulation Studies Involving Methods for Testing Latent Mean Differences

Various methodological questions regarding partial factorial invariance have been studied using simulation methods. For example, Millsap and Kwok (2004) studied the impact of varying degrees of partial invariance on accuracy of selection using composites; Kaplan (1989) and Lo and Thompson (2009) studied the detection of cross-loadings under partial loading invariance; and Gonzalez-Roma, Hernandez, and Gomez-Benito (2006) investigated differential item functioning (DIF) of graded response items. However, to my best knowledge, only a few studies have been conducted that focused on the impact of partial factorial invariance on latent mean comparisons. Of these studies, some applied MIMIC and some applied SMM. Articles which applied SMM are summarized in Table 1. The dissertation of Yang (2008) and the research of Hancock et al. (2000) focused on first-order CFA models, whereas the study of Molenaar et al. (2009) focused on second-order

Table 1

Simulation Studies on Latent Mean Comparisons Using SMM

Reference	Model	Data Generation Factors	Data Analysis	Main Results
Yang (2008)	One-factor standard CFA model	a) model size; b) the proportion and size of non-invariance in loadings and intercepts; c) construct reliability and observed score variance; d) the size of latent mean differences; e) consistency of direction between intercepts and latent mean differences	All the models were correctly specified	a) all the type I error rates was acceptable; b) the latent mean difference size was the main factor affecting the power to detect them; c) the power was higher for the model with complete invariance than the model with varied degree of non-invariance; d) estimation accuracy was not affected by the varied research conditions
Hancock, Lawrence, & Nevitt (2000)	One-factor standard CFA model	a) total sample size; b) disparity of ratio of sample size; c) the size of population generalized variance; d) positive condition or negative condition; e) whether or not the loadings were invariant; f) whether or not loadings were equal for each group; g) magnitude of factor loadings; h) latent mean differences	a) imposing equality constraints on loadings and intercepts; b) freeing the equality constraints on loadings	a) all the type I error rates was acceptable; b) increased sample size, latent mean differences and magnitude of loadings led to larger power; c) in most of the cases, increased disparity of ratio of sample size led to decreased power; d) releasing equality constraints on non-invariant loadings led to slight improvement in power; e) positive condition decreased the power relative to negative condition
Molenaar, Dolan, & Wicherts (2009)	Second-order CFA model with one second-order factor, three first-order factors and first-order cross loadings	a) latent mean differences for the general factor; b) the number of non-invariant indicator intercepts; c) the number of intercepts for specific factors; d) intercorrelations among the first-order factors		a) the power was mainly determined by the effect size; b) more non-invariant intercepts for the specific factors were associated with less power; c) the influence of the number of non-invariance in intercepts for specific factors decreased as the intercorrelations among specific factors increased; d) the power increased as the intercorrelations among first-order factors increased; e) the number of non-invariant indicator intercept hardly influenced the power

CFA models. Not surprisingly, all of these studies indicated that the effect size of the latent mean difference was the main factor that influenced the power to detect this difference. Also, as expected, larger sample size was associated with larger power (Hancock et al., 2000).

Yang (2008) provided a comprehensive study on the impact of partial loading invariance and partial intercept invariance as well as some other important factors on latent mean comparisons. In Yang's study (2008), all models were correctly specified, and results indicated that power for detecting latent mean differences was higher for the model with complete invariance than for the model with noninvariant components and, further, that power did not differ significantly across models with varying degrees of noninvariance. Estimation accuracy was not affected by research conditions given that the models were correctly specified.

In Hancock et al.'s study (2000), noninvariant loadings were either constrained to be equal or freely estimated. They varied sample size, between-group sample size ratio, and the pattern of sample size and population generalized variance. Their results indicated that when the loadings were noninvariant, relaxing equality constraints would increase the power to detect latent mean differences, and in most of the cases, larger disparity between sample sizes for the groups was associated with decreased power. Additionally, positive conditions (the group with larger sample size was associated with larger population generalized variance) had lower power relative to negative conditions (the group with larger sample size was associated with smaller population generalized variance).

Molenaar et al. (2009) focused on covariance and mean structure for second-order CFA models. They investigated the factors that influence the power to detect between-

group differences in the second-order factor. They generated datasets based on a WAIS-III factor model — a second-order CFA model with one second-order factor affecting four specific factors. Their research conditions were selected based on results from the real data sets. In their study, they manipulated four factors: the size of latent mean differences, the number of differences for indicator intercepts, the number of differences for intercepts of specific factors, and the strength of the correlations among first-order factors. Their results indicated that the power to detect general factor mean differences was mainly determined by its effect size. Additionally, they observed lower power for conditions with greater numbers of noninvariant intercepts for first-order factors, and, further, that such influence decreased as the correlations among the first-order factors increased, which means that the impact of the numbers of noninvariant intercepts for first-order factors on power became smaller as the second-order factor accounted for more variance in the first-order factors. They also found that the impact of number of noninvariant indicator intercepts on power to detect the latent mean difference of the second-order factor was very small.

The Purpose of the Current Study

To date, few researchers have conducted latent mean comparisons based on second-order CFA models under partial factorial invariance. Although Molenaar et al. (2009) conducted a simulation study to investigate the latent mean comparison for the second-order factor, they focused only on the influence of partial intercept invariance. Additionally, their research conditions were proposed based on a specific research scenario, so further study is needed to evaluate whether their findings generalize to other situations. Given that complete factorial invariance in terms of both first-order and second-order measurement

structures may be difficult to achieve and, further, there is no absolute standard to determine whether complete factorial invariance is satisfied, examination of the robustness of latent mean comparisons to partial factorial invariance within the framework of second-order CFA models is needed.

As illustrated in the introduction, there are more types of parameters in second-order CFA models than in first-order CFA models. Additionally, the interpretation of the formula relating parameters to the variance–covariance structure and mean vector is more complicated for second-order CFA models than that for first-order CFA models. Thus, the guidelines proposed based on simulation studies for first-order CFA models may not apply to estimates and tests of latent mean comparisons for second-order CFA models. Even if factors found to influence latent mean comparisons in first-order CFA models also apply for latent mean comparisons in second-order CFA models, the strength of influence of these factors may differ. Thus, from a theoretical viewpoint, systematic studies on what factors influence latent mean comparisons in second-order CFA models are also needed.

The purpose of the current thesis is to systematically investigate the influence of partial factorial invariance on between-population comparisons of latent means on the higher-order factor in a second-order CFA model. The impact of noninvariance on evaluations of differences in the second-order factor mean is assessed under six general situations: 1) noninvariance in second-order factor loadings; 2) noninvariance in first-order factor loadings; 3) noninvariance in the combination of first-order and second-order factor loadings; 4) noninvariance in intercepts of first-order factors; 5) noninvariance in indicator intercepts; and 6) noninvariance in the combination of intercepts of first-order factors and indicators

The focal outcomes of the study include the estimation bias, estimated variance and, power in estimating and testing the between-population latent mean difference for the second-order factor. Total sample size and effect size of the difference in second-order factor means were also manipulated, both of which have been shown to influence positively the power of this test (Molenaar et al., 2009). Of greater interest, however, were the specifications of conditions related to noninvariant loadings and intercepts. The research design is given in Figure 5. It was expected that noninvariance in loadings and intercepts as well as specifications of these noninvariant parameters may have a direct influence on estimation bias and estimated variance of latent mean differences for second-order factors, and power may be directly related to estimation bias and estimated variance for a given sample size and effect size.

To better understand the function of estimation bias in mean differences of second-order factors, the signs of relative differences in factor loadings and intercepts were manipulated in the data generation process. Additionally, the numbers of noninvariant first-order factor loadings and indicator intercepts were also manipulated. When noninvariance existed in one second-order factor loading and two first-order factor loadings relating to the same first-order factor, whether or not the noninvariant first-order factor loadings were associated with the referent factor used in scaling the second-order factor and whether or not they were associated with the first-order factor that had the noninvariant second-order factor loading were considered. When noninvariance existed in one intercept of a first-order factor and two intercepts of indicators relating to the same first-order factor, whether or not the indicators with noninvariant intercepts loaded on the referent factor used in

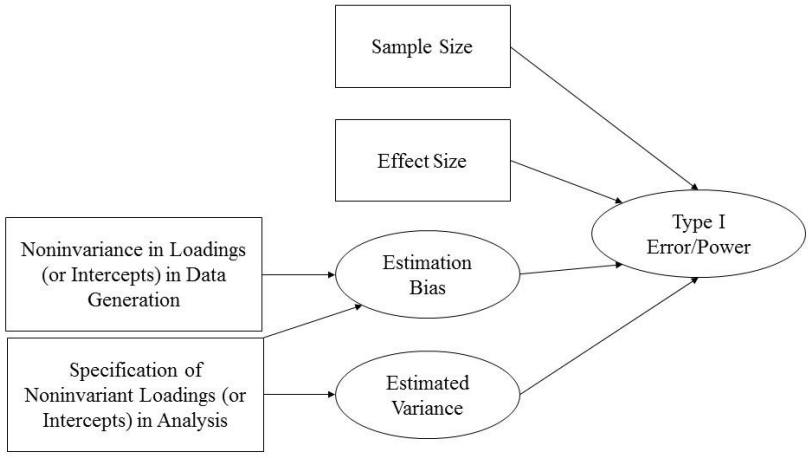


Figure 5 Research Design

scaling the second-order factor and whether or not they loaded on the first-order factor that had noninvariant intercept were considered.

In the data analysis step, noninvariant loadings or intercepts were either freely estimated or constrained to be equal. Additionally, given that some invariant loadings or intercepts may be incorrectly considered to be noninvariant in practice, the impact of freely estimating invariant loadings or intercept on the latent mean comparison was also examined.

Finally, recommendations based on results of the study are offered to aid researchers seeking to examine differences in latent means for the in the second-order CFA models under partial factorial invariance.

Chapter 2: Methods

Overview

Monte Carlo methods were used in the current study to explore the influence of partial loading and intercept invariance on the latent mean comparison of the second-order factor within the framework of higher-order CFA models. A representative model and a set of manipulated research conditions--including population characteristics, sample characteristics, imposition of between-group constraints, and indices for evaluating fit--were designed to address the research questions. Then values of population parameters were chosen and the population variance-covariance matrix and mean vector were formed based on these chosen values and the specified representative model. Mplus 6 (Muthén & Muthén, 2010) Monte Carlo options were used for both data generation and data analysis. From each population, 5,000 sample data sets were generated. The results obtained from fitting the data analysis model to each sample included overall model fit indices and parameter estimates. The results from all of converged replications within a condition were summarized descriptively, including power or Type I error rate, mean of the standardized root mean square residual (SRMR), mean of the root mean square error of approximation (RMSEA), estimation bias, estimated variance, and coverage rate of the 95% confidence interval. Details regarding the representative model, research conditions, the procedures for data generation and analysis, and the results of interest are described next.

Representative Model

The representative model was a multiple group, second-order confirmatory factor analysis model involving both mean and covariance structure, as shown in Figure 6. For all conditions in this study, two populations were specified and the model was configurally

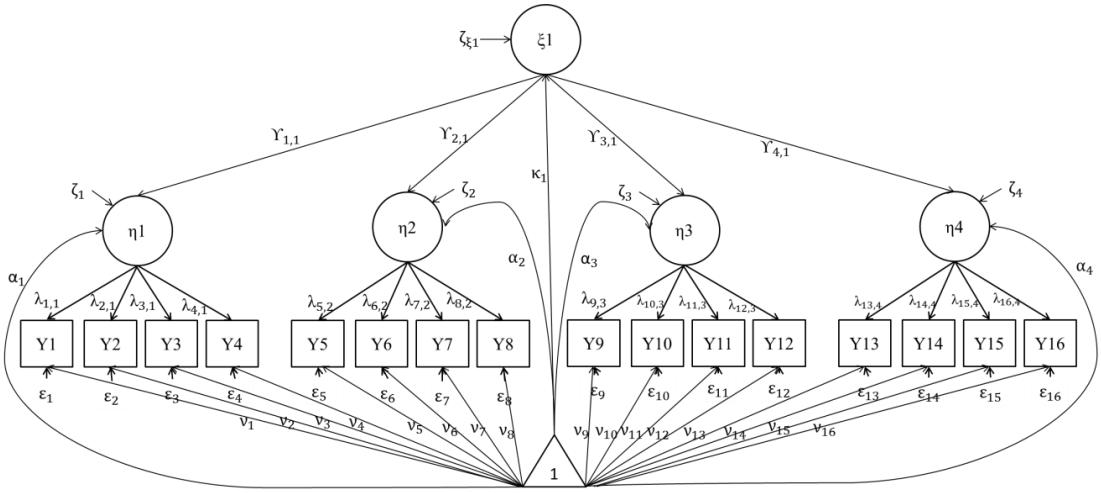


Figure 6 Representative Model

invariant across these populations. As shown in Figure 6, a single second-order factor was hypothesized to explain covariation among four first-order factors, each of which had four indicators with non-zero loadings. Neither cross-loadings nor error covariances exist in the generation models.

For both populations, the variance of the second-order factor was set to 1, such that the estimated latent mean difference can be considered as a standardized effect size. To obtain standardized interpretations of other parameters, all loadings were specified as completely standardized factor loadings, and unit variance for each first-order factor and measured variable were obtained by setting appropriate disturbance or residual variance. To be specific, the disturbance of each first-order factor was set to one minus the square of the corresponding second-order factor loading (i.e., $\sigma^2(\zeta_1) = 1 - \Upsilon_{1,1}^2$), and the residual of each measured variable was set to one minus the sum of its variance explained by the second-order factor (i.e., for measure 1, it is $\Upsilon_{1,1}^2\lambda_{1,1}^2$) and its variance explained by the

corresponding first-order factor (i.e., for measure 1, it is $\sigma^2(\zeta_1)\lambda_{1,1}^2$), which is also equal to one minus the square of the corresponding first-order factor loading (i.e., $\sigma^2(\varepsilon_1)$ = 1 - $\gamma_{1,1}^2\lambda_{1,1}^2 - \sigma^2(\zeta_1)\,\lambda_{1,1}^2$ = 1 - $\lambda_{1,1}^2$).

With respect to the variance and covariance structure, factors were scaled by fixing loadings of referent variables to particular values. The first indicator for each specific factor and the 1st first-order factor were designated as the reference variables and were generated to be invariant across the two populations. Although in realistic situations, reference indicators or factors may be chosen arbitrarily, in the current simulation study, it was assumed that the chosen reference indicators and the reference factor have invariant loadings and intercepts across groups such that the estimation bias would be unrelated with the choice of marker. For both groups, the loadings for the reference variables were set to a given value (the completely standardized factor loading) when generating data; when analyzing data, the loadings for the reference variables were fixed to the same value, such that the same scale was applied for both data generation and analysis models. During data analysis, the choice of the values for the loadings of reference variables would not change the model fit, but would influence parameter estimates. If the loading of the reference variable was fixed to a value (i.e., unity) other than the value specified in the population when generating the data, the difference between the estimated latent mean difference and the population latent mean difference would be a combination of the scaling difference and the real difference.

With respect to the mean structure, factors were scaled by fixing the latent mean of the second-order factor and intercepts of first-order factors in the reference group to particular values and setting equality constraints on intercepts of referent variables across

the two groups. The mean of the second-order factor for the reference group (Group 1) was fixed to 0, such that the mean of the second-order factor for Group 2 was the latent mean difference of this factor between Group 1 and Group 2. For the purpose of scaling first-order factors, the intercepts of all first-order factors for the reference group were fixed to 0 when analyzing data. The intercepts of the first-order factors in Group 2 can be considered as the residual mean differences. The latent mean of the second-order factor and the intercepts of the first-order factors in Group 1 were set to 0 when generating data. Equality constraints were imposed on the intercepts for the reference variables which were generated to be invariant as mentioned above.

In the current study, both the first-order and second-order factor loadings in the reference group (Group 1) remained the same across research conditions. In Group 1:

$$\boldsymbol{\Gamma}' = [.7\ .6\ .7\ .8] \text{ and } \boldsymbol{\Lambda}_y = \begin{bmatrix} .7 & .6 & .6 & .8 \\ .7 & .6 & .6 & .8 \\ .7 & .6 & .6 & .8 \\ .7 & .6 & .6 & .8 \end{bmatrix}$$

where $\boldsymbol{\Gamma}'$ is the vector of second-order factor loadings and $\boldsymbol{\Lambda}_y$ is the matrix containing first-order factor loadings. In Group 1, indicator intercepts were set to 1, and intercepts for first-order factors were set to 0 as mentioned above, which also remained the same across research conditions. For Group 2, the factor loadings and intercepts for the reference variables were the same as those for Group 1. To be specific, in Group 2, $\gamma_{1,1} = .7$ & $\alpha_1 = 0$; $\lambda_{1,1} = .7$ & $\nu_1 = 1$; $\lambda_{5,2} = .7$ & $\nu_5 = 1$; $\lambda_{9,3} = .7$ & $\nu_9 = 1$ and $\lambda_{13,4} = .7$ & $\nu_{13} = 1$.

As mentioned above, the model with only the reference indicator invariant across groups is actually equivalent to the configurally invariant model. Steenkamp and

Baumgartner (1998) proved that at least one indicator besides the reference indicator should have invariant factor loadings and intercepts to make the latent mean comparison meaningful. When unit variance in one group is used to scale the latent factor and the model is identified via at least one pair of invariant measures, the latent mean difference estimate is also unreliable (Thompson & Green, 2013). Assuming the same principle also applies to second-order factor loadings, for each second-order factor, at least one first-order factor besides the referent first-order factor should have invariant second-order factor loadings and intercepts across groups. Thus, in the current design, to make the latent mean comparison of the second-order factor meaningful, for all conditions, it was assumed that the last first-order factor and the last indicator for each first-order factor have invariant loadings and intercepts (in both groups, $\Upsilon_{4,1}$ = .8 & α_4 = 0, $\lambda_{4,1}$ = .8 & ν_4 = 1, $\lambda_{8,2}$ = .8 & ν_8 = 1, $\lambda_{12,3}$ = .8 & ν_{12} = 1, $\lambda_{16,4}$ = .8 & ν_{16} = 1) across groups and, further, these invariant loadings and intercepts were constrained to be equal during analysis.

Research Conditions

The main factors that were manipulated in the current thesis may be distinguished by those manipulated within the data generation phase and those manipulated within the analysis model. The data generation conditions varied were sample size, effect size of latent mean difference for the second-order factor, and patterns of noninvariance in factor loadings or intercepts. When manipulating noninvariant factor loadings or intercepts, conditions were designed to evaluate the following: the unique influence of noninvariance in first-order factor loadings or indicator intercepts on the latent mean comparison of the second-order factor, the unique influence of noninvariance in second-order factor loadings or intercepts of first-order factors on the latent mean comparison of the second-order factor,

and the influence of simultaneous noninvariance in the first-order and second-order factor loadings or simultaneous noninvariance in intercepts of both first-order factors and indicators on the latent mean comparison of the second-order factor, respectively. In the analysis model, model misspecification conditions were introduced in which equality constraints on noninvariant factor loadings or intercepts and equality constraints on invariant loadings or intercepts were manipulated.

The manipulated data generation and analysis factors yielded a total of 360 unique conditions. Design parameters are described below and summarized in Tables 2 and 3, followed by a description of the baseline conditions.

Data Generation Conditions

Data generation conditions included sample size, size of latent mean difference for the second-order factor and patterns of noninvariance in loadings or intercepts. When manipulating the patterns of noninvariance in factor loadings, all intercepts were generated to be invariant across groups; and when manipulating the patterns of noninvariance in intercepts, all loadings were generated to be invariant across groups. Specific data generation conditions were described below.

Sample size. Given that sample size is widely known to influence the power of statistical significance tests, total sample size was varied to include 300 or 600 cases. These sample sizes were chosen to avoid floor effects or ceiling effects regarding power, based on research conditions examined in preliminary analyses for which the difference between them was meaningful. Sample sizes were equal between the two groups.

Size of latent mean difference for the second-order factor. Effect size is also a main factor affecting power of the test of differences in latent means. The latent mean

difference of the second-order factor was 0, .20 (small effect size), or .40 (large effect size). When generating data, the variance of the second-order factor was set to 1 for both groups, so the corresponding latent mean difference can be considered as the standardized effect size. An effect size of 0 is consistent with the null hypothesis of no between-population differences in latent means; rejection of the null hypothesis in this case is a Type I error. Small and large values of standardized effect size were chosen based on the findings of Molenaar et al.'s (2009) simulation study examining the influence of partial intercept invariance on the latent mean comparison of the second-order factor within the framework of higher-order CFA models. In preliminary analyses for the current study, it was found that when the effect size was very large (i.e., larger than .60), the power to detect the latent mean difference of the second-order factor was close to 100% for all research conditions. When the effect size was .40, differences in power among research conditions can be observed, with some conditions achieving desired power levels of .80 or greater.

Direction of differences in noninvariant second-order factor loadings when all first-order factor loadings were invariant. By varying the direction of differences in noninvariant second-order factor loadings while correctly assuming invariance in first-order factor loadings, we can study the unique impact of noninvariant second-order factor loadings on second-order factor mean difference estimation. As shown in Table 2, second-order factor loadings for Group 2 were $\boldsymbol{\Gamma}'$ = [.7 .8 .7 .8] for the cases with a larger second-order loading in Group 2, $\boldsymbol{\Gamma}'$ = [.7 .4 .7 .8] for the cases with a smaller second-order loading in Group 2. For these conditions, the first-order factor loadings were invariant, and the matrix containing first-order factor loadings for both groups was:

Table 2

Manipulated Factors for Type of Partial Invariance in Loadings

Factor	Partial invariance of second-order loadings	Partial invariance of first-order loadings	Partial invariance of both first- and second-order loadings
Total sample size	300 or 600	300 or 600	300 or 600
Effect size of the latent mean difference	0, .20 or .40	0, .20 or .40	0, .20 or .40
Relative differences in noninvariant second-order loadings	In Group 2: $\Upsilon_{2,1}$ = .8 or $\Upsilon_{2,1}$ = .4	NA	In Group 2: $\Upsilon_{2,1}$ = .8 or $\Upsilon_{2,1}$ = .4
Relative differences in noninvariant first-order loadings	NA	In Group 2: $\lambda_{6,2(6,2;7,2;10,3;11;3)}$ = .8 or $\lambda_{6,2(6,2;7,2;10,3;11;3)}$ = .4	In Group 2: $\lambda_{2,1(6,2)(10,3)}$ = .4 & $\lambda_{3,1(7,2)(11,3)}$ = .4
The number of noninvariant first-order loadings	NA	1 or 4	2
The first-order factor that has noninvariant first-order factor loadings	NA	η2 or η2 & η3	η1, η2 or η3
Data analysis procedure	Constraining the noninvariant loadings to be equal or not, or the noninvariant loadings are constrained to be equal but some invariant loadings are freely estimated		Constraining noninvariant loadings to be equal or not

Note: η1 is the reference factor, η2 is the factor associated with noninvariant second-order loadings and η3 is the factor associated with invariant second-order loadings.

$$\mathbf{\Lambda_y} = \begin{bmatrix} .7 & .6 \\ .7 & .6 \\ .7 & .6 \\ .7 & .6 \end{bmatrix} \begin{bmatrix} .6 & .8 \\ .6 & .8 \\ .6 & .8 \\ .6 & .8 \end{bmatrix}$$

Numbers and directions of differences in noninvariant first-order factor loadings when all second-order factor loadings were invariant. To explore the unique impact of noninvariance of first-order factor loadings on the latent mean comparison for the second-order factor, numbers and directions of differences in noninvariant first-order factor loadings were varied while holding second-order factor loadings invariant. When one noninvariant first-order factor loading was involved, it was associated with the second first-order factor; and when four noninvariant first-order loadings were involved, two of them were associated with the second first-order factor and the other two were associated with the third first-order factor. Four levels of noninvariance in first-order factor loadings were created by manipulating both direction of noninvariance (i.e., difference of .2 or -.2) and number of noninvariant first-order loadings (i.e., 1 or 4). Accordingly, first-order factor loadings for the following $\mathbf{\Lambda_y}$s for Group 2 were:

$$\mathbf{\Lambda_y} = \begin{bmatrix} .7 & .6 \\ .7 & .8 \\ .7 & .6 \\ .7 & .6 \end{bmatrix} \begin{bmatrix} .6 & .8 \\ .6 & .8 \\ .6 & .8 \\ .6 & .8 \end{bmatrix}, \mathbf{\Lambda_y} = \begin{bmatrix} .7 & .6 \\ .7 & .4 \\ .7 & .6 \\ .7 & .6 \end{bmatrix} \begin{bmatrix} .6 & .8 \\ .6 & .8 \\ .6 & .8 \\ .6 & .8 \end{bmatrix} \mathbf{\Lambda_y} = \begin{bmatrix} .7 & .6 \\ .7 & .8 \\ .7 & .8 \\ .7 & .6 \end{bmatrix} \begin{bmatrix} .6 & .8 \\ .8 & .8 \\ .8 & .8 \\ .6 & .8 \end{bmatrix}, \mathbf{\Lambda_y} = \begin{bmatrix} .7 & .6 \\ .7 & .4 \\ .7 & .4 \\ .7 & .6 \end{bmatrix} \begin{bmatrix} .6 & .8 \\ .4 & .8 \\ .4 & .8 \\ .6 & .8 \end{bmatrix}$$

At the same time, in Group 1, $\mathbf{\Lambda_y}$ was:

$$\mathbf{\Lambda_y} = \begin{bmatrix} .7 & .6 \\ .7 & .6 \\ .7 & .6 \\ .7 & .6 \end{bmatrix} \begin{bmatrix} .6 & .8 \\ .6 & .8 \\ .6 & .8 \\ .6 & .8 \end{bmatrix}$$

For these conditions, the second-order factor loadings were invariant across groups, with $\mathbf{\Gamma'} = [.7\ .6\ .7\ .8]$.

The combination of noninvariance in both first-order and second-order factor loadings. For these conditions that explore the impact of combinations of noninvariance in both first-order and second-order factor loadings on latent mean comparison for the second-order factor, the numbers and size of noninvariance in first-order loadings was held constant (as shown in Table 2). The direction of differences in noninvariant second-order factor loadings was varied, so as shown in Table 2, in Group 2, $\boldsymbol{\Gamma}'$ = [.7 .8 .7 .8] or $\boldsymbol{\Gamma}'$ = [.7, .4, .7, .8], and in Group 1, $\boldsymbol{\Gamma}'$ = [.7, .6, .7, .8]. Only one of the four first-order factors had noninvariant first-order factor loadings. The first-order factor that had noninvariant first-order factor loadings was varied to be the referent factor, an invariant specific factor, or a noninvariant specific factor, designated respectively by the following $\boldsymbol{\Lambda}_y$s for Group 2:

$$\boldsymbol{\Lambda}_y = \begin{bmatrix} .7 & .4 & .4 & .8 \\ .7 & .6 & .6 & .8 \\ .7 & .6 & .6 & .8 \\ .7 & .6 & .6 & .8 \end{bmatrix} \boldsymbol{\Lambda}_y = \begin{bmatrix} .7 & .6 & .6 & .8 \\ .7 & .4 & .4 & .8 \\ .7 & .6 & .6 & .8 \\ .7 & .6 & .6 & .8 \end{bmatrix}, \boldsymbol{\Lambda}_y = \begin{bmatrix} .7 & .6 & .6 & .8 \\ .7 & .6 & .6 & .8 \\ .7 & .4 & .4 & .8 \\ .7 & .6 & .6 & .8 \end{bmatrix}$$

Direction of differences in noninvariant intercepts of first-order factors when all indicator intercepts were invariant. As shown in Table 3, Group 2's $\boldsymbol{\alpha}'$ was [0 .1 0 0] for the cases with a larger intercept in Group 2, and $\boldsymbol{\alpha}'$ was [0 -.1 0 0] for the cases with a smaller intercept in Group 2. For these conditions, the indicator intercepts were 1s for both of the two groups.

Numbers and directions of differences in noninvariant indicator intercepts when all of the intercepts of first-Order factors were invariant. When one noninvariant indicator intercept was involved, it was associated with the second first-order factor; and when four noninvariant indicator intercepts were involved, two of them were associated with the second first-order factor and the other two were associated with the third first-order factor. Four levels of noninvariance in indicator intercepts were created by

Table 3

Manipulated Factors for Type of Partial Invariance in Intercepts

Factor	Partial invariance of intercepts for the first-order factor	Partial invariance of indicator intercepts	Partial invariance of intercepts for both the first-order factor and indicators
Total sample size	300 or 600	300 or 600	300 or 600
Effect size of the latent mean difference	0, .20 or .40	0, .20 or .40	0, .20 or .40
Relative differences in noninvariant intercepts of the first-order factor	In Group 2: α_2 = .1 or α_2 = -.1	NA	In Group 2: α_2 = .1 or α_2 = -.1
Relative differences in noninvariant indicator intercepts	NA	In Group 2: $\nu_{6(6;7;10;11)}$ = 1.1 or $\nu_{6(6;7;10;11)}$= .9	In Group 2: $\nu_{2(6)(10)}$ = .9 & $\nu_{3(7)(11)}$ = .9
The number of noninvariant indicator intercepts	NA	1 or 4	2
The first-order factor that has measures with noninvariant indicator intercepts	NA	η2 or η2 & η3	η1, η2 or η3
Data analysis procedure	Constraining noninvariant intercepts to be equal or not, or the noninvariant intercepts are constrained to be equal but some invariant intercepts are freely estimated		Constraining noninvariant intercepts to be equal or not

Note: η1 is the reference factor, η2 is the factor associated with measures having noninvariant intercepts and η3 is the factor associated with invariant intercepts.

manipulating both direction of noninvariance (i.e., difference of .1 or -.1) and numbers of noninvariant first-order loadings (i.e., 1 or 4). At the same time, in Group 1, the indicator intercepts were 1s.

For these conditions, the intercepts of first-order factors were invariant across groups, with $\boldsymbol{\alpha}' = [0\ 0\ 0\ 0]$.

The combination of noninvariance in intercepts for both first-order factors and indicators. For these conditions that explore the impact of combinations of noninvariance in intercepts of both first-order factors and indicators on latent mean comparison for the second-order factor, the numbers (i.e., 2) and size of noninvariance (i.e., -.1) in indicator intercepts was held constant (as shown in Table 3). The direction of differences in noninvariant intercepts of first-order factors was varied, so as shown in Table 3, in Group 2, $\boldsymbol{\alpha}' = [0\ .1\ 0\ 0]$ or $\boldsymbol{\alpha}' = [0\ -.1\ 0\ 0]$, and in Group 1, $\boldsymbol{\alpha}' = [0\ 0\ 0\ 0]$. Only one of the four first-order factors had noninvariant indicator intercepts. The first-order factor that had noninvariant indicator intercepts was varied to be the referent factor, an invariant specific factor, or a noninvariant specific factor.

Data Analysis Conditions

For both conditions to explore the impact of partial loading invariance and conditions to explore the impact of partial intercept invariance, the data analysis procedure was varied in the same way. When exploring the influence of partial loading invariance, if a loading was freely estimated, the corresponding intercept was also freely estimated, as would be consistent with recommended practice (e.g., Millsap, 2011; Thompson & Green, 2013). When exploring the impact of partial intercept invariance, loadings were generated

to be invariant and were constrained equal regardless of whether the corresponding intercept was freely estimated. The specific data analysis conditions were described below.

Equality constraints on noninvariant loadings or intercepts. In the analysis of the generated data, noninvariant loadings or intercepts were either constrained to be equal or freely estimated across groups.

Equality constraints on invariant loadings or intercepts. To examine the impact of unnecessarily relaxing between-group constraints for invariant parameters, in addition to the conditions in which both invariant and noninvariant loadings or intercepts were constrained to be equal, some invariant loadings or intercepts were freely estimated while constraining noninvariant loadings or intercepts to be equal (shown in Tables 2-3). To be specific, when noninvariance existed in $\gamma_{2,1}$ or α_2, $\gamma_{2,1}$ or α_2 were constrained to be equal but $\gamma_{3,1}$ or α_3 were freely estimated; when noninvariance existed in $\lambda_{6,2}$ or ν_6, $\lambda_{6,2}$ or ν_6 were constrained to be equal but $\lambda_{7,2}$ or ν_7 were freely estimated; and when noninvariance existed in $\boldsymbol{\lambda_{6,2}}$, $\boldsymbol{\lambda_{7,2}}$, $\boldsymbol{\lambda_{10,3}}$, and $\boldsymbol{\lambda_{11,3}}$, (or ν_6, ν_7, ν_{10} and ν_{13}), $\boldsymbol{\lambda_{6,2}}$, $\boldsymbol{\lambda_{7,2}}$, $\boldsymbol{\lambda_{10,3}}$, and $\boldsymbol{\lambda_{11,3}}$, (or ν_6, ν_7, ν_{10} and ν_{13}) were constrained to be equal but $\boldsymbol{\lambda_{2,1}}$, $\boldsymbol{\lambda_{3,1}}$, $\boldsymbol{\lambda_{14,4}}$, and $\boldsymbol{\lambda_{15,4}}$, (or ν_2, ν_3, ν_{14} and ν_{15}) were freely estimated.

Baseline Conditions

A series of baseline conditions were included in the current study. When exploring the impact of partial loading invariance, complete invariant data with the same parameter values as those illustrated for Group 1 was generated for a given effect size and sample size. In the general baseline case, loadings and intercepts were constrained to be equal. To examine the impact of unnecessarily relaxing between-group constraints for invariant parameters and the unique influence of noninvariance in data generation process, a specific

baseline case for each research condition was also included in which invariant parameters were freely estimated in the analysis phase.

Data Generation and Data Analysis Procedures

Mplus 6 (Muthén & Muthén, 2010) was employed for both data generation and data analysis. The population mean vector and variance-covariance matrix for each group were generated based on the values of parameters illustrated above. For each condition, 5,000 datasets with a given sample size were simulated from each population. The problematic simulations were discarded before summarizing the outcome so the number of simulations for each research condition might vary. To ensure independence across research conditions, a random seed was generated for each data generation condition using a random number generator.

For each generated dataset, a Wald test and a chi-square difference test were used for testing latent mean difference for the second-order factor based on a given data analysis procedure discussed above. When conducting chi-square difference tests, the more restricted model was the model constraining latent means of the second-order factor to be zero in both of the two groups while the less constrained model was the one freely estimating the latent mean of the second-order factor in Group 2. Other parameters of these two models were specified in the same way following the research conditions.

Outcomes of Interest

For a given research condition, the outcomes of interest were summarized across all replications with a proper solution. Results for replications with estimation problems, including non-convergence and Heywood cases (i.e., negative residual variance), were excluded from computation of the summary statistics. To obtain a comprehensive

assessment of the impact of manipulated factors on the estimates and tests of differences in latent means between populations, the summarized outcomes included Type I error rate or power, estimation bias, estimated variance, mean SRMR, and mean RMSEA.

Type I Error Rate

For a given research condition in which the effect size of the latent mean difference was generated to be zero, Type I error rate refers the proportion of replications in which the null hypothesis that the latent mean of the second-order factor in Group 2 is zero was rejected.

Power

For a given research condition in which the effect size of the latent mean difference was non-zero in the population, power refers the proportion of replications in which the null hypothesis that the latent mean of the second-order factor in Group 2 is zero was rejected.

Powers were interpreted for a condition only if the respective Type I error rate falls in the acceptable range of .025-.075, as designated by Bradley's liberal criterion (1978).

Estimation Bias

Estimation bias was the difference between the mean of estimated latent mean differences across replications within a condition and the population latent mean difference. The corresponding equation is:

$$\text{Estimation Bias} = E(\hat{\theta}) - \theta \quad (15)$$

where $E(\hat{\theta})$ denotes the average value of estimated latent mean differences across replications, and θ denotes the true value of the latent mean difference in the population.

Estimated variance

Estimated variance was the variance of estimated latent mean differences across replications.

Coverage Rates of 95% Confidence Interval

Coverage rate of the 95% confidence interval was the proportion of replications within a condition for which the population value of the difference in second-order factor means falls within the computed 95% confidence interval for this difference in means.

Standardized Root Mean Square Residual (SRMR)

SRMR is an absolute model fit index and reflects the mean of absolute correlation residuals. The overall fit indices, such as SRMR and RMSEA, were collected because they are generally recommended for use in judging overall model fit. In the current study, these indices were examined to determine descriptively whether they were sensitive to misspecification of the noninvariant parameters. Hu and Bentler (1999) suggested that the acceptable value for SRMR should be equal or less than .08. The SRMRs obtained from replications with proper solutions were averaged for each condition to obtain the mean of SRMR.

Root Mean Square Error of Approximation (RMSEA)

RMSEA is a parsimony-corrected model fit index. The population RMSEA may be estimated based on fitting a hypothesized model to a sample as:

$$\text{RMSEA} = \sqrt{\frac{\chi^2 - df}{df\ (N-1)}} \tag{17}$$

where χ^2 is the chi-square statistic obtained when fitting a given model to a sample, df is the model degrees of freedom, and *N* is the sample size. As recommended by Browne and Cudeck (1993), the values of less than .05 indicate a very good fit; the values between .05 and .08 indicate a fair fit; and the values larger than .10 indicate a bad fit. The RMSEAs obtained from replications with proper solutions were averaged for each condition to obtain the mean of RMSEA.

Chapter 3: Results

Impact of Partial Loading Invariance on Estimation Bias for the Second-Order Factor Mean Difference

In the baseline conditions used for exploring the impact of partial loading invariance, completely invariant loadings and intercepts were generated and different patterns of loadings and their corresponding intercepts were freely estimated. For these baseline conditions, the data generation conditions were the same for a given sample size and effect size. As shown in Table A1, the number of baseline conditions in which all loadings and intercepts were constrained to be equal was 6, and for these conditions, the mean estimation bias (i.e., $\widehat{\boldsymbol{\Delta\kappa}} - \boldsymbol{\Delta\kappa}$) was .0004 (range of -.0011 to .0021), and the mean absolute estimation bias was .0008. The number of other baseline conditions in Table A1 was 54, and in these conditions, series of patterns of the loadings and their corresponding intercepts were freely estimated. The mean estimation bias for these baseline conditions with freely estimated loadings was .0003 (range of -.0012 to .0026), and the mean absolute estimation bias was .0008. Compared with the baseline conditions in which all loadings and intercepts were constrained to be equal, the estimation bias did not increase or decrease a lot due to removing equality constraints on invariant parameters, and the largest absolute difference between each pair was .0007.

In the research conditions, a series of patterns of noninvariance in factor loadings were varied when generating the data, and these noninvariant loadings were either freely estimated or constrained to be equal when analyzing the data. The results regarding the estimation bias for these research conditions are shown in Tables A2-10. The number of correctly specified models in these research conditions for exploring the influence of partial

loading invariance was 72, and in these models, all of the noninvariant loadings and their corresponding intercepts were freely estimated when analyzing the data. The mean estimation bias for these correctly specified models was -.00003 (range of -.0040 to .0046), and the mean absolute estimation bias was .0012. Thus, compared with the baseline conditions, very little estimation bias was produced due to partial loading invariance as long as no equality constraints were placed on noninvariant loadings.

Each data generation model was either correctly specified or misspecified by constraining noninvariant loadings or intercepts to be equal. Given that no obvious estimation bias was observed for correctly specified models, it was supposed that the estimation bias in correctly specified models reflected random sampling error. To avoid the influence of the sampling error, we primarily focused on the difference in estimation bias of the latent mean difference between the misspecified model and the corresponding correctly specified model (i.e., these model pairs were estimated using the same generated dataset). As shown in Tables A2-4, when the latent mean difference of the second-order factor was generated to be zero, the mean absolute difference in estimation bias was .0003 (maximum of .0006) between the misspecified models in which all of loadings and intercepts were constrained to be equal and the corresponding correctly specified model. For the conditions with an effect size of zero and noninvariant loadings at only one of the two levels, when noninvariant loadings were constrained to be equal but some invariant loadings and their corresponding intercepts were freely estimated, the mean absolute differences in estimation bias was .0002 (maximum of .0014) compared with the corresponding correctly specified model (shown in Tables A2-3). Thus, when the latent mean difference of the second-order factor was zero and noninvariance only existed in

factor loadings, the estimation bias was not influenced by constraining noninvariant loadings to be equal.

In the population, when the latent mean difference of the second-order factor was nonzero, the latent mean of the second-order factor was specified to be larger in Group 2 than in Group 1 (reference group). When the noninvariant loadings were present at one of the two levels and they were falsely constrained to be equal, the nonzero latent mean difference was overestimated if the loading(s) in Group 2 were larger than the loading(s) in Group 1, and vice versa (shown in Tables A5, A6, A8, A9).

The results indicated that the overestimation or underestimation of the latent mean difference of the second-order factor got larger for the cases in which the noninvariant parameters were constrained to be equal but some of the invariant loadings and their corresponding intercepts were freely estimated compared with cases in which both noninvariant and invariant parameters were constrained to equal. To be specific, when noninvariance was only present in second-order factor loadings and the noninvariant second-order factor loading was constrained to be equal but an invariant second-order factor loading was freely estimated, the means of the absolute changes in estimation bias were .0015 and .0027 for the conditions with latent mean differences of .20 and .40, respectively; when noninvariance was only present in one first-order factor loading and this first-order factor loading was constrained to be equal but an invariant first-order factor loading was freely estimated, the means of the absolute changes in estimation bias were .0001 and .0002 for the conditions with latent mean differences of .20 and .40 respectively; and when noninvariance was only present in four first-order factor loadings and these four noninvariant first-order factor loadings were constrained to be equal but four

invariant first-order factor loadings were freely estimated, the means of the absolute changes in estimation bias were .0005 for the conditions with latent mean differences of both .20 and .40.

The direction of the between-group differences in noninvariant loadings did not only influence the sign of the difference of the estimation bias, but may also influence the magnitude of the difference in the estimation bias. As shown in Tables A5, A6, A8 and A9, in almost all of the cases, for a given between-group difference in the pair(s) of noninvariant loadings, the estimation bias seemed to be influenced more by constraining the noninvariant loading(s) with larger value(s) in Group 2 than by constraining the noninvariant loading(s) with larger value(s) in Group 1.

As shown in Tables A5-10, for the conditions in which noninvariance only existed in factor loadings and all of loadings and intercepts were constrained to be equal, the difference in estimation bias was almost twice as large when the latent mean difference of the second-order factor was .40 than when the latent mean difference of the second-order factor was .20 for a given misspecified model. To be specific, when the latent mean difference was .20, the mean absolute difference in estimation bias was .0039 (maximum of .0066); however, when the latent mean difference was .40, the mean absolute difference in estimation bias was .0073 (maximum of .0132).

When noninvariance only existed in the first-order factor loadings and the latent mean difference of the second-order factor was nonzero, larger numbers of noninvariant first-order factor loadings were associated with larger differences in estimation bias of the second-order factor (shown in Tables A6 and A8). As shown in the Figure 1, for the representative model, each second-order factor loading was related to four indicators via a

first-order factor. The results indicated that the differences in estimation bias were a little bit larger for the cases with one noininvariant second-order factor loading than those for the cases with four first-order factor loadings (shown in Tables A5, A6, A8, A9).

Among all of the research conditions used for exploring the impact of noninvariance of factor loadings when both invariant and noninvariant loadings were constrained to be equal, the difference in estimation bias was most obvious when noninvariance existed at both levels, the sign of the relative difference of the noninvariant second-order loadings was the same as the sign of the relative differences of the noninvariant first-order loadings, and the indicators with noninvariant first-order loadings were associated with the referent factor or a factor with invariant second-order loading (shown in Tables A5-10).

Impact of Partial Intercept Invariance on Estimation Bias for the Second-Order Factor Mean Difference

The estimation bias in the baseline conditions for exploring the impact of partial intercept invariance is shown in Table A11. In these baseline conditions, the data were generated to be fully invariant with respect to intercepts and specific series of patterns of intercepts were freely estimated when analyzing the data. There were 6 baseline conditions in which no intercepts were freely estimated; the mean estimation bias across these baseline conditions was .0004 (range of -.0018 to .0041), and the mean of absolute estimation bias was .0020. For the 54 other baseline conditions in which some of these invariant intercepts were freely estimated, the mean of the estimation bias was .0004 (range of -.0019 to .0044), and the mean of the absolute estimation bias was .0020. The same completely invariant replicate datasets were generated for a given sample size and effect size, and then analyzed using various degrees of constraints. Compared with the baseline conditions with every

loading and intercept constrained to be equal, the estimation bias did not increase or decrease a lot due to removing equal constraints on some of these invariant parameters, and the largest absolute difference between each pair was .0007 (shown in Table A11).

Different patterns of noninvariance in intercepts were generated in the research conditions, and when analyzing the data, the noninvariant intercepts were either freely estimated or constrained to be equal. The estimation bias of research conditions for exploring the impact of partial intercept invariance is shown in Tables A12-20. There were 72 models that were correctly specified by freely estimating all noninvariant intercepts. The mean estimation bias for the correctly specified models was .0001 (range of -.0039 to .0043), and the mean of the absolute estimation bias was .0013. Thus, compared with the baseline conditions, noninvariance in intercepts did not influence the estimation bias as long as these noninvariant intercepts were freely estimated.

We still focus on the difference in estimation bias between each misspecified model and the corresponding correctly specified model. When exploring the influence of constraining noninvariant intercepts, the direction of the difference in the estimation bias was also determined by the relative differences in noninvariant intercepts (shown in Tables A12-20). The latent mean difference of the second-order factor was overestimated if the noninvariant intercepts were constrained to be equal and the direction of the difference in noninvariant intercepts was consistent with the direction of the latent mean difference, and vice versa. Compared with the magnitude of the difference in estimation bias when exploring the impact of partial loading invariance, the magnitude of the difference in estimation bias was much larger when noninvariant intercepts were involved. When both invariant and noninvariant intercepts were constrained to be equal, the means of the

absolute difference in estimation bias were .0220 (maximum of .0409), .0215 (maximum of .0416), and .0208 (maximum of .0408) for the conditions with the latent mean difference of the second-order factor of 0, .20 and .40, respectively. When noninvariance only existed in intercepts, the effect size of the latent mean difference of the second-order factor seemed to have no influence on the estimation bias produced by falsely constraining noninvariant intercepts to be equal. Additionally, different from partial loading conditions, the direction of between-group differences in intercepts seemed to have very little influence on the magnitude of differences in the estimation bias.

Compared with the conditions in which both invariant and noninvariant intercepts were constrained to be equal, incorrectly constraining noninvariant intercepts to be equal but freely estimating some invariant intercepts resulted in larger estimation bias. To be specific, when noninvariance was only present in the intercept for a first-order factor, the means of the absolute differences in estimation bias between the analysis models with the noninvariant intercept constrained to be equal but an invariant intercept of another first-order factor was freely estimated (i.e., intercept constraints were applied to the wrong indicator) and the corresponding models with both noninvariant intercepts and invariant intercepts constrained to be equal were .0074, .0072, and .0075 for the conditions with latent mean differences of 0, .20, and .40, respectively (shown in Tables A12, A15 and A18); when noninvariance was only present in one indicator intercept and this indicator intercept was constrained to be equal but an invariant indicator intercept was freely estimated, the means of the absolute changes in estimation bias were .0008 for the conditions with latent mean differences of 0, .20 and .40 respectively; and when noninvariance was only present in four indicator intercepts and these four indicator

intercepts were constrained to be equal but four invariant indicator intercepts were freely estimated, the means of the absolute changes in estimation bias were .0024, .0027, and .0025 for the conditions with latent mean differences of 0, .20 and .40 (shown in Tables A13, A16, and A19).

When noninvariance only existed in indicator intercepts, it was indicated that a larger number of noninvariant indicator intercepts was associated with a larger difference in estimation bias of the second-order factor (shown in Tables A13, A16, and A19). In the representative model, the intercept of a first-order factor had direct influence on four indicators based on the formula relating parameters to indicator means, and when generating the noninvariant intercepts at one of the two levels, the between-group differences were constant. Thus, a comparison can be made between the influence of one noninvariant intercept of the first-order factor and four noninvariant indicator intercepts. As shown in Tables A12, A13, A15, A16, A18 and A19, constraining four noninvariant indicator intercepts resulted in larger estimation bias than constraining one noninvariant intercept of the first-order factor.

Similar to the results regarding noninvariant loadings, for conditions with noninvariant intercepts, the difference in estimation bias was most obvious when noninvariance existed in intercepts of both the first-order factor and indicators, the sign of the relative difference of the noninvariant intercept of the first-order factor was the same as the sign of the relative differences of the noninvariant indicator intercepts, and the indicators with noninvariant indicator intercepts were associated with the referent factor or an invariant factor (shown in Tables A12-20).

The Factors that Influence the Estimated Variance of the Latent Mean Difference of the Second-Order Factor

The main factor that influenced the estimated variance of the latent mean difference in the second-order factor was sample size (shown in Tables A1-20). Some other factors including how the model was specified and the effect size of the latent mean difference to be estimated also had an influence on the estimated variance of the latent mean difference, although the influence of these factors was relatively very small for the research conditions in this study. The estimated variance became slightly larger after removing equality constraints, although the difference was not obvious within the level of precision reported (rounding to the thousandths place). The estimated variance of the latent mean difference also increased a little as the effect size of this latent mean difference increased. The noninvariance in the data generation process and whether or not a model was correctly specified seemed to have very little influence on the estimated variance.

Type I Error Rates

Results regarding Type I error rates are shown in Tables A1-4 and Tables A11-14. For all conditions, Type I error rates were within the limits of .025-.075. Type I error rates obtained using chi-square difference tests were always larger than those obtained via Wald tests, and, as expected, in most cases the discrepancies in Type I error rates were larger for conditions with a total sample size of 300 compared with the differences for conditions with a total sample size of 600.

There were no obvious systematic differences in Type I error rates between baseline conditions and research conditions used to explore the impact of noninvariant loadings on

the latent mean comparison of the second-order factor, and Type I error rates were around .05 for these conditions (shown in Tables A1-4).

For the baseline conditions and the research conditions with correctly specified models used to explore the impact of partial intercept invariance, Type I error rates were also around .05 (shown in Tables A11-14). But As shown in Tables A12-14, when the noninvariant intercepts were falsely constrained to be equal, Type I error rates became larger compared with the corresponding correctly specified models in most of the cases, and the increase in Type I error rates was especially obvious when the estimation bias and the sample size were both large. For example, the largest values for Type I error rates were .073 which occurred when noninvariance existed in intercepts of both the first-order factor and indicators, the sign of the relative difference of the noninvariant intercept of the first-order factor was the same as the sign of the relative differences of the noninvariant indicator intercepts, and the sample size was 600 (shown in Table A14).

Power

For all conditions, power to detect the latent mean difference of the second-order factor obtained using chi-square differences test was larger than or equal to power obtained using Wald tests, and in most of the cases, the differences were larger for the conditions with sample size of 300 than those for the conditions with sample size of 600 (shown in Tables A1, A5-10, 11, and 15-20). Powers reported below were obtained using chi-square difference tests.

Consistent with previous studies, the most dominant factors influencing power for detecting between-group differences in second-order factor means were sample size and effect size. The mean values of the power were .312 (median of .315) and .829 (median

of .831) across conditions with sample size of 300 and latent mean differences of .20 and .40, respectively; and the mean values of the power were .542 (median of .547) and .984 (median of .985) across conditions with sample size of 600 and latent mean differences of .20 and .40, respectively. But our main focus in this study was on the relationship between power and the influence of manipulated factors on the magnitude and direction of estimation bias and the variance of the estimated latent mean difference. As shown in Tables A5-10 and A15-20, overestimation in the magnitude of the latent mean difference of the second-order factor resulted in an increase in power and underestimation of the latent mean difference of the second-order factor resulted in a decrease in power. Empirical powers ranged from .223-.403 and .734-.888 across conditions with sample size of 300 and latent mean differences of .20 and .40, respectively; for sample size of 600, powers ranged from .389-.664 and .959-.995 across conditions with latent mean differences of .20 and .40, respectively.

Scatterplots displaying the relationship between the estimation bias and power for each sample size and effect size are shown in Figure 7. As shown in Figure 7, when the latent mean difference was .20, the estimation bias of the latent mean difference of the second-order factor was almost perfectly correlated with power to detect this between-group difference; and when the latent mean difference was .40, the correlation between the estimation bias and power became a little smaller. To be specific, the correlation was .964 when the latent mean difference was .20, which was the same for both sample sizes; correlations were .912 and .872 when the latent mean difference was .40 and sample sizes were 300 and 600, respectively.

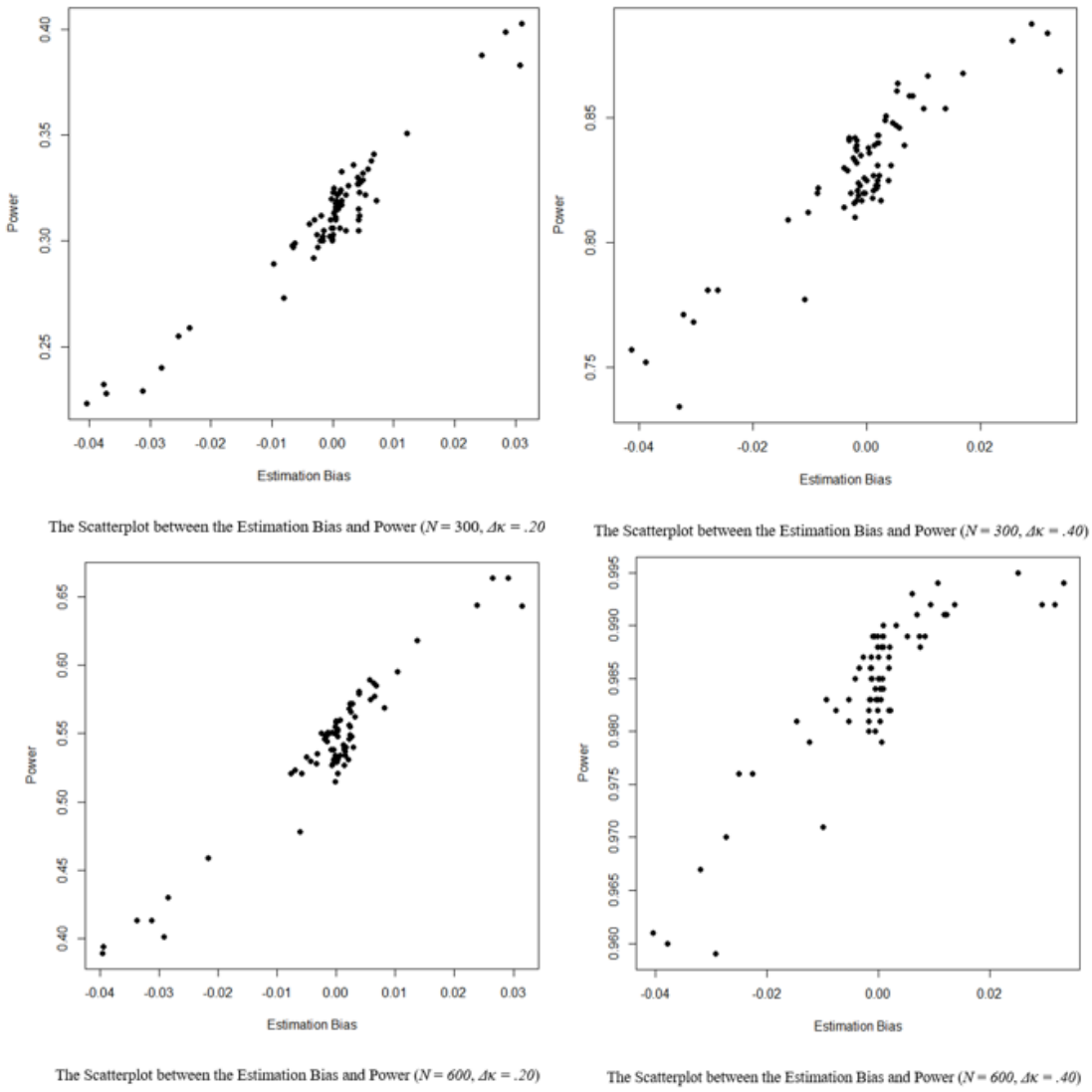


Figure 7 The Relationship between Estimation Bias and Power

The estimated variance also had an influence on power. For a set of baseline conditions with the same generated data, estimated biases were similar but estimated variances varied due to freely estimating some of the loadings or intercepts, and the changes in power can be attributed to the influence of estimated variance. When freely estimating more loadings or intercepts, power decreased as the estimated variance got

larger (shown in Tables A1 and A11). As mentioned previously, the influence of one second-order factor loading or intercept of the first-order factor can be compared with four first-order factor loadings or indicator intercepts in the current representative model, and the results in Tables A1 and A11 indicated that compared with the baseline conditions with all of loadings and intercepts constrained to be equal, in most of the cases, freely estimating one invariant second-order loading or intercept of the first-order factor resulted in a larger increase in estimated variance and a larger decrease in power than freely estimating four invariant first-order loadings or indicator intercepts.

Changes in power for the misspecified models in comparison with those for correctly specified models can be considered as a result of influence of both estimation bias and estimated variance. For example, compared with the power obtained using the correctly specified model, when the model was misspecified by constraining both invariant and noninvariant loadings to be equal and the estimated latent mean difference was smaller than the true latent mean difference, power did not decrease appreciably because the estimated variance decreased during this process (shown in Tables A5-10). For the misspecified models resulting in overestimation (i.e., the estimated value is larger than the true value) in the latent mean difference, when the degree of overestimation in the latent mean difference increased by not only setting noninvariant loadings or intercepts to be equal but also freely estimating some of the invariant loadings or intercepts, power decreased or only increased a little because the increase in estimated variance cancelled out the influence of increase in estimation bias (shown in Tables A5, A6, A8, A9, A15, A16, A18, and A19).

Coverage Rates of 95 % Confidence Interval

For most conditions, the coverage rates of the 95% confidence interval were around .950. For some conditions, however, coverage rates dropped under 93% because of relatively serious estimation bias (i.e., larger than .0400 in absolute value) resulted from incorrectly constraining noninvariant intercepts at both levels to be equal. Most of these conditions with very low coverage rates occurred for conditions with sample size of 600.

RMSEA

The means of the RMSEAs for the baseline conditions are given in Tables A21-22. The results indicated that the means of the RMSEA were .016 or .017 for the baseline conditions with sample size of 300, and they were .009 for the baseline conditions with sample size of 600. For the baseline conditions, the means of the RMSEA were not influenced by freely estimating some of the loadings or intercepts. After constraining the latent mean of the second-order factor in Group 2 to be zero, the means of RMSEA did not change for almost all of the baseline conditions with the effect size of zero; when the effect size was .20, the means of RMSEA increased by .001 or .002 for the baseline conditions with sample size of 300 and by .002 for the baseline conditions with sample size of 600; and when the effect was .40, the means of RMSEA increased .by 004 or .005 for the baseline conditions with sample size of 300 and by .007 for the baseline conditions with sample size of 600.

In the research conditions with correctly specified models, the means of RMSEA were .016 or .017 when the sample size was 300, and .009 or .010 when the sample size was 600 (shown in Tables A23 - 40), which were basically the same as those for the baseline conditions. For the research conditions, after constraining the latent mean of the

second-order factor in Group 2 to be zero, the increase in the means of RMSEA was similar to that for the baseline conditions with the same sample size and effect size.

The means of RMSEAs for the conditions to explore the influence of partial loading invariance are shown in Tables A23-31. Compared with the correctly specified models, constraining a pair of noninvariant second-order factor loadings to be equal yielded increases of .001 in the means of RMSEAs for most conditions with sample size of 300, and increases of .002 for the conditions with sample size of 600 (shown in Tables A23, 26, and 29). After constraining a pair of noninvariant first-order factor loadings to be equal, the means RMSEAs increased by .001 or .002 for the conditions with sample size of 300 and by .002 or .003 for the conditions with sample size of 600 (shown in Tables A24, 27, and A30). After constraining four pairs of noninvariant first-order factor loadings to be equal, the means of RMSEAs increased by .005 or .006 for the conditions with sample size of 300 and by .008 for the conditions with sample size of 600 (shown in Tables A24, A27, and A30). After constraining one pair of noninvariant second-order factor loadings and two pairs of noninvariant first-order factor loadings to be equal, the means of RMSEAs increased by .003 or .004 for the conditions with sample size of 300 and by .005 or .006 for the conditions with sample size of 600 (shown in Tables A25, A28, and A31).

The means of RMSEAs for the conditions to explore the influence of partial intercept invariance are shown in Tables A32-40. As shown in Tables A32-40, compared with the correctly specified models, after constraining a pair of noninvariant intercepts of the first-order factor to be equal, the increase in the means of RMSEAs was zero or .001 for the conditions with sample size of 300 and .001 for the conditions with sample size of 600 (shown in Tables A32, A35, and A38). After constraining a pair of noninvariant indicator

intercepts to be equal, the means of RMSEAs did not change for most of the conditions with sample size of 300 and they increased by .001 for the conditions with sample size of 600 (shown in Tables A33, A36, and A39). After constraining four pairs of noninvariant indicator intercepts to be equal, the means of RMSEAs increased by .002 for most of the conditions with sample size of 300 and by .003 for the conditions with sample size of 600 (shown in Tables A33, A36, and A39). After constraining one pair of noninvariant intercepts of the first-order factor and two pairs of noninvariant indicator intercepts to be equal, the means of RMSEAs increased by .001 or .002 for the conditions with sample size of 300 and by .002 for most conditions with sample size of 600 (shown in Tables A34, A37, and A40).

Compared with the conditions in which both invariant and noninvariant loadings (or intercepts) to be equal, for the conditions with noninvariant loadings (or intercepts) constrained to be equal but some of the invariant loadings (or intercepts) freely estimated, the means of RMSEAs remain the same for most of the conditions and they were changed a little for only a few conditions (shown in Tables A23, A24, A26, A27, A29, A30, A32, A33, A35, A36, A38 and A39). Among the few conditions in which means of RMSEA changed, most were for conditions in which some invariant loadings were freely estimated.

SRMR

With respect to the means of SRMRs, for the baseline conditions with sample size of 300 and all loadings and intercepts constrained to be equal, the means of SRMRs were .077, .080 (or .081), and .089 when the effect sizes of the latent mean difference of the second-order factor were 0, .20, and .40, respectively; for the completely constrained baseline conditions with sample size of 600, the means of SRMRs were .054, .059, and .070

when the effect sizes of the latent mean difference of the second-order factor were 0, .20, and .40, respectively (shown in Tables A21-22). When freely estimating some of the invariant loadings and their corresponding intercepts between groups, the means of SRMR increased by .001 or .002 (as shown in Table A21). When only freely estimating some of the invariant intercepts between groups, the means of SRMRs did not change for most of the cases and they increased by .001 for only a few cases (as shown in Table A22). For the baseline conditions, after constraining the latent mean of the second-order factor in Group 2 to be zero, the means of SRMRs did not change or decreased by .001 with the effect size of zero; when the effect size was .20, the means of SRMRs decreased by .001 or .002 for the sample size of 300 and by .002 or .003 for the sample size of 600; and when the effect was .40, the means of SRMRs decreased by .004 or .005 for sample size of 300 and by .006 or .007 for the sample size of 600.

Compared with the corresponding baseline conditions with the same set of parameters freely estimated, for the correctly specified models in the research conditions, the means of SRMRs increased due to the noninvariance in the generated data and the increase was especially obvious when noninvariant loadings were generated. Different patterns of noninvariance in loadings also had an influence on the means of SRMR. For the correctly specified models in the research conditions, when the sample size was 300, the largest means of SRMRs were .089, .092, and .101 for the conditions with effect sizes of 0, .20, and .40, respectively; when the sample size was 600, the largest means of SRMR were .069, .073, and .084 for the conditions with effect sizes of 0, .20 and .40, respectively (shown in Tables A23-40). For the research conditions, after constraining the latent mean

of the second-order factor in Group 2 to be zero, the decreases in the means of SRMRs were similar to those for the baseline conditions with the same effect size and sample size.

The means of SRMRs for the conditions to explore the influence of partial loading invariance are shown in Tables A23-31. Compared with the correctly specified models, after constraining a pair of noninvariant second-order factor loadings to be equal, the means of SRMRs decreased by .001 or .002 when the direction of the relative difference in the noninvariant loadings was consistent with the direction of the latent mean difference, whereas the decrease was .002 or .003 when the direction of the relative difference in the noninvariant loadings was inconsistent with the direction of the latent mean difference (see Tables A23, A26, and A29). After constraining a pair of noninvariant first-order factor loadings to be equal, the means of SRMRs stayed the same or decreased by.001 when the direction of the relative difference in the noninvariant loadings was consistent with the direction of the latent mean difference, and decreased by .001 or .002 when the direction of the relative difference in the noninvariant loadings was inconsistent with the direction of the latent mean difference (see Tables A24, A27, and A30). When four pairs of noninvariant first-order factor loadings were constrained to be equal, the means of SRMRs decreased by .001 or .002 when the direction of the relative difference in the noninvariant loadings was consistent with the direction of the latent mean difference, and decreased by .004 or .005 when the direction of the relative difference in the noninvariant loadings was inconsistent with the direction of the latent mean difference (shown in Tables A24, A27, and A30). After constraining one pair of noninvariant second-order factor loadings and two pairs of noninvariant first-order factor loadings to be equal, the decrease in the means of SRMRs ranged from .002 to .005, varying as a function of the position and the

relative difference of the noninvariant loadings and the sample size, and the largest decrease usually occurred in the conditions in which the relative difference in the noninvariant second-order factor loadings was consistent with that in the noninvariant first-order factor loadings and the total sample size was 600 (shown in Tables A25, A28, and A31).

The means of SRMRs for the conditions to explore the influence of partial intercept invariance are shown in Tables A32-40. As shown in Tables A32-40, compared with the correctly specified models, after constraining a pair of noninvariant intercepts of the first-order factor to be equal, the means of SRMR remained the same (shown in Tables A32, A35, and A38); after constraining one pair or four pairs of noninvariant indicator intercepts to be equal, the decrease in the means of SRMR was zero or .001 (shown in Tables A33, A36 and A39); and after constraining one pair of noninvariant intercepts of the first-order factor and two pairs of noninvariant indicator intercepts to be equal, the means of SRMR did not change or decreased by .001 (shown in Tables A34, A37, and A40).

Chapter 4: Discussion

Overview

This simulation study was undertaken to examine the performance of several approaches to evaluating the latent mean comparison of a second-order factor under partial factorial invariance. In the few previous studies that have involved comparisons of higher-order factor means, power has been the primary focus (e.g., Hancock et al., 2000; Molenaar et al., 2009). Given that both previous studies (i.e., summarized in Table 1) and the current study indicated that power was mainly determined by sample size and effect size of the latent mean difference, results in this study focused on factors that may influence power for a given sample size and effect size. Consistent with expectations, results of the current study indicated that power was closely related to estimation bias for the difference in second-order factor means for a given sample size and effect size, so the functions through which data generation and analysis conditions influenced the estimation bias of the second-order factor are discussed in detail in this section. Power was influenced by estimated variances of the latent mean difference for the second-order factor. Although the differences among estimated variances were not obvious within the level of precision reported (rounding to the thousandths place), the indirect influence of estimated variance on power was found in some conditions; thus, the factors that may influence the estimated variance also need our attention.

In addition to discussion of the impact of partial factorial invariance on the estimation of between-group differences of the second-order factor in the current research scenario, the underlying mathematical functions are also discussed. Recommendations are

provided for researchers seeking to examine between-group differences of second-order factors under partial factorial invariance.

The Relationship between Loading and Intercept Noninvariance and Estimation Bias of the Second-order Factor

The results indicated that no estimation bias of the latent mean difference of the second-order factor was produced as long as equality constraints were not placed on noninvariant parameters. To be specific, estimation bias was not influenced by noninvariance in loadings or intercepts as long as these noninvariant parameters were freely estimated, which was consistent with Yang's (2008) results. Additionally, freely estimating invariant loadings or intercepts did not influence the estimation bias as long as all parameters constrained to be equal between groups were correctly specified.

As suggested by Yuan, Marshall, and Bentler (2003), model misspecification may cause estimation bias in parameters. In the current study, misspecification of noninvariant loadings or intercepts by constraining them to be equal between-groups resulted in estimation bias of second-order factor means in most cases. Although the differences in estimation bias produced due to misspecification of noninvariant loadings or intercepts were not very large in many research conditions of the current study, the systematic relationships between the pattern of noninvariance and the direction and magnitude of differences in estimation bias were very clear. Two issues described below exist regarding the influence of misspecification of noninvariant factor loadings and intercepts on estimation bias of the latent mean difference: one can be generalized to other CFA models (i.e., first-order CFA models), and the other is focused on the influence of partial invariance

at different levels within a second-order CFA framework. These issues can be examined through related mathematical functions.

The General Influence of Misspecification of Noninvariant Factor Loadings and Intercepts on Estimation Bias for the Latent Mean Difference

The latent mean difference of the second-order factor was overestimated if the direction of differences in noninvariant loadings was consistent with the latent mean difference of the second-order factor; it was underestimated if the noninvariant loadings were in opposite direction of the latent mean difference. When the latent mean difference was zero, constraining noninvariant loadings did not result in estimation bias for the second-order factor. With respect to misspecification of noninvariant intercepts, the estimation bias of the second-order factor was positive if the intercepts in Group 2 were larger than those in the reference group, and vice versa.

For a given size of noninvariance, constraining noninvariant intercepts to be equal resulted in much larger estimation bias than constraining noninvariant loadings to be equal. For a given misspecified model and its corresponding correctly specified model, when exploring the impact of improperly constrained noninvariant loadings, the difference in estimation bias was almost twice as large when the latent mean difference was .40 than that when the latent mean difference was .20; when exploring the impact of improperly constrained noninvariant intercepts, however, the estimation bias was almost the same regardless of the size of the latent mean difference of the second-order factor.

For the misspecified conditions in which noninvariant loadings or intercepts were constrained to be equal, the estimation bias became larger as some of the invariant loadings or intercepts were freely estimated. As invariant loadings or intercepts were freely

estimated, the total number of measures with equality constraints decreased and the relative ratio of measures with improper equality constraints increased, which might result in the increase in estimation bias of the latent mean difference.

The Influence of Misspecification of Noninvariant Factor Loadings and Intercepts at Different Levels on Estimation Bias for the Latent Mean Difference of the Second-Order Factor

There were some issues regarding the impact of partial factorial invariance at different levels on the latent mean comparison within the framework of second-order CFA models. Misspecification of a second-order factor loading or the intercept of a first-order factor resulted in larger estimation bias than misspecification of a first-order factor loading or indicator intercept because second-order factor loadings or intercepts of first-order factors have a direct influence on first-order factors that relate to more than one indicator.

In the current second-order CFA model, a single second-order factor underlied four first-order factors, each of which had four indicators with non-zero loadings, so a comparison can be made between the impact of one noninvariant second-order factor loading or intercept of a first-order factor and the impact of four noninvariant first-order factor loadings or indicator intercepts. When noninvariance only existed in factor loadings at one of the two levels, the pairs of noninvariant loading values were .6 in Group 1 and .8 or .4 in Group 2. The results indicated that the differences in estimation bias were a bit larger for the cases with one noininvariant second-order factor loading than those for the cases with four first-order factor loadings. The differences in noninvariant intercepts were .10 for all of the research conditions. When noninvariance existed in intercepts at one

of the two levels, constraining four noninvariant indicator intercepts resulted in markedly larger estimation bias than constraining one noninvariant intercept of the first-order factor.

The results indicated that among all of the conditions, the difference in estimation bias was most obvious when noninvariance existed at both levels, the sign of the relative difference of the noninvariant intercept of the first-order factor or second-order loadings was the same as the sign of the relative differences of the noninvariant indicator intercepts or first-order loadings, and the indicators with noninvariant indicator intercepts or first-order loadings were associated with the referent factor or a factor with invariant intercepts or second-order loading.

Mathematical Functions Underlying the Influence of Misspecification of Noninvariant Loadings and Intercepts on Estimation Bias for the Latent Mean Difference for the Second-Order Factor

The impact of constrained noninvariant loadings or intercepts on latent mean difference estimation of the second-order factor can be understood via the equation relating parameters to the model implied mean vector (i.e., Equation 10). When estimating parameter values, the known information comprises the sample means and sample variance-covariance matrices in the multiple groups. For a given measure Y, the formula relating the model-implied population mean for a measure, E(*Y*), to the loadings and intercepts is:

$$\text{For Group 1: } E(Y)_{G1} = \nu_{G1} + \lambda_{G1}\alpha_{G1} + \lambda_{G1}\gamma_{G1}\kappa_{G1}$$

$$\text{For Group 2: } E(Y)_{G2} = \nu_{G2} + \lambda_{G2}\alpha_{G2} + \lambda_{G2}\gamma_{G2}\kappa_{G2} \qquad (18)$$

where λ is the first-order factor loading relating the corresponding first-order factor (i.e., ξ) to a measure Y, γ is the second-order factor loading relating the second-order factor (i.e.,

η) to ξ, ν is the indicator intercept, α is the intercept for ξ, κ allows estimation of differences in the latent means of η, and the subscripts G1 and G2 represent the group membership. Multiple model parameters are utilized in estimating the latent mean difference of the second-order factor: sample means, estimates of first- and second-order factor loadings, and estimates of indicator intercepts and intercepts of first-order factors for the measures with equality constraints in analysis. The variance-covariance matrices of the sample and the pattern of constraints in the specified model influences the estimates of loadings and intercepts.

Assuming G1 is the reference group, in order to scale the latent factors, κ_{G1} and α_{G1} are set to 0, so κ_{G2} reflects the latent mean difference in the second-order factor, and α_{G1} reflects the mean difference in the first-order factor that cannot be accounted for by the second-order factor. For measure Y, when generating the data, if noninvariance only exists in factor loadings ($\nu_{G1} = \nu_{G2} = \nu$, $\alpha_{G2} = 0$, but $\lambda_{G1} \neq \lambda_{G2}$ or $\gamma_{G1} \neq \gamma_{G2}$), the population mean difference in measure Y will be $\lambda_{G2}\gamma_{G2}\kappa_{G2}$. If we set equality constraints on loadings and intercepts for this measure, its contribution to the latent mean difference will be $\lambda_{G2}\gamma_{G2}\kappa_{G2}/\lambda_{estimated}\gamma_{estimated}$ ($\lambda_{estimated}$ falls between λ_{G1} and λ_{G2}; $\gamma_{estimated}$ falls between γ_{G1} and γ_{G2}). This equation offers an explanation for the following results: 1) whether or not the contribution of measure Y on the latent mean difference estimation is overestimated or underestimated depends on the relative difference between λ_{G1} and λ_{G2} (or γ_{G1} and γ_{G2}); 2) when κ_{G2} was .40 (i.e., two times .20), the impact of constraining noninvariant loadings to be equal on the estimation bias was two times that for conditions with κ_{G2} of .20; and 3) when the latent mean difference (κ_{G2}) was zero, constraining noninvariant loadings to be equal did not influence the estimation bias.

For measure Y, when generating the data, if noninvariance only exists in intercepts ($\lambda_{G1} = \lambda_{G2} = \lambda$, $\gamma_{G1} = \gamma_{G2} = \gamma$, but $\nu_{G1} \neq \nu_{G2}$ or $\alpha_{G2} \neq 0$), the population mean difference in measure Y will be $\lambda_{G2}\gamma_{G2}\kappa_{G2} + \nu_{G2} - \nu_{G1} + \lambda_{G2}\alpha_{G2}$. If we set equality constraints on loadings and intercepts for this measure, its contribution to the latent mean difference will be $\lambda_{G2}\gamma_{G2}\kappa_{G2} / \lambda_{estimated}\gamma_{estimated} + (\nu_{G2} - \nu_{G1}) / \lambda_{estimated}\gamma_{estimated} + \lambda_{G2}\alpha_{G2} / \lambda_{estimated}\gamma_{estimated}$. Given that loadings were generated to be equal, the differences between $\lambda_{estimated}$ and λ_{G2} (or $\gamma_{estimated}$ and γ_{G2}) were negligible, and the contribution of measure Y can be written as $\kappa_{G2} + (\nu_{G2} - \nu_{G1}) / \lambda_{estimated}\gamma_{estimated} + \lambda_{G2}\alpha_{G2} / \lambda_{estimated}\gamma_{estimated}$, which offered explanations for the following results: 1) the relative difference between ν_{G1} and ν_{G2} and the sign of α_{G2} had an influence on the contribution of measure Y on the latent mean difference estimation; 2) the effect size of the latent mean difference did not influence the estimation bias resulting from constraining noninvariant intercepts to be equal; 3) constraining four noninvariant indicator intercepts resulted in larger estimation bias than constraining one noninvariant intercept of a first-order factor because λ_{G2} was less than 1, which decreased the term $\alpha_{G2} / \lambda_{estimated}\gamma_{estimated}$.

The magnitude of estimation bias was very small for conditions in which noninvariant loadings were constrained to be equal. The reason was that the ratio between $\lambda_{G2}\gamma_{G2}$ and $\lambda_{estimated}\gamma_{estimated}$ was usually not much larger or smaller than 1 unless the magnitude of noninvariance in factor loadings was very large. But the magnitude of estimation bias was very obvious for conditions in which noninvariant intercepts were constrained to be equal because in the current study, the relative differences in intercepts were specified as .10, which differed markedly from the latent mean difference (.20 or .40). In other words, if the latent mean difference was 2 or 4 (i.e., 10 times the latent mean

difference generated in the current study) but the size of noninvariance in loadings or intercepts was held the same as the current study, the estimation bias (i.e., E ($\hat{\theta}$) – θ) resulting from constraining noninvariant loadings to be equal would be 10 times that for the current study, but the estimation bias resulting from constraining noninvariant intercepts to be equal would be the same as the current study.

The situation for understanding the results for relative estimation bias (i.e., (E ($\hat{\theta}$) – θ) / θ) was opposite. Changing effect size would not influence the relative estimation bias resulting from constraining noninvariant loadings to be equal but would influence the relative estimation bias resulted from constraining noninvariant intercepts to be equal, holding the size of noninvariance constant.

Power (Type I Error Rates)

Both Wald tests and chi-square difference tests were applied to examine the latent mean difference of the second-order factor. Power or Type I error rates obtained using chi-square difference tests were always larger than those obtained via Wald tests, and in most of the cases, the differences were larger for the conditions with sample size of 300 than those for the conditions with sample size of 600.

The results indicated that Type I error rates were within the limits of .025-.075 for all of the conditions. Type I error rates almost fell beyond the required criteria for the conditions with very large estimation bias (i.e., around .0400 in absolute value) and large sample size (i.e., 600).

Consistent with previous studies, power to detect the latent mean difference of the second-order factor was mainly determined by sample size and effect size of the difference. In previous studies, the main focus was on relationships between power and the

combination of data characteristics and specifications of parameters (shown in Table 1). But in the current study, the investigation focused on how noninvariance in data generation process and specifications of loadings or intercepts influenced power via estimation bias and estimated variance. As expected, power to estimate the between-group difference in means of the second-order factor was closely related to estimation bias of this latent mean difference for a given effect size and sample size. The correlations between power and estimation bias were larger for conditions with effect size of .20 than those for conditions with effect size of .40 because empirical powers were more variable for conditions with effect size of .20 (shown in Figures 7-10).Power to detect the latent mean difference was also influenced to a lesser degree by the variance of estimated between-group differences across replications. When the effect size was .20, the impact of estimated variance on power was a little more obvious for the conditions with larger sample size (i.e., 600); when the effect size was .40, such impact became minimal for the conditions with sample size of 600. Imposing equality constraints on loadings or intercepts regardless of invariance decreased the estimated variance, which further increased power.

RMSEA and SRMR

The performance of RMSEA differed from that of SRMR in several ways in the current research scenario. First, for correctly specified models, the mean SRMR was influenced by more factors than RMSEA. For a given sample size, the mean RMSEA was almost the same across all conditions in which the model was correctly specified. However, for a given sample size, mean SRMRs varied for correctly specified models because the SRMR was influenced by the data generation model including the effect size of the latent mean difference of the second-order factor and noninvariance in factor loadings.

Second, the change of SRMR was more obvious than that of RMSEA after freely estimating some invariant factor loadings. The change of a given goodness of fit index (i.e., RMSEA or SRMR) between the model with more equality constraints on invariant parameters (i.e., loadings or intercepts) and the corresponding baseline model reflects the sensitivity of this goodness of fit index to random variation in invariance tests (Chen, 2007). In the current study, freely estimating some of the invariant loadings or intercepts increased the model complexity, but it did not influence RMSEA for most of the conditions, which was consistent with the simulation study conducted by Chen (2007). In Chen's study (2007), the change of RMSEA was no more than .001 in a series of model comparisons between the model with more equality constraints and the corresponding baseline model when all parameters were generated to be invariant. For SRMR, the means did not change obviously after freely estimating some invariant intercepts, but they did change obviously after freely estimating some invariant loadings and their corresponding intercepts; this was consistent with Chen's (2007) finding that SRMR was more sensitive to random variation in factor loadings than in intercepts. However, different from Chen's (2007) study in which the mean SRMR became larger after imposing equality constraints on invariant parameters, it was found that the more constrained model had smaller mean SRMR in the current study. The reason for this discrepancy between these two studies was unclear.

Third, RMSEA was more sensitive than SRMR for detecting model misspecification due to improper equality constraints including the nonzero latent mean difference. Although the means of the RMSEA suggested good fit for all research conditions, the changes in RMSEA were sensitive to model misspecification. For a given sample size and misspecification, the mean increase in RMSEA compared with the

corresponding correctly specified model was also nearly the same across conditions regardless of how the data were generated. Additionally, the change in RMSEA was more sensitive to the misspecification of parameters that involved in model implied variance and covariance matrices (i.e., constraining noninvariant loadings to be equal) than the misspecification of parameters that only involved in model implied mean vectors (i.e., constraining noninvariant intercepts to be equal). It was also noted that the increase in RMSEA had nothing to with the estimation bias of the latent mean difference for the second-order factor, which suggests that the extent of model misfit was not directly related to the magnitude of the estimation bias for a given parameter. For example, as mentioned previously, constraining noninvariant loadings to be equal resulted in a larger increase in RMSEA than constraining noninvariant intercepts to be equal, but constraining noninvariant intercepts to be equal resulted in larger estimation bias for the difference in factor means than constraining noninvariant loadings to be equal in the current study. Another example was that constraining noninvariant second-order factor loadings to be equal and constraining noninvariant first-order factor loadings to be equal had similar impacts on RMSEAs but had different impacts on estimation bias of the latent mean difference. However, SRMR seemed not to be a sensitive index to detect noninvariance in the current research scenario. It was supposed that smaller values of SRMR indicate better fit, but constraining noninvariant parameters to be equal between groups led to a decrease in SRMR. SRMR was sensitive to removing equality constraints in general. In fact, freeing equality constraints led to an increase in SRMR regardless of whether these equality constraints were correctly or incorrectly specified.

Recommendations

Given that estimation bias can be predicted via the mathematical function given above, researchers might consider calculating a rough estimate for the estimation bias before conducting the latent mean comparison. Given that the population parameters are unknown in practice, researchers could analyze the model with only equality constraints on parameters for measures that were expected to be invariant (i.e., based on previous studies or on strong theoretical grounds) to obtain loading or intercept estimates for other measures as proxies for population parameters. Each indicator with equality constraints on loadings and intercepts had an impact on the between-group difference estimate in the means of the latent factor that related to this indicator. The results indicated that misspecification of noninvariant loadings or intercepts may not result in an obvious misfit in model in terms of RMSEAs, so it was necessary to know the impact of including measures for which loadings or intercepts differ (i.e., these measures may be considered as invariant based on results of model comparisons) on the latent mean difference estimate. As shown above, the contribution of each measure on the latent mean difference estimate was $\lambda_{G2}\gamma_{G2}\kappa_{G2} / \lambda_{estimated}\gamma_{estimated} + (\nu_{G2} - \nu_{G1}) / \lambda_{estimated}\gamma_{estimated} + \lambda_{G2}\alpha_{G2} / \lambda_{estimated}\gamma_{estimated}$, so the rough estimates of the extent to which the latent mean difference was overestimated or underestimated can be obtained.

The rough estimates of the estimation bias for the current research conditions are given in the Appendix B. It was supposed that the contribution of a set of measures on the latent mean difference estimate was the average value of the contribution of each measure. For the measures generated to be invariant, their contributions to the latent mean difference estimate was the true value of the latent mean difference. For the measures with constrained

noninvariant loadings, the average loading values between Group 1 and Group 2 were used as the estimated loadings (i.e., $\lambda_{estimated}$ and $\gamma_{estimated}$) for convenience. For the measures with constrained noninvariant intercepts, given that the bias in estimated loadings was very small, the population values of loadings were used as estimated loadings (i.e., $\lambda_{estimated}$ and $\gamma_{estimated}$). As shown in the Appendix, the rough estimates of estimation bias were larger than the actual estimation bias (i.e., differences in estimation bias between the misspecified models and the corresponding correctly specified models) for most of the conditions, and the differences between the rough estimates of estimation bias and the actual estimation bias were especially large for conditions in which noninvariant intercepts were constrained to be equal. When noninvariance existed at both levels and the signs of noninvariant loadings or intercepts were opposite for the two levels, the actual estimation bias may be larger than the rough estimates.

The influence of estimated variance on power to detect the latent mean difference should also be considered. When researchers plan to freely estimate many loadings or intercepts to reduce estimation bias, a small simulation study can be conducted to gain some idea of the reduction in power when these parameters are freely estimated.

In the current study, chi-square differences tests obtained larger power than the corresponding Wald tests. Thus, chi-square tests are recommended if larger power is desired.

Several goodness of fit indices including comparative fit index (CFI), Tucker–Lewis Index (TLI), Normed Fit Index (NNFI), Gamma hat, McDonald's, Non-Centrality Index [Mc]), RMSEA, and SRMR, have been used for evaluating invariance (Cheung & Rensvold, 2002; Chen, 2007). Changes of RMESA and SRMR were examined in the

current simulation study, and RMSEA was more sensitive than SRMR for detecting noninvariance in second-order CFA models based on the results of the current study.

Limitations

First, as with any simulation study, there are many other general model specifications that might be examined. This study focused on variations of a CFA model with a single second-order factor and four first-order factors with four indicators for each; in the future, researchers could compare the impact of partial factorial invariance on latent mean comparisons for alternative models that vary in size or structure, such as bifactor models. Second, in the current study, latent mean comparisons were conducted based on known equality constraints. It is necessary to further study the latent mean comparison of the second-order factor based on equality constraints obtained from various levels of invariance tests such that the influence of potential mistakes made from the process of detecting noninvariance in loadings or intercepts in second-order CFA models can be examined. Third, in the current study, the models were either under partial loading invariance or partial intercept invariance, so the impact of more complicated patterns of noninvariance on latent mean comparisons can be studied in the future. Fourth, in the current study, the majority of the loadings or intercepts were generated to be invariant and constrained to be equal in analysis, which may make the estimation bias and estimated variance of the latent mean difference of the second-order factor not large enough. Some extreme conditions including very large proportions of noninvariant parameters can be focused on in future studies. Fifth, the second-order factor was a strong factor in the current study, thus explaining much of the variances in first-order factors and measures, but in some empirical studies, both first-order and second-order factor loadings might be smaller

than the specified parameter values in this study. Thus, sensitivity analysis can be conducted to evaluate whether the magnitude of loading parameters would influence the conclusions of this study. Finally, descriptive methods were applied when summarizing the outcomes of this simulation study, and significance tests and more complicated models (i.e., path models) may be needed to further understand which factors have impact on outcome variables, including estimation bias, estimated variance, and power.

Significance and Conclusions

This study systemically investigated the impact of partial factorial invariance on the latent mean comparison of a second-order factor within higher-order CFA models, which has not been done in previous studies. In the current study, noninvariance in factor loadings or intercepts was manipulated at one of the two levels or both levels, and its impact on the latent mean comparison of the second-order factor was examined in terms of estimation bias, estimated variance, and power. Additionally, a series of baseline conditions were devised such that the influence of data characteristics and model specifications on the latent mean comparison can be distinguished.

In previous simulation studies involving the impact of misspecification of loadings or intercepts on latent mean comparisons (e.g., Hancock, Lawrence, & Nevitt, 2000), the magnitude of estimation bias for the target parameter (i.e., latent mean differences) was rarely focused on. In the current study, the findings suggested the relationships between misspecification of different patterns of noninvariant loadings or intercepts and the direction and magnitude of estimation bias for the latent mean difference of the second-order factor, which were further explained using mathematical functions.

In previous simulation studies regarding invariance tests (e.g., Hancock, Lawrence, & Nevitt, 2000; Molenaar, Dolan, & Wicherts, 2009; Yang, 2008), only the specification of noninvariant parameters were manipulated. In this study, research conditions in which invariant loadings or intercepts were erroneously specified as noninvariant were also considered to gain understanding of their impact on estimation bias, power, RMSEA, and SRMR.

Different from previous studies involving latent mean comparisons in which the impact of data generation conditions on power to detect the latent mean difference was directly examined, in this study, it was expected that noninvariance in loadings and intercepts as well as model specifications had a direct influence on estimation bias and estimated variance of the latent mean difference which would further influence the power to detect this difference; this expectation was supported by the current simulation study.

Finally, several recommendations are provided for researchers seeking to examine the latent mean difference under partial factorial invariance. First, based on the mathematical functions developed and illustrated in the current study, rough estimates of estimation bias resulted from constraining a given loading or intercept can be obtained, which may serve as another criterion for future researchers to determine whether or not imposing equality constraints. In empirical studies, sometimes researchers may need different criteria to determine invariance of a measure and the rough estimate of estimation bias produced due to constraining this measure to be invariant can serve as such a criterion. As mentioned previously, in practice, in order to calculate the rough estimation bias that would result from imposing equality constraints on a given loading or intercept, researchers need to use the estimated values of this loading or intercept as its population values in the

two groups. The estimated values of the loading or intercept of interest can be obtained from the model with only loadings and intercepts that were expected to be invariant (i.e., based on previous studies or on strong theoretical grounds) constrained to be equal. Second, the reduction in power resulting from freely estimating a set of parameters should be considered, and a simulation study may be warranted before deciding to freely estimate them. Third, chi-square difference tests are recommended over Wald tests when larger power is desired. Fourth, in addition to chi-square difference tests, the sensitivity of the change of RMSEA due to imposing equality constraint on a given set of parameters offers some utility for detecting noninvariance of these parameters, including the latent mean difference in second-order CFA models.

References

Bollen, K. A. (1989). *Structural equations with latent variables.* New York: Wiley.

Brown, T. A. (2006*). Confirmatory factor analysis for applied research.* Guilford Publications.

Browne, M. W., & Cudeck, R. (1993). Alternative ways of assessing model fit. Sage focus editions, *154*, 136-162.

Byrne, B. M., Shavelson, R. J., & Muthén, B. (1989). Testing for the equivalence of factor covariance and mean structures: The issue of partial measurement invariance. *Psychological Bulletin*, *105*, 456-466.

Byrne, B. M., & Stewart, S. M. (2006). Teacher's corner: the MACS approach to testing for multigroup invariance of a second-order structure: a walk through the process. *Structural Equation Modeling, 13*(2), 287-321

Chen, F. F., Sousa, K. H., & West, S. G. (2005). Teacher's corner: Testing measurement invariance of second-order factor models. *Structural Equation Modeling, 12*(3), 471-492.

Chen, F. F., West, S. G., & Sousa, K. H. (2006). A comparison of bifactor and second-order models of quality of life. *Multivariate Behavioral Research, 41*(2), 189-225.

Chen, F. F. (2007). Sensitivity of goodness of fit indexes to lack of measurement invariance. *Structural Equation Modeling*, *14*(3), 464-504.

Cheung, G.W., & Rensvold, R. B. (2002). Evaluating goodness-of-fit indexes for testing measurement invariance. *Structural Equation Modeling, 9*, 233–255.

Dimitrov, D. M. (2010). Testing for factorial invariance in the context of construct validation. *Measurement and Evaluation in Counseling and Development, 43*(2), 121-149.

Gonzalez-Roma, V., Hernandez, A., & Gomez-Benito, J. (2006). Power and Type I error of the mean and covariance structure analysis model for detecting differential item functioning in graded response items. *Multivariate Behavioral Research, 41*(1), 29-53.

Hancock, G. R., Lawrence, F. R., & Nevitt, J. (2000). Type I error and power of latent mean methods and MANOVA in factorially invariant and noninvariant latent variable systems. *Structural Equation Modeling, 7,* 534-556.

Horn, J. L., & McArdle, J. J. (1992). A practical and theoretical guide to measurement invariance in aging research. *Experimental Aging Research, 18*(3), 117-144.

Jöreskog, K. G., & Goldberger, A. S. (1975). Estimation of a model with multiple indicators and multiple causes of a single latent variable. *Journal of the American Statistical Association, 70,* 631–639.

Kaplan, D. (1989). Power of the likelihood ratio test in multiple group confirmatory factor analysis under partial measurement invariance. *Educational and Psychological Measurement, 49,* 579-586.

Lo, W. J., & Thompson, M. S. (April, 2009). Using goodness-of-fit indices and alternative search procedures for identifying partial measurement invariance. Presented at the annual meeting of the American Educational Research Association, San Diego, CA.

Mellenbergh, G. J. (1989). Item bias and item response theory. *International Journal of Educational Research, 13*(2), 127-143.

Meredith, W. (1993). Measurement invariance, factor analysis and factorial invariance. *Psychometrika, 58,* 525-543.

Meredith, W., & Millsap, R. E. (1992). On the misuse of manifest variables in the detection of measurement bias. *Psychometrika, 57*(2), 289-311.

Millsap, R. E. (2007). Invariance in measurement and prediction revisited. *Psychometrika, 72*(4), 461-473.

Millsap, R. E. (2011). *Statistical approaches to measurement invariance.* New York: Routledge.

Millsap, R. E., & Kwok, O. M. (2004). Evaluating the impact of partial factorial invariance on selection in two populations. *Psychological Methods, 9*(1), 93-115.

Molenaar, D., Dolan, C. V., & Wicherts, J. M. (2009). The power to detect sex differences in IQ test scores using multi-group covariance and means structure analyses. *Intelligence, 37,* 396–404.

Muthén, L. K., & Muthén, B. O. (2010). Mplus software (Version 6). Los Angeles, CA: Muthén & Muthén.

Reise, S. P., Widaman, K. F., & Pugh, R. H. (1993). Confirmatory factor analysis and item response theory: Two approaches for exploring measurement invariance. *Psychological Bulletin, 114*(3), 552-566.

Schmitt, N., & Kuljanin, G. (2008). Measurement invariance: Review of practice and implications. *Human Resource Management Review, 18*(4), 210-222.

Sörbom, D. (1974). A general method for studying differences in factor means and factor structure between groups. *British Journal of Mathematical and Statistical Psychology, 27,* 229-239.

Steenkamp, J. B. E., & Baumgartner, H. (1998). Assessing measurement invariance in cross-national consumer research. *Journal of Consumer Research, 25*(1), 78-107.

Thompson, M. S. & Green, S. B. (2013). Evaluating between-group differences in latent variable means. In G. R. Hancock & R. O. Muller (Editors). *Structural Equation Modeling: A second course, 2nd ed.* (pp. 163-218). Information Age Publishing, Inc.

Vandenberg, R. J., & Lance, C. E. (2000). A review and synthesis of the measurement invariance literature: Suggestions, practices, and recommendations for organizational research. *Organizational Research Methods*, *3*(1), 4-70.

Yang, Y. (2008). *Partial invariance in loadings and intercepts—Their interplays and implications for latent mean comparisons.* (Unpublished doctoral dissertation). The University of Nebraska, Lincoln.

Yoon, M., & Millsap, R. E. (2007). Detecting violations of factorial invariance using data-based specification searches: A Monte Carlo study. *Structural Equation Modeling, 14*(3), 435-463.

Yuan, K. H., Marshall, L. L., & Bentler, P. M. (2003). Assessing the effect of model misspecifications on parameter estimates in structural equation models. *Sociological Methodology*, *33*(1), 241-265.

APPENDIX A

DETAILED RESULTS OF THE SIMULATION STUDY

Table A1

Type I Error Rate or Power, Bias, Estimated Variance, and Coverage Rate of the Estimated Latent Mean Difference of the Second-Order Factor for the Baseline Conditions to Explore the Influence of Partial Loading Invariance

Freely estimated parameters	*N*	*Δκ = .00*				*Δκ = .20*				*Δκ = .40*			
		α	$b(\hat{\phi})$	$\sigma^2_{\hat{\phi}}$	%	Power	$b(\hat{\phi})$	$\sigma^2_{\hat{\phi}}$	%	Power	$b(\hat{\phi})$	$\sigma^2_{\hat{\phi}}$	%
None	300	.055 (.050)	.0004	.019	.950	.323 (.309)	.0002	.020	.949	.843 (.832)	.0021	.020	.951
	600	.052 (.050)	-.0011	.009	.950	.559 (.551)	-.0001	.009	.952	.988 (.988)	.0008	.010	.954
$\gamma_{2,1}$	300	.056 (.051)	.0009	.020	.949	.313 (.301)	.0002	.021	.947	.823 (.814)	.0019	.021	.953
	600	.054 (.052)	-.0009	.010	.948	.538 (.530)	-.0004	.010	.950	.984 (.983)	.0004	.010	.954

Table A1 Continued

Freely estimated parameters	*N*	*Δκ = .00*				*Δκ = .20*				*Δκ = .40*			
		α	$b(\hat{\phi})$	$\sigma^2_{\hat{\phi}}$	%	Power	$b(\hat{\phi})$	$\sigma^2_{\hat{\phi}}$	%	Power	$b(\hat{\phi})$	$\sigma^2_{\hat{\phi}}$	%
$\gamma_{3,1}$	300	.055 (.052)	-.0007	.020	.948	.302 (.291)	-.0005	.021	.947	.817 (.808)	.0026	.022	.951
	600	.053 (.051)	-.0012	.010	.949	.515 (.509)	-.0002	.010	.951	.979 (.979)	.0005	.010	.948
$\boldsymbol{\lambda}_{6,2}$	300	.053 (.049)	.0005	.019	.951	.325 (.310)	.0002	.020	.947	.843 (.831)	.0021	.020	.949
	600	.052 (.050)	-.0011	.009	.950	.555 (.548)	-.0002	.009	.952	.990 (.989)	.0008	.010	.955

Table A1 Continued

Freely estimated parameters	*N*	*Δκ = .00*				*Δκ = .20*				*Δκ = .40*			
		α	$b(\hat{\phi})$	$\sigma^2_{\hat{\phi}}$	%	Power	$b(\hat{\phi})$	$\sigma^2_{\hat{\phi}}$	%	Power	$b(\hat{\phi})$	$\sigma^2_{\hat{\phi}}$	%
$\boldsymbol{\lambda}_{7,2}$	300	.054 (.050)	.0005	.019	.950	.323 (.309)	.0001	.020	.949	.843 (.833)	.0020	.020	.950
	600	.052 (.049)	-.0012	.009	.951	.558 (.549)	-.0001	.009	.951	.988 (.987)	.0007	.010	.956
$\boldsymbol{\lambda}_{(6,2;7,2;10,3;11;3)}$	300	.054 (.050)	.0004	.019	.950	.320 (.307)	-.0002	.020	.947	.840 (.828)	.0021	.020	.949
	600	.054 (.053)	-.0011	.010	.947	.550 (.544)	-.0004	.009	.951	.988 (.988)	.0005	.010	.953

Table A1 Continued

Freely estimated parameters	*N*	*Δκ = .00*				*Δκ = .20*				*Δκ = .40*			
		α	$b(\hat{\phi})$	$\sigma^2_{\hat{\phi}}$	%	Power	$b(\hat{\phi})$	$\sigma^2_{\hat{\phi}}$	%	Power	$b(\hat{\phi})$	$\sigma^2_{\hat{\phi}}$	%
$\boldsymbol{\lambda}_{(\mathbf{2},\mathbf{1};\mathbf{3},\mathbf{1};\mathbf{14},\mathbf{4};\mathbf{15},\mathbf{4})}$	300	.053 (.049)	.0007	.019	.951	.318 (.305)	.0004	.020	.949	.831 (.821)	.0020	.021	.951
	600	.055 (.052)	-.0012	.010	.948	.548 (.539)	.0002	.009	.954	.989 (.988)	.0007	.010	.957
$\gamma_{2,1}$ and $\boldsymbol{\lambda}_{(2,1;3,1)}$	300	.056 (.050)	.0009	.020	.950	.310 (.297)	.0004	.021	.947	.822 (.812)	.0018	.021	.953
	600	.055 (.053)	-.0010	.010	.947	.534 (.528)	-.0002	.010	.953	.984 (.983)	.0004	.010	.954

Table A1 Continued

Freely estimated parameters	N	$\Delta\kappa$ = .00				$\Delta\kappa$ = .20				$\Delta\kappa$ = .40			
		α	$b(\hat{\phi})$	$\sigma^2_{\hat{\phi}}$	%	Power	$b(\hat{\phi})$	$\sigma^2_{\hat{\phi}}$	%	Power	$b(\hat{\phi})$	$\sigma^2_{\hat{\phi}}$	%
$\gamma_{2,1}$ and $\boldsymbol{\lambda}_{(6,2;7,2)}$	300	.056 (.051)	.0009	.020	.949	.313 (.302)	.0002	.021	.947	.823 (.814)	.0019	.021	.953
	600	.054 (.052)	-.0009	.010	.948	.538 (.530)	-.0004	.010	.950	.984 (.983)	.0004	.010	.954
$\gamma_{2,1}$ and $\boldsymbol{\lambda}_{(10,3;11,3)}$	300	.056 (.051)	.0008	.020	.949	.306 (.294)	.0000	.021	.947	.827 (.818)	.0022	.021	.954
	600	.055 (.053)	-.0009	.010	.947	.528 (.521)	-.0006	.010	.952	.984 (.983)	.0003	.010	.950

Note: N denotes total sample size. Δκ denotes the latent mean difference of the second-order factor. Type I error rates (α) and powers outside the parentheses were obtained using the chi-square difference test, and those inside the parentheses were obtained via the Wald test for the latent mean difference; both were based on α = .05. $\hat{\phi}$ denotes the estimated latent mean difference of the second-order factor, so $b(\hat{\phi})$ is the bias and $\sigma^2_{\hat{\phi}}$ is the variance of this parameter.

Table A2

Type I Error Rate, Bias, Estimated Variance, and Coverage Rate of the Estimated Latent Mean Difference of the Second-Order Factor for the Exploration of the Influence of Partial Invariance of Second-Order Factor Loadings

N	$\gamma_{2,1(G2)} - \gamma_{2,1(G1)}$	$\gamma_{2,1}$ is freely estimated				$\gamma_{2,1}$ is constrained to be equal				$\gamma_{2,1}$ is constrained to be equal but $\gamma_{3,1}$ is freely estimated			
		α	$b(\hat{\phi})$	$\sigma^2_{\hat{\phi}}$	%	α	$b(\hat{\phi})$	$\sigma^2_{\hat{\phi}}$	%	α	$b(\hat{\phi})$	$\sigma^2_{\hat{\phi}}$	%
300	.20	.054 (.049)	-.0007	.019	.951	.051 (.047)	-.0012	.018	.953	.050 (.046)	-.0006	.019	.954
	-.20	.053 (.046)	.0007	.019	.954	.053 (.050)	.0002	.019	.950	.053 (.049)	-.0007	.021	.951
600	.20	.051 (.049)	.0001	.010	.951	.053 (.050)	.0000	.009	.950	.052 (.051)	.0000	.009	.949
	-.20	.053 (.051)	-.0016	.010	.949	.052 (.050)	-.0012	.009	.950	.051 (.048)	-.0020	.010	.952

Note: N denotes total sample size. $\gamma_{2,1}$ is the second-order loading relating the 2nd first-order factor to the second-order factor and G2 (G1) denotes group membership. Type I error rates (α) outside the parentheses were obtained using the chi-square difference test, and those inside the parentheses were obtained via the Wald test for the latent mean difference; both were based on α = .05. $\hat{\phi}$ denotes the estimated latent mean difference of the second-order factor, so $b(\hat{\phi})$ is the bias and $\sigma^2_{\hat{\phi}}$ is the variance of this parameter.

Table A3

Type I Error Rate, Bias, Estimated Variance, and Coverage Rate of the Estimated Latent Mean Difference of the Second-Order Factor for the Exploration of the Influence of Partial Invariance of First-Order Factor Loadings

N	The number of noninvariant first-order loadings	Relative Difference	Freely estimating $\boldsymbol{\lambda}_{6,2(6,2;7,2;10,3;11;3)}$				Constraining $\boldsymbol{\lambda}_{6,2(6,2;7,2;10,3;11;3)}$ to be equal				Constraining $\boldsymbol{\lambda}_{6,2(6,2;7,2;10,3;11;3)}$ to be equal but freely estimating $\boldsymbol{\lambda}_{7,2(2,1;3,1;14,4;15,4)}$			
			α	$b(\hat{\phi})$	$\sigma^2_{\hat{\phi}}$	%	α	$b(\hat{\phi})$	$\sigma^2_{\hat{\phi}}$	%	α	$b(\hat{\phi})$	$\sigma^2_{\hat{\phi}}$	%
300	1	.20	.060 (.054)	-.0003	.019	.946	.058 (.055)	-.0003	.019	.945	.058 (.054)	-.0003	.019	.946
		-.20	.051 (.048)	-.0026	.019	.952	.052 (.048)	-.0026	.019	.952	.053 (.049)	-.0026	.019	.951
	4	.20	.045 (.042)	-.0013	.018	.958	.046 (.041)	-.0012	.018	.959	.048 (.043)	-.0012	.019	.957
		-.20	.052 (.049)	-.0015	.019	.951	.054 (.050)	-.0013	.019	.950	.056 (.052)	-.0013	.020	.948

Table A3 Continued

N	The number of noninvariant first-order loadings	Relative Difference	Freely estimating $\boldsymbol{\lambda}_{6,2(6,2;7,2;10,3;11;3)}$				Constraining $\boldsymbol{\lambda}_{6,2(6,2;7,2;10,3;11;3)}$ to be equal				Constraining $\boldsymbol{\lambda}_{6,2(6,2;7,2;10,3;11;3)}$ to be equal but freely estimating $\boldsymbol{\lambda}_{7,2(2,1;3,1;14,4;15,4)}$			
			α	$b(\hat{\phi})$	$\sigma^2_{\hat{\phi}}$	%	α	$b(\hat{\phi})$	$\sigma^2_{\hat{\phi}}$	%	α	$b(\hat{\phi})$	$\sigma^2_{\hat{\phi}}$	%
600	1	.20	.052 (.048)	-.0011	.009	.952	.050 (.048)	-.0011	.009	.952	.050 (.048)	-.0011	.009	.952
		-.20	.049 (.048)	-.0007	.009	.952	.049 (.046)	-.0007	.009	.954	.050 (.047)	-.0008	.009	.953
	4	.20	.052 (.050)	-.0004	.009	.950	.052 (.050)	-.0003	.009	.950	.052 (.049)	-.0004	.009	.951
		-.20	.048 (.045)	-.0019	.009	.955	.049 (.046)	-.0019	.009	.954	.048 (.046)	-.0018	.010	.954

Note: N denotes total sample size. Relative Difference refers the value of noninvariant loading(s) in Group 2 minus that in Group 1. Type I error rates (α) outside the parentheses were obtained using the chi-square difference test, and those inside the parentheses were obtained via the Wald test for the latent mean difference; both were based on α = .05. $\hat{\phi}$ denotes the estimated latent mean difference of the second-order factor, so $b(\hat{\phi})$ is the bias and $\sigma^2_{\hat{\phi}}$ is the variance of this parameter.

Table A4

Type I Error Rate, Bias, Estimated Variance, and Coverage Rate of the Estimated Latent Mean Difference of the Second-Order Factor for the Exploration of the Influence of Partial Invariance of Both First- and Second-Order Loadings

N	The first-order factor that has noninvariant parameters	Relative differences in noninvariant second-order loadings	Freely estimating noninvariant loadings				Constraining noninvariant loadings to be equal			
			α	$b(\hat{\phi})$	$\sigma^2_{\hat{\phi}}$	%	α	$b(\hat{\phi})$	$\sigma^2_{\hat{\phi}}$	%
300	η1	.20	.052 (.045)	.0014	.020	.955	.050 (.045)	.0011	.018	.955
		-.20	.049 (.045)	-.0011	.020	.955	.049 (.045)	-.0009	.019	.955
	η2	.20	.050 (.046)	.0005	.019	.954	.051 (.047)	.0000	.018	.953
		-.20	.050 (.045)	-.0004	.019	.955	.051 (.048)	-.0007	.019	.952
	η3	.20	.050 (.047)	.0000	.020	.953	.054 (.049)	.0003	.018	.951
		-.20	.058 (.052)	.0024	.020	.948	.060 (.056)	.0022	.020	.944

Table A4 Continued

N	The first-order factor that has noninvariant parameters	Relative differences in noninvariant second-order loadings	Freely estimating noninvariant loadings				Constraining noninvariant loadings to be equal			
			α	$b(\hat{\phi})$	$\sigma^2_{\hat{\phi}}$	%	α	$b(\hat{\phi})$	$\sigma^2_{\hat{\phi}}$	%
600	η1	.20	.051 (.049)	.0002	.010	.951	.049 (.047)	.0001	.009	.953
		-.20	.051 (.048)	.0036	.010	.952	.049 (.047)	.0030	.009	.953
	η2	.20	.053 (.051)	.0016	.010	.949	.055 (.053)	.0016	.009	.947
		-.20	.055 (.053)	-.0005	.010	.947	.056 (.053)	-.0008	.010	.947
	η3	.20	.052 (.051)	.0003	.010	.949	.055 (.054)	-.0007	.009	.946
		-.20	.049 (.048)	.0022	.010	.952	.050 (.048)	.0018	.009	.952

Note: *N* denotes total sample size. η1 is the reference first-order factor, η2 is the first-order factor associated with noninvariant second-order loadings, and η3 is the first-order factor associated with invariant second-order loadings. Type I error rates (α) outside the parentheses were obtained using the chi-square difference test, and those inside the parentheses were obtained via the Wald test for the latent mean difference; both were based on α = .05. $\hat{\phi}$ denotes the estimated latent mean difference of the second-order factor, so $b(\hat{\phi})$ is the bias and $\sigma^2_{\hat{\phi}}$ is the variance of this parameter.

Table A5

Power (Δκ = .20), Bias, Estimated Variance, and Coverage Rate of the Estimated Latent Mean Difference of the Second-Order Factor for the Exploration of the Influence of Partial Invariance of Second-Order Factor Loadings

N	$\gamma_{2,1(G2)}$ - $\gamma_{2,1(G1)}$	$\gamma_{2,1}$ is freely estimated				$\gamma_{2,1}$ is constrained to be equal				$\gamma_{2,1}$ is constrained to be equal but $\gamma_{3,1}$ is freely estimated			
		Power	$b(\hat{\phi})$	$\sigma^2_{\hat{\phi}}$	%	Power	$b(\hat{\phi})$	$\sigma^2_{\hat{\phi}}$	%	Power	$b(\hat{\phi})$	$\sigma^2_{\hat{\phi}}$	%
300	.20	.305 (.292)	.0021	.020	.951	.338 (.324)	.0063	.019	.952	.319 (.307)	.0071	.020	.951
	-.20	.300 (.288)	-.0021	.019	.956	.297 (.280)	-.0065	.018	.957	.273 (.261)	-.0081	.020	.955
600	.20	.531 (.524)	.0003	.009	.956	.587 (.581)	.0064	.009	.956	.569 (.562)	.0082	.009	.953
	-.20	.529 (.521)	.0001	.009	.954	.530 (.521)	-.0043	.009	.954	.478 (.470)	-.0061	.010	.954

Note: N denotes total sample size. $\gamma_{2,1}$ is the second-order loading relating the 2nd first-order factor to the second-order factor and G2 (G1) denotes group membership. Δκ denotes the latent mean difference of the second-order factor. Powers outside the parentheses were obtained using the chi-square difference test, and those inside the parentheses were obtained via the Wald test for the latent mean difference; both were based on α = .05. $\hat{\phi}$ denotes the estimated latent mean difference of the second-order factor, so $b(\hat{\phi})$ is the bias and $\sigma^2_{\hat{\phi}}$ is the variance of this parameter.

Table A6

Power (Δκ = .20), Bias, Estimated Variance, and Coverage Rate of the Estimated Latent Mean Difference of the Second-Order Factor for the Exploration of the Influence of Partial Invariance of First-Order Factor Loadings

N	The number of noninvariant first-order loadings	Relative Difference	Freely estimating $\boldsymbol{\lambda}_{6,2(6,2;7,2;10,3;11;3)}$				Constraining $\boldsymbol{\lambda}_{6,2(6,2;7,2;10,3;11;3)}$ to be equal				Constraining $\boldsymbol{\lambda}_{6,2(6,2;7,2;10,3;11;3)}$ to be equal but freely estimating $\boldsymbol{\lambda}_{7,2(2,1;3,1;14,4;15,4)}$			
			Power	$b(\hat{\phi})$	$\sigma^2_{\hat{\phi}}$	%	Power	$b(\hat{\phi})$	$\sigma^2_{\hat{\phi}}$	%	Power	$b(\hat{\phi})$	$\sigma^2_{\hat{\phi}}$	%
300	1	.20	.314 (.299)	.0003	.019	.949	.318 (.305)	.0013	.019	.948	.317 (.306)	.0014	.019	.950
		-.20	.323 (.309)	.0012	.019	.956	.322 (.308)	.0007	.019	.956	.319 (.307)	.0006	.019	.956
	4	.20	.311 (.301)	.0005	.019	.950	.329 (.316)	.0049	.019	.951	.322 (.310)	.0054	.019	.952
		-.20	.328 (.315)	.0046	.019	.953	.324 (.310)	.0013	.019	.952	.315 (.299)	.0009	.020	.950

Table A6 Continued

N	The number of noninvariant first-order loadings	Relative Difference	Freely estimating $\boldsymbol{\lambda}_{6,2(6,2;7,2;10,3;11;3)}$				Constraining $\boldsymbol{\lambda}_{6,2(6,2;7,2;10,3;11;3)}$ to be equal				Constraining $\boldsymbol{\lambda}_{6,2(6,2;7,2;10,3;11;3)}$ to be equal but freely estimating $\boldsymbol{\lambda}_{7,2(2,1;3,1;14,4;15,4)}$			
			Power	$b(\hat{\phi})$	$\sigma^2_{\hat{\phi}}$	%	Power	$b(\hat{\phi})$	$\sigma^2_{\hat{\phi}}$	%	Power	$b(\hat{\phi})$	$\sigma^2_{\hat{\phi}}$	%
600	1	.20	.551 (.543)	-.0007	.010	.948	.552 (.543)	.0002	.010	.949	.553 (.544)	.0003	.010	.950
		-.20	.551 (.543)	-.0014	.009	.954	.549 (.542)	-.0020	.009	.953	.546 (.538)	-.0020	.009	.952
	4	.20	.544 (.535)	-.0016	.009	.951	.572 (.567)	.0027	.009	.953	.562 (.555)	.0032	.010	.953
		-.20	.560 (.550)	.0006	.009	.955	.550 (.542)	-.0025	.009	.957	.535 (.527)	-.0032	.009	.957

Note: N denotes total sample size. Relative Difference refers the value of noninvariant loading(s) in Group 2 minus that in Group 1. Δκ denotes the latent mean difference of the second-order factor. Powers outside the parentheses were obtained using the chi-square difference test, and those inside the parentheses were obtained via the Wald test for the latent mean difference; both were based on α = .05. $\hat{\phi}$ denotes the estimated latent mean difference of the second-order factor, so $b(\hat{\phi})$ is the bias and $\sigma^2_{\hat{\phi}}$ is the variance of this parameter.

Table A7

Power (Δκ = .20), Bias, Estimated Variance, and Coverage Rate of the Estimated Latent Mean Difference of the Second-Order Factor for the Exploration of the Influence of Partial Invariance of Both First- and Second-Order Loadings

N	The first-order factor that has noninvariant parameters	Relative differences in noninvariant second-order loadings	Freely estimating noninvariant loadings				Constraining noninvariant loadings to be equal			
			Power	$b(\hat{\phi})$	$\sigma^2_{\hat{\phi}}$	%	Power	$b(\hat{\phi})$	$\sigma^2_{\hat{\phi}}$	%
300	η1	.20	.303 (.293)	.0000	.020	.951	.333 (.319)	.0015	.018	.953
		-.20	.292 (.279)	-.0032	.020	.948	.289 (.274)	-.0097	.019	.945
	η2	.20	.302 (.290)	-.0016	.020	.953	.322 (.312)	.0022	.018	.952
		-.20	.305 (.293)	-.0015	.020	.951	.298 (.287)	-.0067	.020	.952
	η3	.20	.300 (.288)	-.0016	.021	.947	.336 (.325)	.0034	.019	.952
		-.20	.300 (.287)	-.0001	.021	.953	.299 (.285)	-.0063	.020	.947

Table A7 Continued

N	The first-order factor that has noninvariant parameters	Relative differences in noninvariant second-order loadings	Freely estimating noninvariant loadings				Constraining noninvariant loadings to be equal			
			Power	$b(\hat{\phi})$	$\sigma^2_{\hat{\phi}}$	%	Power	$b(\hat{\phi})$	$\sigma^2_{\hat{\phi}}$	%
600	η1	.20	.531 (.525)	.0021	.010	.951	.581 (.572)	.0038	.009	.948
		-.20	.537 (.531)	.0015	.010	.951	.533 (.526)	-.0050	.010	.952
	η2	.20	.540 (.533)	.0016	.010	.946	.589 (.583)	.0057	.009	.947
		-.20	.540 (.533)	.0013	.010	.948	.528 (.519)	-.0034	.010	.944
	η3	.20	.531 (.525)	-.0001	.010	.952	.579 (.573)	.0039	.009	.951
		-.20	.534 (.524)	.0007	.010	.954	.521 (.512)	-.0059	.010	.954

Note: *N* denotes total sample size. η1 is the reference first-order factor, η2 is the first-order factor associated with noninvariant second-order loadings, and η3 is the first-order factor associated with invariant second-order loadings. Δκ denotes the latent mean difference of the second-order factor. Powers outside the parentheses were obtained using the chi-square difference test, and those inside the parentheses were obtained via the Wald test for the latent mean difference; both were based on α = .05. $\hat{\phi}$ denotes the estimated latent mean difference of the second-order factor, so $b(\hat{\phi})$ is the bias and $\sigma^2_{\hat{\phi}}$ is the variance of this parameter.

Table A8

Power (Δκ = .40), Bias, Estimated Variance, and Coverage Rate of the Estimated Latent Mean Difference of the Second-Order Factor for the Exploration of the Influence of Partial Invariance of Second-Order Factor Loadings

N	$\gamma_{2,1(G2)} - \gamma_{2,1(G1)}$	$\gamma_{2,1}$ is freely estimated				$\gamma_{2,1}$ is constrained to be equal				$\gamma_{2,1}$ is constrained to be equal but $\gamma_{3,1}$ is freely estimated			
		Power	$b(\hat{\phi})$	$\sigma^2_{\hat{\phi}}$	%	Power	$b(\hat{\phi})$	$\sigma^2_{\hat{\phi}}$	%	Power	$b(\hat{\phi})$	$\sigma^2_{\hat{\phi}}$	%
300	.20	.820 (.813)	-.0002	.022	.944	.867 (.860)	.0108	.020	.946	.854 (.845)	.0138	.021	.944
	-.20	.826 (.816)	-.0004	.021	.951	.822 (.812)	-.0085	.020	.945	.777 (.766)	-.0108	.022	.947
600	.20	.985 (.985)	.0002	.010	.949	.994 (.994)	.0107	.010	.955	.992 (.991)	.0136	.010	.957
	-.20	.983 (.982)	.0004	.011	.948	.982 (.981)	-.0076	.010	.947	.971 (.969)	-.0100	.011	.943

Note: N denotes total sample size. $\gamma_{2,1}$ is the second-order loading relating the 2nd first-order factor to the second-order factor and G2 (G1) denotes group membership. Δκ denotes the latent mean difference of the second-order factor. Powers outside the parentheses were obtained using the chi-square difference test, and those inside the parentheses were obtained via the Wald test for the latent mean difference; both were based on α = .05. $\hat{\phi}$ denotes the estimated latent mean difference of the second-order factor, so $b(\hat{\phi})$ is the bias and $\sigma^2_{\hat{\phi}}$ is the variance of this parameter.

Table A9

Power (Δκ = .40), Bias, Estimated Variance, and Coverage Rate of the Estimated Latent Mean Difference of the Second-Order Factor for the Exploration of the Influence of Partial Invariance of First-Order Factor Loadings

N	The number of noninvariant first-order loadings	Relative Difference	Freely estimating $\boldsymbol{\lambda}_{6,2(6,2;7,2;10,3;11;3)}$				Constraining $\boldsymbol{\lambda}_{6,2(6,2;7,2;10,3;11;3)}$ to be equal				Constraining $\boldsymbol{\lambda}_{6,2(6,2;7,2;10,3;11;3)}$ to be equal but freely estimating $\boldsymbol{\lambda}_{7,2(2,1;3,1;14,4;15,4)}$			
			Power	$b(\hat{\phi})$	$\sigma^2_{\hat{\phi}}$	%	Power	$b(\hat{\phi})$	$\sigma^2_{\hat{\phi}}$	%	Power	$b(\hat{\phi})$	$\sigma^2_{\hat{\phi}}$	%
300	1	.20	.830 (.819)	-.0040	.020	.950	.834 (.826)	-.0024	.020	.952	.833 (.827)	-.0021	.020	.953
		-.20	.839 (.830)	.0014	.021	.945	.836 (.828)	.0005	.021	.945	.838 (.827)	.0003	.021	.946
	4	.20	.820 (.811)	-.0027	.021	.942	.846 (.836)	.0058	.021	.944	.839 (.831)	.0067	.021	.947
		-.20	.849 (.840)	.0032	.020	.948	.842 (.832)	-.0030	.020	.946	.829 (.819)	-.0034	.021	.948

Table A9 Continued

N	The number of noninvariant first-order loadings	Relative Difference	Freely estimating $\boldsymbol{\lambda}_{6,2(6,2;7,2;10,3;11;3)}$				Constraining $\boldsymbol{\lambda}_{6,2(6,2;7,2;10,3;11;3)}$ to be equal				Constraining $\boldsymbol{\lambda}_{6,2(6,2;7,2;10,3;11;3)}$ to be equal but freely estimating $\boldsymbol{\lambda}_{7,2(2,1;3,1;14,4;15,4)}$			
			Power	$b(\hat{\phi})$	$\sigma^2_{\hat{\phi}}$	%	Power	$b(\hat{\phi})$	$\sigma^2_{\hat{\phi}}$	%	Power	$b(\hat{\phi})$	$\sigma^2_{\hat{\phi}}$	%
600	1	.20	.987 (.987)	-.0027	.010	.950	.989 (.989)	-.0010	.010	.951	.989 (.988)	-.0007	.010	.951
		-.20	.989 (.988)	.0009	.010	.952	.988 (.987)	-.0002	.010	.953	.989 (.988)	-.0002	.010	.952
	4	.20	.985 (.983)	-.0012	.010	.948	.991 (.990)	.0069	.010	.949	.989 (.988)	.0073	.010	.949
		-.20	.987 (.986)	.0000	.010	.953	.983 (.983)	-.0053	.010	.947	.981 (.981)	-.0054	.010	.948

Note: N denotes total sample size. Relative Difference refers the value of noninvariant loading(s) in Group 2 minus that in Group 1. $\Delta\kappa$ denotes the latent mean difference of the second-order factor. Powers outside the parentheses were obtained using the chi-square difference test, and those inside the parentheses were obtained via the Wald test for the latent mean difference; both were based on $\alpha = .05$. $\hat{\phi}$ denotes the estimated latent mean difference of the second-order factor, so $b(\hat{\phi})$ is the bias and $\sigma^2_{\hat{\phi}}$ is the variance of this parameter.

Table A10

Power (Δκ = .40), Bias, Estimated Variance, and Coverage Rate of the Estimated Latent Mean Difference of the Second-Order Factor for the Exploration of the Influence of Partial Invariance of Both First- and Second-Order Loadings

N	The first-order factor that has noninvariant parameters	Relative differences in noninvariant second-order loadings	Freely estimating noninvariant loadings				Constraining noninvariant loadings to be equal			
			Power	$b(\hat{\phi})$	$\sigma^2_{\hat{\phi}}$	%	Power	$b(\hat{\phi})$	$\sigma^2_{\hat{\phi}}$	%
300	η1	.20	.823 (.812)	.0017	.021	.948	.864 (.856)	.0055	.019	.950
		-.20	.825 (.815)	.0038	.021	.948	.820 (.809)	-.0086	.020	.948
	η2	.20	.823 (.814)	-.0011	.021	.949	.861 (.852)	.0054	.020	.954
		-.20	.820 (.808)	-.0006	.021	.946	.812 (.801)	-.0103	.020	.945
	η3	.20	.825 (.815)	.0000	.021	.947	.859 (.850)	.0081	.020	.950
		-.20	.816 (.805)	-.0022	.021	.946	.809 (.799)	-.0138	.021	.942

Table A10 Continued

N	The first-order factor that has noninvariant parameters	Relative differences in noninvariant second-order loadings	Freely estimating noninvariant loadings				Constraining noninvariant loadings to be equal			
			Power	$b(\hat{\phi})$	$\sigma^2_{\hat{\phi}}$	%	Power	$b(\hat{\phi})$	$\sigma^2_{\hat{\phi}}$	%
600	η1	.20	.984 (.983)	-.0006	.011	.950	.990 (.990)	.0032	.010	.949
		-.20	.983 (.983)	-.0015	.011	.948	.981 (.980)	-.0147	.010	.943
	η2	.20	.985 (.984)	.0001	.010	.949	.993 (.992)	.0061	.010	.951
		-.20	.983 (.982)	-.0004	.010	.951	.983 (.983)	-.0094	.010	.946
	η3	.20	.982 (.981)	.0019	.011	.949	.992 (.991)	.0093	.010	.950
		-.20	.983 (.982)	-.0001	.010	.953	.979 (.978)	-.0124	.010	.949

Note: *N* denotes total sample size. η1 is the reference first-order factor, η2 is the first-order factor associated with noninvariant second-order loadings, and η3 is the first-order factor associated with invariant second-order loadings. Δκ denotes the latent mean difference of the second-order factor. Powers outside the parentheses were obtained using the chi-square difference test, and those inside the parentheses were obtained via the Wald test for the latent mean difference; both were based on α = .05. $\hat{\phi}$ denotes the estimated latent mean difference of the second-order factor, so $b(\hat{\phi})$ is the bias and $\sigma^2_{\hat{\phi}}$ is the variance of this parameter.

Table A11

Type I Error Rate or Power, Bias, Estimated Variance, and Coverage Rate of the Estimated Latent Mean Difference of the Second-Order Factor for the Baseline Conditions to Explore the Influence of Partial Intercept Invariance

Freely estimated parameters	*N*	*Δκ = .00*				*Δκ = .20*				*Δκ = .40*			
		α	$b(\hat{\phi})$	$\sigma^2_{\hat{\phi}}$	%	Power	$b(\hat{\phi})$	$\sigma^2_{\hat{\phi}}$	%	Power	$b(\hat{\phi})$	$\sigma^2_{\hat{\phi}}$	%
None	300	.053 (.047)	-.0018	.018	.953	.327 (.314)	.0041	.019	.952	.841 (.834)	-.0018	.020	.946
	600	.052 (.050)	.0008	.009	.950	.572 (.563)	.0023	.009	.947	.987 (.986)	-.0013	.010	.950
α_2	300	.051 (.046)	-.0012	.019	.954	.315 (.300)	.0042	.020	.954	.824 (.811)	-.0015	.021	.944
	600	.052 (.050)	.0004	.010	.950	.549 (.542)	.0023	.010	.950	.982 (.981)	-.0017	.010	.949

Table A11 Continued

Freely estimated parameters	*N*	*Δκ = .00*				*Δκ = .20*				*Δκ = .40*			
		α	$b(\hat{\phi})$	$\sigma^2_{\hat{\phi}}$	%	Power	$b(\hat{\phi})$	$\sigma^2_{\hat{\phi}}$	%	Power	$b(\hat{\phi})$	$\sigma^2_{\hat{\phi}}$	%
α_3	300	.053 (.049)	-.0014	.020	.951	.305 (.292)	.0042	.020	.953	.810 (.800)	-.0021	.022	.944
	600	.054 (.051)	.0008	.010	.949	.540 (.531)	.0029	.010	.949	.983 (.982)	-.0016	.011	.952
$\boldsymbol{\nu_6}$	300	.054 (.049)	-.0019	.018	.951	.327 (.315)	.0042	.019	.954	.841 (.832)	-.0018	.020	.945
	600	.052 (.050)	.0009	.009	.950	.568 (.562)	.0022	.009	.949	.986 (.986)	-.0014	.010	.949

Table A11 Continued

Freely estimated parameters	*N*	*Δκ = .00*				*Δκ = .20*				*Δκ = .40*			
		α	$b(\hat{\phi})$	$\sigma^2_{\hat{\phi}}$	%	Power	$b(\hat{\phi})$	$\sigma^2_{\hat{\phi}}$	%	Power	$b(\hat{\phi})$	$\sigma^2_{\hat{\phi}}$	%
$\boldsymbol{\nu_7}$	300	.054 (.049)	-.0018	.018	.951	.328 (.314)	.0043	.019	.952	.839 (.830)	-.0018	.020	.944
	600	.052 (.051)	.0008	.009	.949	.568 (.563)	.0022	.009	.950	.986 (.985)	-.0013	.010	.950
$\boldsymbol{\nu_{(6;7;10;11)}}$	300	.054 (.050)	-.0018	.019	.950	.323 (.311)	.0044	.019	.954	.837 (.828)	-.0018	.021	.945
	600	.052 (.050)	.0009	.009	.950	.566 (.559)	.0024	.010	.949	.986 (.984)	-.0015	.010	.951

Table A11 Continued

Freely estimated parameters	N	$\Delta\kappa = .00$				$\Delta\kappa = .20$				$\Delta\kappa = .40$			
		α	$b(\hat{\phi})$	$\sigma^2_{\hat{\phi}}$	%	Power	$b(\hat{\phi})$	$\sigma^2_{\hat{\phi}}$	%	Power	$b(\hat{\phi})$	$\sigma^2_{\hat{\phi}}$	%
$\boldsymbol{\nu}_{(\mathbf{2};\mathbf{3};\mathbf{14};\mathbf{15})}$	300	.054 (.048)	-.0017	.019	.952	.323 (.309)	.0044	.020	.950	.832 (.824)	-.0017	.021	.948
	600	.054 (.053)	.0010	.009	.947	.556 (.549)	.0022	.010	.949	.985 (.984)	-.0013	.010	.947
α_2 and $\boldsymbol{\nu}_{(2;3)}$	300	.051 (.048)	-.0012	.019	.952	.312 (.297)	.0044	.020	.956	.821 (.811)	-.0016	.021	.947
	600	.050 (.049)	.0006	.010	.951	.546 (.538)	.0022	.010	.950	.980 (.980)	-.0018	.011	.949

Table A11 Continued

Freely estimated parameters	N	$\Delta\kappa$ = .00				$\Delta\kappa$ = .20				$\Delta\kappa$ = .40			
		α	$b(\hat{\phi})$	$\sigma^2_{\hat{\phi}}$	%	Power	$b(\hat{\phi})$	$\sigma^2_{\hat{\phi}}$	%	Power	$b(\hat{\phi})$	$\sigma^2_{\hat{\phi}}$	%
α_2 and $\boldsymbol{\nu}_{(6;7)}$	300	.051 (.046)	-.0012	.019	.954	.315 (.300)	.0042	.020	.954	.824 (.811)	-.0015	.021	.944
	600	.052 (.050)	.0004	.010	.950	.549 (.542)	.0023	.010	.950	.982 (.981)	-.0017	.010	.949
α_2 and $\boldsymbol{\nu}_{(10;11)}$	300	.051 (.046)	-.0011	.019	.954	.310 (.297)	.0042	.020	.954	.819 (.808)	-.0015	.021	.943
	600	.051 (.050)	.0005	.010	.950	.548 (.540)	.0026	.010	.949	.981 (.980)	-.0018	.010	.949

Note: N denotes total sample size. Δκ denotes the latent mean difference of the second-order factor. Type I error rates (α) and powers outside the parentheses were obtained using the chi-square difference test, and those inside the parentheses were obtained via the Wald test for the latent mean difference; both were based on α = .05. $\hat{\phi}$ denotes the estimated latent mean difference of the second-order factor, so $b(\hat{\phi})$ is the bias and $\sigma^2_{\hat{\phi}}$ is the variance of this parameter.

Table A12

Type I Error Rate, Bias, Estimated Variance, and Coverage Rate of the Estimated Latent Mean Difference of the Second-Order Factor for the Exploration of the Influence of Partial Invariance of Intercepts of First-Order Factors

N	$\alpha_{2(G2)} - \alpha_{2(G1)}$	α_2 is freely estimated				α_2 is constrained to be equal				α_2 is constrained to be equal but α_3 is freely estimated			
		α	$b(\hat{\phi})$	$\sigma^2_{\hat{\phi}}$	%	α	$b(\hat{\phi})$	$\sigma^2_{\hat{\phi}}$	%	α	$b(\hat{\phi})$	$\sigma^2_{\hat{\phi}}$	%
300	.10	.049 (.044)	.0002	.019	.956	.056 (.050)	.0245	.019	.950	.059 (.055)	.0326	.020	.945
	-.10	.057 (.051)	.0011	.020	.949	.061 (.057)	-.0232	.019	.943	.061 (.055)	-.0299	.020	.945
600	.10	.052 (.050)	-.0008	.010	.950	.061 (.057)	.0238	.009	.943	.061 (.059)	.0313	.010	.941
	-.10	.050 (.049)	.0006	.010	.951	.055 (.053)	-.0242	.009	.947	.063 (.061)	-.0313	.010	.939

Note: N denotes total sample size. α_2 is the intercept of the first-order factor and G2 (G1) denotes group membership. Type I error rates (α) outside the parentheses were obtained using the chi-square difference test, and those inside the parentheses were obtained via the Wald test for the latent mean difference; both were based on α = .05. $\hat{\phi}$ denotes the estimated latent mean difference of the second-order factor, so $b(\hat{\phi})$ is the bias and $\sigma^2_{\hat{\phi}}$ is the variance of this parameter.

Table A13

Type I Error Rate, Bias, Estimated Variance, and Coverage Rate of the Estimated Latent Mean Difference of the Second-Order Factor for the Exploration of the Influence of Partial Invariance of Indicator Intercepts

N	The number of noninvariant Indicator Intercepts	Relative Difference	Freely estimating $\boldsymbol{\nu_{6(6;7;10;11)}}$				Constraining $\boldsymbol{\nu_{6(6;7;10;11)}}$ to be equal				Constraining $\boldsymbol{\nu_{6(6;7;10;11)}}$ to be equal but freely estimating $\boldsymbol{\nu_{7(2;3;14;15)}}$			
			α	$b(\hat{\phi})$	$\sigma^2_{\hat{\phi}}$	%	α	$b(\hat{\phi})$	$\sigma^2_{\hat{\phi}}$	%	α	$b(\hat{\phi})$	$\sigma^2_{\hat{\phi}}$	%
300	1	.10	.050 (.045)	.0022	.019	.955	.049 (.045)	.0080	.019	.955	.050 (.046)	.0088	.019	.954
		-.10	.051 (.047)	.0026	.018	.953	.051 (.046)	-.0032	.018	.954	.050 (.046)	-.0040	.018	.954
	4	.10	.052 (.049)	.0007	.019	.951	.053 (.049)	.0285	.019	.951	.055 (.049)	.0307	.019	.951
		-.10	.047 (.045)	-.0025	.019	.955	.055 (.051)	-.0305	.018	.949	.056 (.051)	-.0330	.019	.949

Table A13 Continued

N	The number of noninvariant Indicator Intercepts	Relative Difference	Freely estimating $\boldsymbol{\nu}_{\mathbf{6(6;7;10;11)}}$				Constraining $\boldsymbol{\nu}_{\mathbf{6(6;7;10;11)}}$ to be equal				Constraining $\boldsymbol{\nu}_{\mathbf{6(6;7;10;11)}}$ to be equal but freely estimating $\boldsymbol{\nu}_{\mathbf{7(2;3;14;15)}}$			
			α	$b(\hat{\phi})$	$\sigma^2_{\hat{\phi}}$	%	α	$b(\hat{\phi})$	$\sigma^2_{\hat{\phi}}$	%	α	$b(\hat{\phi})$	$\sigma^2_{\hat{\phi}}$	%
600	1	.10	.051 (.048)	-.0005	.009	.952	.050 (.048)	.0055	.009	.952	.051 (.048)	.0063	.009	.952
		-.10	.051 (.050)	-.0004	.009	.950	.052 (.050)	-.0063	.009	.950	.053 (.050)	-.0070	.009	.950
	4	.10	.053 (.050)	-.0006	.009	.950	.062 (.060)	.0276	.009	.940	.063 (.061)	.0301	.009	.939
		-.10	.054 (.052)	.0003	.010	.948	.066 (.063)	-.0282	.010	.937	.068 (.065)	-.0305	.010	.935

Note: N denotes total sample size. Relative Difference refers the value of noninvariant indicator intercept(s) in Group 2 minus that in Group 1. Type I error rates (α) outside the parentheses were obtained using the chi-square difference test, and those inside the parentheses were obtained via the Wald test for the latent mean difference; both were based on α = .05. $\hat{\phi}$ denotes the estimated latent mean difference of the second-order factor, so $b(\hat{\phi})$ is the bias and $\sigma^2_{\hat{\phi}}$ is the variance of this parameter.

Table A14

Type I Error Rate, Bias, Estimated Variance, and Coverage Rate of the Estimated Latent Mean Difference of the Second-Order Factor for the Exploration of the Influence of Partial Invariance of Intercepts of Both First-Order Factors and Indicators

N	The first-order factor that has noninvariant parameters	Relative differences in noninvariant intercepts of the first-order factor	Freely estimating noninvariant intercepts				Constraining noninvariant intercepts to be equal			
			α	$b(\hat{\phi})$	$\sigma^2_{\hat{\phi}}$	%	α	$b(\hat{\phi})$	$\sigma^2_{\hat{\phi}}$	%
300	η1	.10	.051 (.046)	-.0024	.020	.954	.052 (.047)	.0050	.019	.953
		-.10	.053 (.048)	-.0037	.020	.952	.063 (.059)	-.0446	.019	.941
	η2	.10	.055 (.051)	-.0002	.020	.949	.059 (.053)	.0129	.019	.947
		-.10	.057 (.053)	-.0010	.020	.947	.063 (.059)	-.0371	.019	.941
	η3	.10	.045 (.042)	-.0026	.019	.958	.048 (.044)	.0065	.018	.956
		-.10	.054 (.050)	.0001	.020	.950	.064 (.059)	-.0402	.019	.941

Table A14 Continued

N	The first-order factor that has noninvariant parameters	Relative differences in noninvariant intercepts of the first-order factor	Freely estimating noninvariant intercepts				Constraining noninvariant intercepts to be equal			
			α	$b(\hat{\phi})$	$\sigma^2_{\hat{\phi}}$	%	α	$b(\hat{\phi})$	$\sigma^2_{\hat{\phi}}$	%
600	η1	.10	.054 (.052)	-.0008	.010	.948	.057 (.056)	.0070	.010	.944
		-.10	.054 (.052)	-.0004	.010	.948	.073 (.070)	-.0411	.009	.930
	η2	.10	.057 (.055)	-.0010	.010	.945	.058 (.056)	.0115	.010	.944
		-.10	.051 (.049)	-.0014	.010	.951	.073 (.071)	-.0373	.009	.929
	η3	.10	.051 (.049)	-.0007	.010	.951	.057 (.054)	.0079	.009	.946
		-.10	.049 (.048)	-.0004	.010	.952	.073 (.071)	-.0413	.009	.929

Note: N denotes total sample size. η1 is the reference first-order factor, η2 is the first-order factor having noninvariant intercepts, and η3 is the first-order factor having invariant intercepts. Type I error rates (α) outside the parentheses were obtained using the chi-square difference test, and those inside the parentheses were obtained via the Wald test for the latent mean difference; both were based on α = .05. $\hat{\phi}$ denotes the estimated latent mean difference of the second-order factor, so $b(\hat{\phi})$ is the bias and $\sigma^2_{\hat{\phi}}$ is the variance of this parameter.

Table A15

Power (Δκ = .20), Bias, Estimated Variance, and Coverage Rate of the Estimated Latent Mean Difference of the Second-Order Factor for the Exploration of the Influence of Partial Invariance of Intercepts of First-Order Factors

N	$\alpha_{2(G2)}$ -$\alpha_{2(G1)}$	α_2 is freely estimated				α_2 is constrained to be equal				α_2 is constrained to be equal but α_3 is freely estimated			
		Power	$b(\hat{\phi})$	$\sigma^2_{\hat{\phi}}$	%	Power	$b(\hat{\phi})$	$\sigma^2_{\hat{\phi}}$	%	Power	$b(\hat{\phi})$	$\sigma^2_{\hat{\phi}}$	%
300	.10	.317 (.302)	.0009	.020	.955	.388 (.373)	.0245	.019	.954	.383 (.371)	.0308	.021	.949
	-.10	.310 (.295)	-.0004	.020	.951	.259 (.247)	-.0236	.019	.943	.229 (.218)	-.0312	.021	.940
600	.10	.531 (.524)	-.0005	.010	.946	.644 (.639)	.0238	.010	.940	.643 (.637)	.0314	.011	.937
	-.10	.555 (.548)	.0023	.010	.953	.459 (.449)	-.0217	.009	.942	.401 (.394)	-.0291	.010	.934

Note: N denotes total sample size. α_2 is the intercept of the first-order factor and G2 (G1) denotes group membership. Δκ denotes the latent mean difference of the second-order factor. Powers outside the parentheses were obtained using the chi-square difference test, and those inside the parentheses were obtained via the Wald test for the latent mean difference; both were based on α = .05. $\hat{\phi}$ denotes the estimated latent mean difference of the second-order factor, so $b(\hat{\phi})$ is the bias and $\sigma^2_{\hat{\phi}}$ is the variance of this parameter.

Table A16

Power (Δκ = .20), Bias, Estimated Variance, and Coverage Rate of the Estimated Latent Mean Difference of the Second-Order Factor for the Exploration of the Influence of Partial Invariance of Indicator Intercepts

N	The number of noninvariant Indicator Intercepts	Relative Difference	Freely estimating $\boldsymbol{\nu_{6(6;7;10;11)}}$				Constraining $\boldsymbol{\nu_{6(6;7;10;11)}}$ to be equal				Constraining $\boldsymbol{\nu_{6(6;7;10;11)}}$ to be equal but freely estimating $\boldsymbol{\nu_{7(2;3;14;15)}}$			
			Power	$b(\hat{\phi})$	$\sigma^2_{\hat{\phi}}$	%	Power	$b(\hat{\phi})$	$\sigma^2_{\hat{\phi}}$	%	Power	$b(\hat{\phi})$	$\sigma^2_{\hat{\phi}}$	%
300	1	.10	.312 (.301)	-.0019	.020	.947	.330 (.315)	.0041	.020	.949	.332 (.316)	.0049	.020	.949
		-.10	.326 (.312)	.0026	.020	.947	.310 (.297)	-.0031	.019	.946	.308 (.294)	-.0039	.020	.945
	4	.10	.316 (.302)	.0004	.019	.950	.399 (.385)	.0284	.019	.949	.403 (.387)	.0310	.020	.948
		-.10	.319 (.308)	.0015	.020	.948	.255 (.242)	-.0254	.020	.940	.240 (.230)	-.0282	.020	.939

Table A16 Continued

N	The number of noninvariant Indicator Intercepts	Relative Difference	Freely estimating $\boldsymbol{\nu_{6(6;7;10;11)}}$				Constraining $\boldsymbol{\nu_{6(6;7;10;11)}}$ to be equal				Constraining $\boldsymbol{\nu_{6(6;7;10;11)}}$ to be equal but freely estimating $\boldsymbol{\nu_{7(2;3;14;15)}}$			
			Power	$b(\hat{\phi})$	$\sigma^2_{\hat{\phi}}$	%	Power	$b(\hat{\phi})$	$\sigma^2_{\hat{\phi}}$	%	Power	$b(\hat{\phi})$	$\sigma^2_{\hat{\phi}}$	%
600	1	.10	.551 (.544)	-.0002	.009	.951	.575 (.569)	.0058	.009	.951	.577 (.568)	.0065	.009	.950
		-.10	.550 (.541)	-.0012	.009	.955	.523 (.517)	-.0070	.009	.954	.521 (.513)	-.0077	.009	.952
	4	.10	.548 (.539)	-.0017	.009	.949	.664 (.655)	.0265	.009	.943	.664 (.657)	.0291	.010	.942
		-.10	.538 (.531)	-.0009	.009	.955	.430 (.424)	-.0284	.009	.942	.413 (.405)	-.0312	.009	.942

Note: N denotes total sample size. Relative Difference refers the value of noninvariant indicator intercept(s) in Group 2 minus that in Group 1. Δκ denotes the latent mean difference of the second-order factor. Powers outside the parentheses were obtained using the chi-square difference test, and those inside the parentheses were obtained via the Wald test for the latent mean difference; both were based on α = .05. $\hat{\phi}$ denotes the estimated latent mean difference of the second-order factor, so $b(\hat{\phi})$ is the bias and $\sigma^2_{\hat{\phi}}$ is the variance of this parameter.

Table A17

Power (Δκ = .20), Bias, Estimated Variance, and Coverage Rate of the Estimated Latent Mean Difference of the Second-Order Factor for the Exploration of the Influence of Partial Invariance of Intercepts of Both First-Order Factors and Indicators

N	The first-order factor that has noninvariant parameters	Relative differences in noninvariant intercepts of the first-order factor	Freely estimating noninvariant intercepts				Constraining noninvariant intercepts to be equal			
			Power	$b(\hat{\phi})$	$\sigma^2_{\hat{\phi}}$	%	Power	$b(\hat{\phi})$	$\sigma^2_{\hat{\phi}}$	%
300	η1	.10	.306 (.292)	-.0003	.020	.951	.341 (.332)	.0068	.019	.952
		-.10	.306 (.296)	.0011	.020	.951	.223 (.211)	-.0405	.019	.942
	η2	.10	.301 (.289)	-.0001	.020	.952	.351 (.335)	.0122	.019	.957
		-.10	.303 (.288)	-.0026	.020	.948	.228 (.218)	-.0373	.019	.938
	η3	.10	.297 (.283)	-.0025	.020	.950	.334 (.320)	.0058	.020	.947
		-.10	.306 (.296)	.0011	.020	.948	.232 (.219)	-.0376	.020	.935

Table A17 Continued

N	The first-order factor that has noninvariant parameters	Relative differences in noninvariant intercepts of the first-order factor	Freely estimating noninvariant intercepts				Constraining noninvariant intercepts to be equal			
			Power	$b(\hat{\phi})$	$\sigma^2_{\hat{\phi}}$	%	Power	$b(\hat{\phi})$	$\sigma^2_{\hat{\phi}}$	%
600	η1	.10	.527 (.520)	-.0007	.010	.948	.585 (.580)	.0068	.010	.950
		-.10	.534 (.525)	.0015	.010	.952	.394 (.389)	-.0394	.009	.928
	η2	.10	.542 (.535)	.0012	.010	.944	.618 (.610)	.0137	.010	.943
		-.10	.538 (.531)	.0013	.010	.952	.413 (.404)	-.0338	.009	.933
	η3	.10	.527 (.520)	.0014	.010	.947	.595 (.586)	.0104	.010	.947
		-.10	.521 (.515)	.0003	.010	.949	.389 (.378)	-.0396	.010	.927

Note: N denotes total sample size. η1 is the reference first-order factor, η2 is the first-order factor having noninvariant intercepts, and η3 is the first-order factor having invariant intercepts. Δκ denotes the latent mean difference of the second-order factor. Powers outside the parentheses were obtained using the chi-square difference test, and those inside the parentheses were obtained via the Wald test for the latent mean difference; both were based on α = .05. $\hat{\phi}$ denotes the estimated latent mean difference of the second-order factor, so $b(\hat{\phi})$ is the bias and $\sigma^2_{\hat{\phi}}$ is the variance of this parameter.

Table A18

Power (Δκ = .40), Bias, Estimated Variance, and Coverage Rate of the Estimated Latent Mean Difference of the Second-Order Factor for the Exploration of the Influence of Partial Invariance of Intercepts of First-Order Factors

N	$\alpha_{2(G2)}$ -$\alpha_{2(G1)}$	α_2 is freely estimated				α_2 is constrained to be equal				α_2 is constrained to be equal but α_3 is freely estimated			
		Power	$b(\hat{\phi})$	$\sigma^2_{\hat{\phi}}$	%	Power	$b(\hat{\phi})$	$\sigma^2_{\hat{\phi}}$	%	Power	$b(\hat{\phi})$	$\sigma^2_{\hat{\phi}}$	%
300	.10	.821 (.813)	.0012	.021	.950	.881 (.873)	.0256	.020	.948	.869 (.861)	.0340	.022	.946
	-.10	.814 (.803)	-.0039	.021	.948	.781 (.771)	-.0262	.021	.940	.734 (.724)	-.0330	.022	.939
600	.10	.985 (.984)	.0007	.010	.951	.995 (.995)	.0251	.010	.948	.994 (.993)	.0332	.011	.950
	-.10	.984 (.984)	.0005	.011	.949	.976 (.975)	-.0226	.010	.935	.959 (.958)	-.0292	.011	.932

Note: N denotes total sample size. α_2 is the intercept of the first-order factor and G2 (G1) denotes group membership. Δκ denotes the latent mean difference of the second-order factor. Powers outside the parentheses were obtained using the chi-square difference test, and those inside the parentheses were obtained via the Wald test for the latent mean difference; both were based on α = .05. $\hat{\phi}$ denotes the estimated latent mean difference of the second-order factor, so $b(\hat{\phi})$ is the bias and $\sigma^2_{\hat{\phi}}$ is the variance of this parameter.

Table A19

Power (Δκ = .40), Bias, Estimated Variance, and Coverage Rate of the Estimated Latent Mean Difference of the Second-Order Factor for the Exploration of the Influence of Partial Invariance of Indicator Intercepts

N	The number of noninvariant Indicator Intercepts	Relative Difference	Freely estimating $\boldsymbol{\nu_{6(6;7;10;11)}}$				Constraining $\boldsymbol{\nu_{6(6;7;10;11)}}$ to be equal				Constraining $\boldsymbol{\nu_{6(6;7;10;11)}}$ to be equal but freely estimating $\boldsymbol{\nu_{7(2;3;14;15)}}$			
			Power	$b(\hat{\phi})$	$\sigma^2_{\hat{\phi}}$	%	Power	$b(\hat{\phi})$	$\sigma^2_{\hat{\phi}}$	%	Power	$b(\hat{\phi})$	$\sigma^2_{\hat{\phi}}$	%
300	1	.10	.835 (.827)	-.0010	.021	.946	.848 (.838)	.0046	.021	.945	.847 (.838)	.0052	.021	.945
		-.10	.851 (.842)	.0034	.020	.948	.842 (.832)	-.0021	.020	.946	.841 (.831)	-.0030	.020	.947
	4	.10	.840 (.832)	.0019	.021	.946	.888 (.883)	.0290	.020	.947	.884 (.877)	.0319	.021	.946
		-.10	.838 (.827)	-.0019	.021	.946	.781 (.769)	-.0280	.021	.936	.768 (.757)	-.0305	.021	.933

Table A19 Continued

N	The number of noninvariant Indicator Intercepts	Relative Difference	Freely estimating $\boldsymbol{\nu}_{\mathbf{6(6;7;10;11)}}$				Constraining $\boldsymbol{\nu}_{\mathbf{6(6;7;10;11)}}$ to be equal				Constraining $\boldsymbol{\nu}_{\mathbf{6(6;7;10;11)}}$ to be equal but freely estimating $\boldsymbol{\nu}_{\mathbf{7(2;3;14;15)}}$			
			Power	$b(\hat{\phi})$	$\sigma^2_{\hat{\phi}}$	%	Power	$b(\hat{\phi})$	$\sigma^2_{\hat{\phi}}$	%	Power	$b(\hat{\phi})$	$\sigma^2_{\hat{\phi}}$	%
600	1	.10	.986 (.986)	.0018	.010	.950	.988 (.988)	.0075	.010	.949	.989 (.988)	.0083	.010	.949
		-.10	.988 (.987)	.0020	.010	.946	.986 (.985)	-.0035	.010	.945	.985 (.985)	-.0042	.010	.945
	4	.10	.987 (.987)	.0019	.010	.947	.992 (.992)	.0293	.010	.945	.992 (.991)	.0317	.010	.941
		-.10	.988 (.988)	.0008	.010	.954	.976 (.974)	-.0251	.010	.941	.970 (.968)	-.0273	.010	.939

Note: N denotes total sample size. Relative Difference refers the value of noninvariant indicator intercept(s) in Group 2 minus that in Group 1. $\Delta\kappa$ denotes the latent mean difference of the second-order factor. Powers outside the parentheses were obtained using the chi-square difference test, and those inside the parentheses were obtained via the Wald test for the latent mean difference; both were based on $\alpha = .05$. $\hat{\phi}$ denotes the estimated latent mean difference of the second-order factor, so $b(\hat{\phi})$ is the bias and $\sigma^2_{\hat{\phi}}$ is the variance of this parameter.

Table A20

Power (Δκ = .40), Bias, Estimated Variance, and Coverage Rate of the Estimated Latent Mean Difference of the Second-Order Factor for the Exploration of the Influence of Partial Invariance of Intercepts of Both First-Order Factors and Indicators

N	The first-order factor that has noninvariant parameters	Relative differences in noninvariant intercepts of the first-order factor	Freely estimating noninvariant intercepts				Constraining noninvariant intercepts to be equal			
			Power	$b(\hat{\phi})$	$\sigma^2_{\hat{\phi}}$	%	Power	$b(\hat{\phi})$	$\sigma^2_{\hat{\phi}}$	%
300	η1	.10	.825 (.817)	.0020	.021	.949	.859 (.852)	.0075	.020	.950
		-.10	.817 (.805)	-.0009	.021	.948	.757 (.748)	-.0414	.020	.928
	η2	.10	.831 (.820)	.0043	.021	.950	.868 (.860)	.0169	.020	.951
		-.10	.827 (.817)	.0012	.021	.945	.771 (.756)	-.0322	.021	.935
	η3	.10	.818 (.809)	.0011	.022	.945	.854 (.847)	.0101	.021	.949
		-.10	.817 (.809)	-.0017	.021	.945	.752 (.740)	-.0389	.021	.931

Table A20 Continued

N	The first-order factor that has noninvariant parameters	Relative differences in noninvariant intercepts of the first-order factor	Freely estimating noninvariant intercepts				Constraining noninvariant intercepts to be equal			
			Power	$b(\hat{\phi})$	$\sigma^2_{\hat{\phi}}$	%	Power	$b(\hat{\phi})$	$\sigma^2_{\hat{\phi}}$	%
600	η1	.10	.982 (.981)	-.0002	.011	.945	.989 (.989)	.0052	.010	.950
		-.10	.981 (.980)	.0003	.011	.945	.961 (.960)	-.0405	.010	.919
	η2	.10	.985 (.984)	.0000	.011	.950	.991 (.991)	.0122	.010	.946
		-.10	.984 (.984)	.0009	.011	.949	.967 (.966)	-.0320	.010	.929
	η3	.10	.982 (.981)	.0021	.010	.948	.991 (.990)	.0118	.010	.950
		-.10	.980 (.979)	-.0006	.011	.945	.960 (.959)	-.0378	.011	.921

Note: N denotes total sample size. η1 is the reference first-order factor, η2 is the first-order factor having noninvariant intercepts, and η3 is the first-order factor having invariant intercepts. Δκ denotes the latent mean difference of the second-order factor. Powers outside the parentheses were obtained using the chi-square difference test, and those inside the parentheses were obtained via the Wald test for the latent mean difference; both were based on α = .05. $\hat{\phi}$ denotes the estimated latent mean difference of the second-order factor, so $b(\hat{\phi})$ is the bias and $\sigma^2_{\hat{\phi}}$ is the variance of this parameter.

Table A21

Mean of RMSEA / SRMR for the Baseline Conditions to Explore the Influence of Partial Loading Invariance

Freely estimated parameters	Sample Size	*Δκ = .00*		*Δκ = .20*		*Δκ = .40*	
		No zero constraint on the mean of ξ in G2	Constraining the mean of ξ in G2 to be zero	No zero constraint on the mean of ξ in G2	Constraining the mean of ξ in G2 to be zero	No zero constraint on the mean of ξ in G2	Constraining the mean of ξ in G2 to be zero
None	300	.016 / .077	.016 / .077	.016 / .080	.017 / .079	.017 / .089	.022 / .084
	600	.009 / .054	.009 / .054	.009 / .059	.011 / .056	.009 / .070	.016 / .063
$\gamma_{2,1}$	300	.016 / .078	.016 / .078	.016 / .081	.017 / .080	.017 / .090	.021 / .085
	600	.009 / .055	.009 / .055	.009 / .059	.011 / .057	.009 / .070	.016 / .064

Table A21 Continued

Freely estimated parameters	Sample Size	*Δκ = .00*		*Δκ = .20*		*Δκ = .40*	
		No zero constraint on the mean of ξ in G2	Constraining the mean of ξ in G2 to be zero	No zero constraint on the mean of ξ in G2	Constraining the mean of ξ in G2 to be zero	No zero constraint on the mean of ξ in G2	Constraining the mean of ξ in G2 to be zero
$\gamma_{3,1}$	300	.016 / .078	.016 / .078	.016 / .081	.017 / .080	.017 / .090	.021 / .085
	600	009 / .055	009 / .055	.009 / .059	.011 / .057	.009 / .070	.016 / .064
$\boldsymbol{\lambda_{6,2}}$	300	.016 / .078	.016 / .077	.016 / .081	.018 / .079	.017 / .090	.022 / .085
	600	.009 / .055	.009 / .054	.009 / .059	.011 / .057	.009 / .070	.016 / .064

Table A21 Continued

Freely estimated parameters	Sample Size	$\Delta\kappa$ = .00		$\Delta\kappa$ = .20		$\Delta\kappa$ = .40	
		No zero constraint on the mean of ξ in G2	Constraining the mean of ξ in G2 to be zero	No zero constraint on the mean of ξ in G2	Constraining the mean of ξ in G2 to be zero	No zero constraint on the mean of ξ in G2	Constraining the mean of ξ in G2 to be zero
$\boldsymbol{\lambda}_{7,2}$	300	.016 / .078	.016 / .077	.016 / .081	.018 / .079	.017 / .090	.022 / .085
	600	.009 / .055	.009 / .054	.009 / .059	.011 / .057	.009 / .070	.016 / .064
$\boldsymbol{\lambda}_{(6,2;7,2;10,3;11;3)}$	300	.016 / .079	.016 / .078	.016 / .082	.018 / .080	.017 / .091	.022 / .086
	600	.009 / .056	.009 / .055	.009 / .060	.011 / .057	.009 / .071	.016 / .064

Table A21 Continued

Freely estimated parameters	Sample Size	$\Delta\kappa$ = .00		$\Delta\kappa$ = .20		$\Delta\kappa$ = .40	
		No zero constraint on the mean of ξ in G2	Constraining the mean of ξ in G2 to be zero	No zero constraint on the mean of ξ in G2	Constraining the mean of ξ in G2 to be zero	No zero constraint on the mean of ξ in G2	Constraining the mean of ξ in G2 to be zero
$\boldsymbol{\lambda}_{(\mathbf{2,1};\mathbf{3,1};\mathbf{14,4};\mathbf{15,4})}$	300	.016 / .079	.016 / .078	.016 / .082	.018 / .080	.017 / .091	.022 / .086
	600	.009 / .056	.009 / .055	.009 / .060	.011 / .057	.009 / .071	.016 / .064
$\gamma_{2,1}$ and $\boldsymbol{\lambda}_{(2,1;3,1)}$	300	.016 / .079	.016 / .079	.016 / .082	.018 / .081	.017 / .091	.021 / .086
	600	.009 / .056	.009 / .055	.009 / .060	.011 / .058	.009 / .071	.016 / .064

Table A21 Continued

Freely estimated parameters	Sample Size	$\Delta\kappa$ = .00		$\Delta\kappa$ = .20		$\Delta\kappa$ = .40	
		No zero constraint on the mean of ξ in G2	Constraining the mean of ξ in G2 to be zero	No zero constraint on the mean of ξ in G2	Constraining the mean of ξ in G2 to be zero	No zero constraint on the mean of ξ in G2	Constraining the mean of ξ in G2 to be zero
$\gamma_{2,1}$ and	300	.016 / .079	.016 / .078	.016 / .082	.018 / .080	.017 / .091	.022 / .086
$\boldsymbol{\lambda}_{(6,2;7,2)}$	600	.009 / .056	.009 / .055	.009 / .060	.011 / .058	.009 / .071	.016 / .064
$\gamma_{2,1}$ and	300	.016 / .079	.016 / .079	.016 / .082	.017 / .081	.017 / .091	.021 / .086
$\boldsymbol{\lambda}_{(10,3;11,3)}$	600	.009 / .056	.009 / .055	.009 / .060	.011 / .058	.009 / .071	.016 / .064

Table A22

Mean of RMSEA / SRMR for the Baseline Conditions to Explore the Influence of Partial Intercept Invariance

Freely estimated parameters	Sample Size	$\Delta\kappa$ = .00		$\Delta\kappa$ = .20		$\Delta\kappa$ = .40	
		No zero constraint on the mean of ξ in G2	Constraining the mean of ξ in G2 to be zero	No zero constraint on the mean of ξ in G2	Constraining the mean of ξ in G2 to be zero	No zero constraint on the mean of ξ in G2	Constraining the mean of ξ in G2 to be zero
None	300	.017 / .077	.016 / .077	.017 / .081	.018 / .079	.017 / .089	.022 / .084
	600	.009 / .054	.009 / .054	.009 / .059	.011 / .056	.009 / .070	.016 / .063
α_2	300	.017 / .078	.017 / .077	.017 / .081	.018 / .079	.017 / .089	.021 / .084
	600	.009 / .054	.009 / .054	.009 / .059	.011 / .057	.009 / .070	.016 / .063

Table A22 Continued

Freely estimated parameters	Sample Size	*Δκ = .00*		*Δκ = .20*		*Δκ = .40*	
		No zero constraint on the mean of ξ in G2	Constraining the mean of ξ in G2 to be zero	No zero constraint on the mean of ξ in G2	Constraining the mean of ξ in G2 to be zero	No zero constraint on the mean of ξ in G2	Constraining the mean of ξ in G2 to be zero
α_3	300	.017 / .078	.017 / .077	.017 / .081	.018 / .079	.017 / .089	.021 / .084
	600	.009 / .054	.009 / .054	.009 / .059	.011 / .057	.009 / .070	.016 / .063
$\boldsymbol{\nu_6}$	300	.017 / .077	.017 / .077	.017 / .081	.018 / .079	.017 / .089	.022 / .084
	600	.009 / .054	.009 / .054	.009 / .059	.011 / .056	.009 / .070	.016 / .063

Table A22 Continued

Freely estimated parameters	Sample Size	*Δκ = .00*		*Δκ = .20*		*Δκ = .40*	
		No zero constraint on the mean of ξ in G2	Constraining the mean of ξ in G2 to be zero	No zero constraint on the mean of ξ in G2	Constraining the mean of ξ in G2 to be zero	No zero constraint on the mean of ξ in G2	Constraining the mean of ξ in G2 to be zero
$\boldsymbol{\nu_7}$	300	.017 / .077	.017 / .077	.017 / .081	.018 / .079	.017 / .089	.022 / .084
	600	.009 / .054	.009 / .054	.009 / .059	.011 / .056	.009 / .070	.016 / .063
$\boldsymbol{\nu_{(6;7;10;11)}}$	300	.017 / .078	.017 / .077	.017 / .081	.018 / .079	.017 / .089	.022 / .085
	600	.009 / .055	.009 / .054	.009 / .059	.011 / .057	.009 / .070	.016 / .063

Table A22 Continued

Freely estimated parameters	Sample Size	*Δκ = .00*		*Δκ = .20*		*Δκ = .40*	
		No zero constraint on the mean of ξ in G2	Constraining the mean of ξ in G2 to be zero	No zero constraint on the mean of ξ in G2	Constraining the mean of ξ in G2 to be zero	No zero constraint on the mean of ξ in G2	Constraining the mean of ξ in G2 to be zero
$\boldsymbol{\nu}_{(2;3;14;15)}$	300	.017 / .078	.017 / .077	.017 / .081	.018 / .079	.017 / .089	.022 / .085
	600	.009 / .055	.009 / .054	.009 / .059	.011 / .057	.009 / .070	.016 / .063
α_2 and $\boldsymbol{\nu}_{(2;3)}$	300	.017 / .078	.017 / .077	.017 / .081	.018 / .079	.017 / .089	.021 / .085
	600	.009 / .055	.009 / .054	.009 / .059	.011 / .057	.009 / .070	.016 / .063

Table A22 Continued

Freely estimated parameters	Sample Size	$\Delta\kappa$ = .00		$\Delta\kappa$ = .20		$\Delta\kappa$ = .40	
		No zero constraint on the mean of ξ in G2	Constraining the mean of ξ in G2 to be zero	No zero constraint on the mean of ξ in G2	Constraining the mean of ξ in G2 to be zero	No zero constraint on the mean of ξ in G2	Constraining the mean of ξ in G2 to be zero
α_2 and $\boldsymbol{\nu}_{(6;7)}$	300	.017 / .078	.017 / .077	.017 / .081	.018 / .079	.017 / .089	.021 / .085
	600	.009 / .055	.009 / .054	.009 / .059	.011 / .057	.009 / .070	.016 / .063
α_2 and $\boldsymbol{\nu}_{(10;11)}$	300	.017 / .078	.017 / .077	.017 / .081	.018 / .079	.017 / .089	.021 / .085
	600	.009 / .055	.009 / .054	.009 / .059	.011 / .057	.009 / .070	.016 / .063

Table A23

Mean of RMSEA / SRMR for the Exploration of the Influence of Partial Invariance of Second-Order Factor Loading (Δκ = .00)

N	$\gamma_{2,1(G2)} - \gamma_{2,1(G1)}$	$\gamma_{2,1}$ is freely estimated		$\gamma_{2,1}$ is constrained to be equal		$\gamma_{2,1}$ is constrained to be equal but $\gamma_{3,1}$ is freely estimated	
		No zero constraint on the mean of ξ in G2	Constraining the mean of ξ in G2 to be zero	No zero constraint on the mean of ξ in G2	Constraining the mean of ξ in G2 to be zero	No zero constraint on the mean of ξ in G2	Constraining the mean of ξ in G2 to be zero
300	.20	.017 / .082	.017 / .082	.018 / .081	.018 / .081	.018 / .082	.018 / .081
	-.20	.017 / .083	.016 / .083	.017 / .081	.017 / .080	.017 / .081	.017 / .081
600	.20	.009 / .061	.009 / .061	.011 / .059	.011 / .059	.011 / .060	.011 / .060
	-.20	.009 / .061	.009 / .061	.011 / .059	.011 / .059	.011 / .059	.011 / .059

Note: N denotes total sample size. *Δκ denotes the latent mean difference of the second-order factor*. *N* denotes total sample size. $\gamma_{2,1}$ is the second-order loading relating the 2nd first-order factor to the second-order factor and G2 (G1) denotes group membership. ξ denotes the second-order factor. For each condition, the first value refers RMSEA and the second one refers SRMR.

Table A24

Mean of RMSEA / SRMR for the Exploration of the Influence of Partial Invariance of First-Order Factor Loading (Δκ = .00)

Sample size	The number of noninvariant first-order loadings	Relative differences in $\boldsymbol{\lambda}_{6,2(6,2;7,2;10,3;11;3)}$	Freely estimating $\boldsymbol{\lambda}_{6,2(6,2;7,2;10,3;11;3)}$		Constraining $\boldsymbol{\lambda}_{6,2(6,2;7,2;10,3;11;3)}$ to be equal		Constraining $\boldsymbol{\lambda}_{6,2(6,2;7,2;10,3;11;3)}$ to be equal but Freely estimating $\boldsymbol{\lambda}_{7,2(2,1;3,1;14,4;15,4)}$	
			Freely estimating the mean of ξ in G2	Constraining the mean of ξ in G2 to be zero	Freely estimating the mean of ξ in G2	Constraining the mean of ξ in G2 to be zero	Freely estimating the mean of ξ in G2	Constraining the mean of ξ in G2 to be zero
300	1	.20	.017 / .080	.017 / .079	.018 / .079	.018 / .079	.018 / .080	.018 / .079
		-.20	.016 / .080	.016 / .080	.018 / .079	.018 / .079	.018 / .080	.018 / .079
	4	.20	.017 / .089	.017 / .088	.023 / .087	.022 / .086	.023 / .088	.023 / .088
		-.20	.016 / .089	.016 / .088	.022 / .085	.022 / .084	.022 / .086	.022 / .085

Table A24 Continued

Sample size	The number of noninvariant first-order loadings	Relative differences in $\boldsymbol{\lambda}_{6,2(6,2;7,2;10,3;11;3)}$	Freely estimating $\boldsymbol{\lambda}_{6,2(6,2;7,2;10,3;11;3)}$		Constraining $\boldsymbol{\lambda}_{6,2(6,2;7,2;10,3;11;3)}$ to be equal		Constraining $\boldsymbol{\lambda}_{6,2(6,2;7,2;10,3;11;3)}$ to be equal but Freely estimating $\boldsymbol{\lambda}_{7,2(2,1;3,1;14,4;15,4)}$	
			Freely estimating the mean of ξ in G2	Constraining the mean of ξ in G2 to be zero	Freely estimating the mean of ξ in G2	Constraining the mean of ξ in G2 to be zero	Freely estimating the mean of ξ in G2	Constraining the mean of ξ in G2 to be zero
600	1	.20	.010 / .057	.009 / .057	.012 / .057	.012 / .056	.012 / .057	.012 / .057
		-.20	.009 / .058	.009 / .057	.011 / .056	.011 / .056	.011 / .057	.011 / .056
	4	.20	.009 / .069	.009 / .069	.017 / .067	.017 / .067	.018 / .068	.018 / .068
		-.20	.009 / .068	.009 / .068	.017 / .063	.017 / .063	.017 / .065	.017 / .064

Note: Δκ denotes the latent mean difference of the second-order factor. The subscript refers the group number. ξ denotes the second-order factor. G2 refers Group 2, which was the reference group. For each condition, the first value refers RMSEA and the second one refers SRMR.

Table A25

Mean of RMSEA / SRMR for the Exploration of the Influence of Partial Invariance of Both First- and Second-Order Loadings ($\Delta\kappa$ = .00)

Sample size	The first-order factor that has noninvariant parameters	Relative differences in noninvariant second-order loadings	Freely estimating noninvariant loadings		Constraining noninvariant loadings to be equal	
			Freely estimating the mean of ξ in G2	Constraining the mean of ξ in G2 to be zero	Freely estimating the mean of ξ in G2	Constraining the mean of ξ in G2 to be zero
300	η1	.20	.017 / .087	.017 / .086	.020 / .083	.020 / .083
		-.20	.017 / .089	.017 / .088	.020 / .085	.020 / .084
	η2	.20	.017 / .084	.017 / .084	.020 / .082	.020 / .081
		-.20	.017 / .089	.017 / .089	.020 / .085	.020 / .084
	η3	.20	.017 / .087	.017 / .087	.020 / .084	.020 / .083
		-.20	.017 / .089	.017 / .089	.020 / .085	.020 / .084

Table A25 Continued

Sample size	The first-order factor that has noninvariant parameters	Relative differences in noninvariant second-order loadings	Freely estimating noninvariant loadings		Constraining noninvariant loadings to be equal	
			Freely estimating the mean of ξ in G2	Constraining the mean of ξ in G2 to be zero	Freely estimating the mean of ξ in G2	Constraining the mean of ξ in G2 to be zero
600	η1	.20	.009 / .067	.009 / .066	.015 / .063	.015 / .062
		-.20	.009 / .069	.009 / .068	.014 / .064	.014 / .063
	η2	.20	.009 / .063	.009 / .063	.014 / .061	.014 / .060
		-.20	.009 / .069	.009 / .069	.015 / .064	.015 / .063
	η3	.20	.009 / .067	.009 / .066	.015 / .063	.015 / .062
		-.20	.009 / .068	.009 / .068	.014 / .064	.014 / .063

Note: Δκ denotes the latent mean difference of the second-order factor. The subscript refers the group number. ξ denotes the second-order factor. G2 refers Group 2, which was the reference group. For each condition, the first value refers RMSEA and the second one refers SRMR.

Table A26

Mean of RMSEA / SRMR for the Exploration of the Influence of Partial Invariance of Second-Order Factor Loading (Δκ = .20)

N	$\gamma_{2,1(G2)}$ - $\gamma_{2,1(G1)}$	$\gamma_{2,1}$ is freely estimated		$\gamma_{2,1}$ is constrained to be equal		$\gamma_{2,1}$ is constrained to be equal but $\gamma_{3,1}$ is freely estimated	
		No zero constraint on the mean of ξ in G2	Constraining the mean of ξ in G2 to be zero	No zero constraint on the mean of ξ in G2	Constraining the mean of ξ in G2 to be zero	No zero constraint on the mean of ξ in G2	Constraining the mean of ξ in G2 to be zero
300	.20	.016 / .086	.018 / .084	.017 / .084	.019 / .083	.017 / .085	.019 / .083
	-.20	.017 / .086	.018 / .084	.018 / .083	.019 / .082	.018 / .084	.019 / .083
600	.20	.009 / .065	.011 / .063	.011 / .064	.013 / .062	.010 / .064	.012 / .063
	-.20	.009 / .065	.011 / .063	.011 / .062	.012 / .060	.011 / .063	.012 / .061

Note: N denotes total sample size. *Δκ denotes the latent mean difference of the second-order factor. N* denotes total sample size. $\gamma_{2,1}$ is the second-order loading relating the 2nd first-order factor to the second-order factor and G2 (G1) denotes group membership. ξ denotes the second-order factor. For each condition, the first value refers RMSEA and the second one refers SRMR.

Table A27

Mean of RMSEA / SRMR for the Exploration of the Influence of Partial Invariance of First-Order Factor Loading (Δκ = .20)

Sample size	The number of noninvariant first-order loadings	Relative differences in $\boldsymbol{\lambda}_{6,2(6,2;7,2;10,3;11;3)}$	Freely estimating $\boldsymbol{\lambda}_{6,2(6,2;7,2;10,3;11;3)}$		Constraining $\boldsymbol{\lambda}_{6,2(6,2;7,2;10,3;11;3)}$ to be equal		Constraining $\boldsymbol{\lambda}_{6,2(6,2;7,2;10,3;11;3)}$ to be equal but Freely estimating $\boldsymbol{\lambda}_{7,2(2,1;3,1;14,4;15,4)}$	
			Freely estimating the mean of ξ in G2	Constraining the mean of ξ in G2 to be zero	Freely estimating the mean of ξ in G2	Constraining the mean of ξ in G2 to be zero	Freely estimating the mean of ξ in G2	Constraining the mean of ξ in G2 to be zero
300	1	.20	.016 / .083	.018 / .081	.018 / .082	.020 / .081	.018 / .083	.019 / .081
		-.20	.017 / .083	.018 / .082	.018 / .082	.019 / .081	.018 / .083	.019 / .081
	4	.20	.017 / .092	.018 / .091	.022 / .090	.024 / .089	.023 / .092	.024/ .090
		-.20	.016 / .091	.018 / .089	.022 / .087	.023 / .085	.022 / .089	.023 / .087

Table A27 Continued

Sample size	The number of noninvariant first-order loadings	Relative differences in $\boldsymbol{\lambda}_{6,2(6,2;7,2;10,3;11;3)}$	Freely estimating $\boldsymbol{\lambda}_{6,2(6,2;7,2;10,3;11;3)}$		Constraining $\boldsymbol{\lambda}_{6,2(6,2;7,2;10,3;11;3)}$ to be equal		Constraining $\boldsymbol{\lambda}_{6,2(6,2;7,2;10,3;11;3)}$ to be equal but Freely estimating $\boldsymbol{\lambda}_{7,2(2,1;3,1;14,4;15,4)}$	
			Freely estimating the mean of ξ in G2	Constraining the mean of ξ in G2 to be zero	Freely estimating the mean of ξ in G2	Constraining the mean of ξ in G2 to be zero	Freely estimating the mean of ξ in G2	Constraining the mean of ξ in G2 to be zero
600	1	.20	.009 / .062	.011 / .060	.012 / .061	.014 / .059	.012 / .061	.013 / .059
		-.20	.009 / .062	.011 / .059	.011 / .060	.013 / .058	.011 / .061	.013 / .059
	4	.20	.009 / .073	.011 / .072	.017 / .072	.019 / .070	.018 / .072	.019 / .071
		-.20	.009 / .071	.011 / .069	.017 / .067	.018 / .065	.017 / .068	.019 / .066

Note: Δκ denotes the latent mean difference of the second-order factor. The subscript refers the group number. ξ denotes the second-order factor. G2 refers Group 2, which was the reference group. For each condition, the first value refers RMSEA and the second one refers SRMR.

Table A28

Mean of RMSEA / SRMR for the Exploration of the Influence of Partial Invariance of Both First- and Second-Order Loadings (Δκ = .20)

Sample size	The first-order factor that has noninvariant parameters	Relative differences in noninvariant second-order loadings	Freely estimating noninvariant loadings		Constraining noninvariant loadings to be equal	
			Freely estimating the mean of ξ in G2	Constraining the mean of ξ in G2 to be zero	Freely estimating the mean of ξ in G2	Constraining the mean of ξ in G2 to be zero
300	η1	.20	.017 / .090	.018 / .089	.021 / .087	.022 / .085
		-.20	.017 / .091	.018 / .090	.020 / .087	.021 / .085
	η2	.20	.017 / .088	.018 / .086	.020 / .085	.021 / .084
		-.20	.017 / .092	.018 / .090	.021 / .087	.022 / .085
	η3	.20	.017 / .090	.018 / .089	.021 / .087	.022 / .085
		-.20	.016 / .091	.018 / .090	.020 / .087	.021 / .085

Table A28 Continued

Sample size	The first-order factor that has noninvariant parameters	Relative differences in noninvariant second-order loadings	Freely estimating noninvariant loadings		Constraining noninvariant loadings to be equal	
			Freely estimating the mean of ξ in G2	Constraining the mean of ξ in G2 to be zero	Freely estimating the mean of ξ in G2	Constraining the mean of ξ in G2 to be zero
600	η1	.20	.009 / .070	.011 / .069	.015 / .067	.017 / .065
		-.20	.009 / .072	.011 / .070	.014 / .067	.016 / .065
	η2	.20	.009 / .067	.011 / .065	.014 / .065	.016 / .063
		-.20	.009 / .072	.011 / .070	.015 / .067	.017 / .065
	η3	.20	.009 / .070	.011 / .068	.015 / .067	.017 / .065
		-.20	.009 / .072	.011 / .070	.014 / .067	.016 / .065

Note: Δκ denotes the latent mean difference of the second-order factor. The subscript refers the group number. ξ denotes the second-order factor. G2 refers Group 2, which was the reference group. For each condition, the first value refers RMSEA and the second one refers SRMR.

Table A29

Mean of RMSEA / SRMR for the Exploration of the Influence of Partial Invariance of Second-Order Factor Loading (Δκ = .40)

N	$\gamma_{2,1(G2)}$ - $\gamma_{2,1(G1)}$	$\gamma_{2,1}$ is freely estimated		$\gamma_{2,1}$ is constrained to be equal		$\gamma_{2,1}$ is constrained to be equal but $\gamma_{3,1}$ is freely estimated	
		No zero constraint on the mean of ξ in G2	Constraining the mean of ξ in G2 to be zero	No zero constraint on the mean of ξ in G2	Constraining the mean of ξ in G2 to be zero	No zero constraint on the mean of ξ in G2	Constraining the mean of ξ in G2 to be zero
300	.20	.017 / .095	.021 / .090	.018 / .094	.023 / .089	.018 / .094	.023 / .090
	-.20	.017 / .093	.021 / .089	.018 / .091	.022 / .086	.018 / .092	.022 / .087
600	.20	.009 / .077	.016 / .071	.011 / .076	.018 / .070	.011 / .076	.018 / .071
	-.20	.009 / .074	.016 / .068	.011 / .072	.017 / .066	.011 / .073	.017 / .066

Note: N denotes total sample size. *Δκ denotes the latent mean difference of the second-order factor*. *N* denotes total sample size. $\gamma_{2,1}$ is the second-order loading relating the 2nd first-order factor to the second-order factor and G2 (G1) denotes group membership. ξ denotes the second-order factor. For each condition, the first value refers RMSEA and the second one refers SRMR.

Table A30

Mean of RMSEA / SRMR for the Exploration of the Influence of Partial Invariance of First-Order Factor Loading ($\Delta\kappa$ = .40)

Sample size	The number of noninvariant first-order loadings	Relative differences in $\boldsymbol{\lambda}_{6,2(6,2;7,2;10,3;11;3)}$	Freely estimating $\boldsymbol{\lambda}_{6,2(6,2;7,2;10,3;11;3)}$		Constraining $\boldsymbol{\lambda}_{6,2(6,2;7,2;10,3;11;3)}$ to be equal		Constraining $\boldsymbol{\lambda}_{6,2(6,2;7,2;10,3;11;3)}$ to be equal but Freely estimating $\boldsymbol{\lambda}_{7,2(2,1;3,1;14,4;15,4)}$	
			Freely estimating the mean of ξ in G2	Constraining the mean of ξ in G2 to be zero	Freely estimating the mean of ξ in G2	Constraining the mean of ξ in G2 to be zero	Freely estimating the mean of ξ in G2	Constraining the mean of ξ in G2 to be zero
300	1	.20	.017 / .092	.022 / .087	.019 / .091	.024 / .086	.019 / .091	.024 / .087
		-.20	.017 / .092	.022 / .087	.018 / .091	.023 / .086	.019 / .091	.023 / .086
	4	.20	.017 / .101	.022 / .097	.022 / .099	.027 / .095	.023 / .100	.028 / .096
		-.20	.017 / .099	.022 / .094	.022 / .095	.027 / .090	.022 / .096	.027 / .091

Table A30 Continued

Sample size	The number of noninvariant first-order loadings	Relative differences in $\boldsymbol{\lambda}_{6,2(6,2;7,2;10,3;11;3)}$	Freely estimating $\boldsymbol{\lambda}_{6,2(6,2;7,2;10,3;11;3)}$		Constraining $\boldsymbol{\lambda}_{6,2(6,2;7,2;10,3;11;3)}$ to be equal		Constraining $\boldsymbol{\lambda}_{6,2(6,2;7,2;10,3;11;3)}$ to be equal but Freely estimating $\boldsymbol{\lambda}_{7,2(2,1;3,1;14,4;15,4)}$	
			Freely estimating the mean of ξ in G2	Constraining the mean of ξ in G2 to be zero	Freely estimating the mean of ξ in G2	Constraining the mean of ξ in G2 to be zero	Freely estimating the mean of ξ in G2	Constraining the mean of ξ in G2 to be zero
600	1	.20	.009 / .073	.016 / .066	.012 / .072	.019 / .066	.012 / .072	.019 / .066
		-.20	.009 / .072	.016 / .066	.012 / .071	.019 / .065	.012 / .072	.019 / .065
	4	.20	.010 / .084	.017 / .079	.018 / .082	.024 / .077	.018 / .083	.024 / .078
		-.20	.009 / .080	.016 / .074	.017 / .076	.023 / .070	.017 / .077	.023 / .071

Note: Δκ denotes the latent mean difference of the second-order factor. The subscript refers the group number. ξ denotes the second-order factor. G2 refers Group 2, which was the reference group. For each condition, the first value refers RMSEA and the second one refers SRMR.

Table A31

Mean of RMSEA / SRMR for the Exploration of the Influence of Partial Invariance of Both First- and Second-Order Loadings ($\Delta\kappa$ = .40)

Sample size	The first-order factor that has noninvariant parameters	Relative differences in noninvariant second-order loadings	Freely estimating noninvariant loadings		Constraining noninvariant loadings to be equal	
			Freely estimating the mean of ξ in G2	Constraining the mean of ξ in G2 to be zero	Freely estimating the mean of ξ in G2	Constraining the mean of ξ in G2 to be zero
300	η1	.20	.017 / .099	.022 / .094	.021 / .096	.026 / .091
		-.20	.016 / .098	.021 / .093	.020 / .094	.024 / .089
	η2	.20	.017 / .096	.021 / .092	.020 / .094	.025 / .090
		-.20	.017 / .099	.021 / .094	.020 / .094	.025 / .089
	η3	.20	.016 / .099	.021 / .094	.020 / .096	.026 / .091
		-.20	.016 / .098	.021 / .094	.020 / .094	.025 / .089

Table A31 Continued

Sample size	The first-order factor that has noninvariant parameters	Relative differences in noninvariant second-order loadings	Freely estimating noninvariant loadings		Constraining noninvariant loadings to be equal	
			Freely estimating the mean of ξ in G2	Constraining the mean of ξ in G2 to be zero	Freely estimating the mean of ξ in G2	Constraining the mean of ξ in G2 to be zero
600	η1	.20	.009 / .081	.016 / .075	.015 / .078	.022 / .072
		-.20	.010 / .079	.016 / .074	.015 / .075	.021 / .069
	η2	.20	.009 / .078	.016 / .072	.014 / .076	.021 / .070
		-.20	.009 / .081	.016 / .075	.015 / .076	.021 / .070
	η3	.20	.010 / .081	.016 / .075	.015 / .078	.022 / .072
		-.20	.009 / .080	.016 / .074	.015 / .076	.021 / .070

Note: Δκ denotes the latent mean difference of the second-order factor. The subscript refers the group number. ξ denotes the second-order factor. G2 refers Group 2, which was the reference group. For each condition, the first value refers RMSEA and the second one refers SRMR.

Table A32

Mean of RMSEA / SRMR for the Exploration of the Influence of Partial Invariance of Intercepts of First-Order Factors (Δκ = .00)

N	$\alpha_{2(G2)}$ -$\alpha_{2(G1)}$	α_2 is freely estimated		α_2 is constrained to be equal		α_2 is constrained to be equal but α_3 is freely estimated	
		No zero constraint on the mean of ξ in G2	Constraining the mean of ξ in G2 to be zero	No zero constraint on the mean of ξ in G2	Constraining the mean of ξ in G2 to be zero	No zero constraint on the mean of ξ in G2	Constraining the mean of ξ in G2 to be zero
300	.10	.016 / .078	.016 / .077	.017 / .078	.017 / .077	.017 / .078	.017 / .077
	-.10	.016 / .078	.016 / .077	.017 / .078	.017 / .077	.017 / .078	.017 / .077
600	.10	.009 / .055	.009 / .055	.010 / .055	.010 / .054	.010 / .055	.010 / .054
	-.10	.009 / .055	.009 / .055	.010 / .055	.010 / .054	.010 / .055	.010 / .054

Note: N denotes total sample size. *Δκ denotes the latent mean difference of the second-order factor*. *N* denotes total sample size. α_2 is the intercept of the first-order factor and G2 (G1) denotes group membership. ξ denotes the second-order factor. For each condition, the first value refers RMSEA and the second one refers SRMR.

Table A33

Mean of RMSEA / SRMR for the Exploration of the Influence of Partial Invariance of Indicator Intercepts (Δκ = .00)

Sample size	The number of noninvariant indicator Intercepts	Relative differences in $\boldsymbol{\nu}_{\mathbf{6(6;7;10;11)}}$	Freely estimating $\boldsymbol{\nu}_{\mathbf{6(6;7;10;11)}}$		Constraining $\boldsymbol{\nu}_{\mathbf{6(6;7;10;11)}}$ to be equal		Constraining $\boldsymbol{\nu}_{\mathbf{6(6;7;10;11)}}$ to be equal but Freely estimating $\boldsymbol{\nu}_{\mathbf{7(2;3;14;15)}}$	
			Freely estimating the mean of ξ in G2	Constraining the mean of ξ in G2 to be zero	Freely estimating the mean of ξ in G2	Constraining the mean of ξ in G2 to be zero	Freely estimating the mean of ξ in G2	Constraining the mean of ξ in G2 to be zero
300	1	.10	.017 / .078	.017 / .077	.017 / .078	.017 / .077	.017 / .078	.017 / .077
		-.10	.017 / .078	.016 / .077	.017 / .078	.017 / .077	.017 / .078	.017 / .077
	4	.10	.017 / .079	.017 / .078	.019 / .078	.019 / .077	.019 / .078	.019 / .078
		-.10	.017 / .079	.017 / .078	.019 / .078	.019 / .077	.019 / .078	.019 / .078

Table A33 Continued

Sample size	The number of noninvariant indicator Intercepts	Relative differences in $\boldsymbol{\nu}_{\mathbf{6(6;7;10;11)}}$	Freely estimating $\boldsymbol{\nu}_{\mathbf{6(6;7;10;11)}}$		Constraining $\boldsymbol{\nu}_{\mathbf{6(6;7;10;11)}}$ to be equal		Constraining $\boldsymbol{\nu}_{\mathbf{6(6;7;10;11)}}$ to be equal but Freely estimating $\boldsymbol{\nu}_{\mathbf{7(2;3;14;15)}}$	
			Freely estimating the mean of ξ in G2	Constraining the mean of ξ in G2 to be zero	Freely estimating the mean of ξ in G2	Constraining the mean of ξ in G2 to be zero	Freely estimating the mean of ξ in G2	Constraining the mean of ξ in G2 to be zero
600	1	.10	.009 / .055	.009 / .054	.010 / .054	.010 / .054	.010 / .055	.010 / .054
		-.10	.009 / .055	.009 / .054	.010 / .054	.010 / .054	.010 / .055	.010 / .054
	4	.10	.009 / .056	.009 / .055	.012 / .055	.012 / .054	.012 / .055	.012 / .055
		-.10	.009 / .056	.009 / .055	.012 / .055	.012 / .054	.012 / .055	.012 / .055

Note: Δκ denotes the latent mean difference of the second-order factor. The subscript refers the group number. ξ denotes the second-order factor. G2 refers Group 2, which was the reference group. For each condition, the first value refers RMSEA and the second one refers SRMR.

Table A34

Mean of RMSEA / SRMR for the Exploration of the Influence of Partial Invariance of Intercepts of Both Levels (Δκ = .00)

Sample size	The first-order factor that has noninvariant parameters	Relative differences in noninvariant intercepts of the first-order factor	Freely estimating noninvariant intercepts		Constraining noninvariant intercepts to be equal	
			Freely estimating the mean of ξ in G2	Constraining the mean of ξ in G2 to be zero	Freely estimating the mean of ξ in G2	Constraining the mean of ξ in G2 to be zero
300	η1	.10	.016 / .079	.016 / .078	.018 / .078	.018 / .077
		-.10	.016 / .078	.016 / .078	.018 / .078	.018 / .077
	η2	.10	.017 / .078	.017 / .077	.018 / .077	.018 / .077
		-.10	.017 / .079	.017 / .078	.019 / .078	.019 / .077
	η3	.10	.016 / .078	.016 / .078	.018 / .078	.018 / .077
		-.10	.017 / .079	.017 / .078	.018 / .078	.018 / .077

Table A34 Continued

Sample size	The first-order factor that has noninvariant parameters	Relative differences in noninvariant intercepts of the first-order factor	Freely estimating noninvariant intercepts		Constraining noninvariant intercepts to be equal	
			Freely estimating the mean of ξ in G2	Constraining the mean of ξ in G2 to be zero	Freely estimating the mean of ξ in G2	Constraining the mean of ξ in G2 to be zero
600	η1	.10	.010 / .056	.010 / .055	.012 / .055	.012 / .055
		-.10	.009 / .056	.009 / .055	.011 / .055	.011 / .054
	η2	.10	.009 / .055	.009 / .055	.011 / .055	.011 / .054
		-.10	.010 / .056	.010 / .056	.012 / .055	.012 / .055
	η3	.10	.009 / .056	.009 / .055	.011 / .055	.011 / .055
		-.10	.009 / .056	.009 / .055	.011 / .055	.011 / .054

Note: Δκ denotes the latent mean difference of the second-order factor. The subscript refers the group number. ξ denotes the second-order factor. G2 refers Group 2, which was the reference group. For each condition, the first value refers RMSEA and the second one refers SRMR.

Table A35

Mean of RMSEA / SRMR for the Exploration of the Influence of Partial Invariance of Intercepts of First-Order Factors (Δκ = .20)

N	$\alpha_{2(G2)}$ -$\alpha_{2(G1)}$	α_2 is freely estimated		α_2 is constrained to be equal		α_2 is constrained to be equal but α_3 is freely estimated	
		No zero constraint on the mean of ξ in G2	Constraining the mean of ξ in G2 to be zero	No zero constraint on the mean of ξ in G2	Constraining the mean of ξ in G2 to be zero	No zero constraint on the mean of ξ in G2	Constraining the mean of ξ in G2 to be zero
300	.10	.017 / .082	.018 / .080	.017 / .082	.019 / .080	.017 / .082	.018 / .080
	-.10	.017 / .080	.018 / .079	.017 / .080	.018 / .078	.017 / .080	.018 / .079
600	.10	.009 / .060	.011 / .058	.010 / .060	.012 / .057	.010 / .060	.012 / .058
	-.10	.009 / .058	.011 / .056	.010 / .058	.011 / .056	.010 / .058	.011 / .056

Note: N denotes total sample size. *Δκ denotes the latent mean difference of the second-order factor. N* denotes total sample size. α_2 is the intercept of the first-order factor and G2 (G1) denotes group membership. ξ denotes the second-order factor. For each condition, the first value refers RMSEA and the second one refers SRMR.

Table A36

Mean of RMSEA / SRMR for the Exploration of the Influence of Partial Invariance of Indicator Intercepts (Δκ = .20)

Sample size	The number of noninvariant indicator Intercepts	Relative differences in $\boldsymbol{\nu}_{\mathbf{6(6;7;10;11)}}$	Freely estimating $\boldsymbol{\nu}_{\mathbf{6(6;7;10;11)}}$		Constraining $\boldsymbol{\nu}_{\mathbf{6(6;7;10;11)}}$ to be equal		Constraining $\boldsymbol{\nu}_{\mathbf{6(6;7;10;11)}}$ to be equal but Freely estimating $\boldsymbol{\nu}_{\mathbf{7(2;3;14;15)}}$	
			Freely estimating the mean of ξ in G2	Constraining the mean of ξ in G2 to be zero	Freely estimating the mean of ξ in G2	Constraining the mean of ξ in G2 to be zero	Freely estimating the mean of ξ in G2	Constraining the mean of ξ in G2 to be zero
300	1	.10	.017 / .081	.018 / .079	.017 / .081	.019 / .079	.017 / .081	.019 / .079
		-.10	.017 / .081	.018 / .079	.017 / .080	.018 / .079	.017 / .081	.018 / .079
	4	.10	.017 / .083	.018 / .081	.019 / .082	.020 / .080	.019 / .083	.020 / .081
		-.10	.017 / .081	.018 / .079	.019 / .080	.019 / .079	.019 / .081	.020 / .079

Table A36 Continued

Sample size	The number of noninvariant indicator Intercepts	Relative differences in $\boldsymbol{\nu}_{\mathbf{6(6;7;10;11)}}$	Freely estimating $\boldsymbol{\nu}_{\mathbf{6(6;7;10;11)}}$		Constraining $\boldsymbol{\nu}_{\mathbf{6(6;7;10;11)}}$ to be equal		Constraining $\boldsymbol{\nu}_{\mathbf{6(6;7;10;11)}}$ to be equal but Freely estimating $\boldsymbol{\nu}_{\mathbf{7(2;3;14;15)}}$	
			Freely estimating the mean of ξ in G2	Constraining the mean of ξ in G2 to be zero	Freely estimating the mean of ξ in G2	Constraining the mean of ξ in G2 to be zero	Freely estimating the mean of ξ in G2	Constraining the mean of ξ in G2 to be zero
600	1	.10	.009 / .059	.011 / .057	.010 / .059	.012 / .057	.010 / .059	.012 / .057
		-.10	.009 / .059	.011 / .056	.010 / .058	.012 / .056	.010 / .058	.012 / .056
	4	.10	.009 / .062	.011 / .059	.012 / .061	.014 / .058	.012 / .061	.014 / .058
		-.10	.009 / .058	.011 / .057	.012 / .058	.014 / .056	.012 / .058	.014 / .056

Note: Δκ denotes the latent mean difference of the second-order factor. The subscript refers the group number. ξ denotes the second-order factor. G2 refers Group 2, which was the reference group. For each condition, the first value refers RMSEA and the second one refers SRMR.

Table A37

Mean of RMSEA / SRMR for the Exploration of the Influence of Partial Invariance of Intercepts of Both Levels (Δκ = .20)

Sample size	The first-order factor that has noninvariant parameters	Relative differences in noninvariant intercepts of the first-order factor	Freely estimating noninvariant intercepts		Constraining noninvariant intercepts to be equal	
			Freely estimating the mean of ξ in G2	Constraining the mean of ξ in G2 to be zero	Freely estimating the mean of ξ in G2	Constraining the mean of ξ in G2 to be zero
300	η1	.10	.017 / .082	.018 / .080	.018 / .081	.020 / .079
		-.10	.017 / .080	.018 / .079	.018 / .080	.019 / .078
	η2	.10	.017 / .082	.018 / .080	.018 / .081	.019 / .079
		-.10	.017 / .081	.018 / .079	.019 / .080	.019 / .079
	η3	.10	.016 / .082	.017 / .080	.018 / .081	.019 / .079
		-.10	.017 / .080	.018 / .079	.018 / .080	.019 / .078

Table A37 Continued

Sample size	The first-order factor that has noninvariant parameters	Relative differences in noninvariant intercepts of the first-order factor	Freely estimating noninvariant intercepts		Constraining noninvariant intercepts to be equal	
			Freely estimating the mean of ξ in G2	Constraining the mean of ξ in G2 to be zero	Freely estimating the mean of ξ in G2	Constraining the mean of ξ in G2 to be zero
600	η1	.10	.009 / .060	.011 / .058	.011 / .060	.013 / .057
		-.10	.009 / .058	.011 / .056	.011 / .057	.012 / .056
	η2	.10	.009 / .060	.011 / .057	.010 / .059	.013 / .057
		-.10	.009 / .059	.011 / .057	.012 / .058	.013 / .056
	η3	.10	.009 / .060	.011 / .058	.011 / .060	.013 / .057
		-.10	.009 / .058	.011 / .056	.011 / .057	.012 / .056

Note: Δκ denotes the latent mean difference of the second-order factor. The subscript refers the group number. ξ denotes the second-order factor. G2 refers Group 2, which was the reference group. For each condition, the first value refers RMSEA and the second one refers SRMR.

Table A38

Mean of RMSEA / SRMR for the Exploration of the Influence of Partial Invariance of Intercepts of First-Order Factors (Δκ = .40)

N	$\alpha_{2(G2)} - \alpha_{2(G1)}$	α_2 is freely estimated		α_2 is constrained to be equal		α_2 is constrained to be equal but α_3 is freely estimated	
		No zero constraint on the mean of ξ in G2	Constraining the mean of ξ in G2 to be zero	No zero constraint on the mean of ξ in G2	Constraining the mean of ξ in G2 to be zero	No zero constraint on the mean of ξ in G2	Constraining the mean of ξ in G2 to be zero
300	.10	.017 / .091	.021 / .086	.017 / .091	.023 / .086	.017 / .091	.022 / .086
	-.10	.016 / .088	.021 / .084	.017 / .088	.021 / .084	.017 / .088	.021 / .084
600	.10	.009 / .073	.016 / .065	.010 / .072	.018 / .065	.010 / .072	.017 / .065
	-.10	.009 / .068	.016 / .062	.010 / .068	.016 / .062	.010 / .068	.015 / .062

Note: N denotes total sample size. *Δκ denotes the latent mean difference of the second-order factor. N* denotes total sample size. α_2 is the intercept of the first-order factor and G2 (G1) denotes group membership. ξ denotes the second-order factor. For each condition, the first value refers RMSEA and the second one refers SRMR.

Table A39

Mean of RMSEA / SRMR for the Exploration of the Influence of Partial Invariance of Indicator Intercepts (Δκ = .40)

Sample size	The number of noninvariant indicator Intercepts	Relative differences in $\boldsymbol{\nu}_{\mathbf{6(6;7;10;11)}}$	Freely estimating $\boldsymbol{\nu}_{\mathbf{6(6;7;10;11)}}$		Constraining $\boldsymbol{\nu}_{\mathbf{6(6;7;10;11)}}$ to be equal		Constraining $\boldsymbol{\nu}_{\mathbf{6(6;7;10;11)}}$ to be equal but Freely estimating $\boldsymbol{\nu}_{\mathbf{7(2;3;14;15)}}$	
			Freely estimating the mean of ξ in G2	Constraining the mean of ξ in G2 to be zero	Freely estimating the mean of ξ in G2	Constraining the mean of ξ in G2 to be zero	Freely estimating the mean of ξ in G2	Constraining the mean of ξ in G2 to be zero
300	1	.10	.016 / .090	.021 / .085	.017 / .090	.022 / .085	.017 / .090	.022 / .085
		-.10	.017 / .089	.022 / .084	.017 / .089	.022 / .084	.017 / .089	.022 / .084
	4	.10	.017 / .093	.022 / .087	.019 / .092	.024 / .086	.019 / .092	.025 / .087
		-.10	.017 / .088	.021 / .084	.018 / .088	.023 / .083	.019 / .088	.023 / .084

Table A39 Continued

Sample size	The number of noninvariant indicator Intercepts	Relative differences in $\nu_{6(6;7;10;11)}$	Freely estimating $\nu_{6(6;7;10;11)}$		Constraining $\nu_{6(6;7;10;11)}$ to be equal		Constraining $\nu_{6(6;7;10;11)}$ to be equal but Freely estimating $\nu_{7(2;3;14;15)}$	
			Freely estimating the mean of ξ in G2	Constraining the mean of ξ in G2 to be zero	Freely estimating the mean of ξ in G2	Constraining the mean of ξ in G2 to be zero	Freely estimating the mean of ξ in G2	Constraining the mean of ξ in G2 to be zero
600	1	.10	.009 / .071	.016 / .064	.010 / .071	.017 / .064	.010 / .071	.017 / .064
		-.10	.009 / .070	.016 / .063	.010 / .070	.017 / .063	.010 / .070	.017 / .063
	4	.10	.009 / .074	.016 / .066	.012 / .073	.020 / .066	.012 / .073	.020 / .066
		-.10	.009 / .068	.016 / .063	.012 / .068	.018 / .062	.012 / .068	.018 / .062

Note: Δκ denotes the latent mean difference of the second-order factor. The subscript refers the group number. ξ denotes the second-order factor. G2 refers Group 2, which was the reference group. For each condition, the first value refers RMSEA and the second one refers SRMR.

Table A40

Mean of RMSEA / SRMR for the Exploration of the Influence of Partial Invariance of Intercepts of Both Levels (Δκ = .40)

Sample size	The first-order factor that has noninvariant parameters	Relative differences in noninvariant intercepts of the first-order factor	Freely estimating noninvariant intercepts		Constraining noninvariant intercepts to be equal	
			Freely estimating the mean of ξ in G2	Constraining the mean of ξ in G2 to be zero	Freely estimating the mean of ξ in G2	Constraining the mean of ξ in G2 to be zero
300	η1	.10	.017 / .091	.022 / .086	.018 / .090	.024 / .085
		-.10	.017 / .087	.022 / .083	.018 / .087	.022 / .083
	η2	.10	.017 / .091	.022 / .086	.018 / .090	.023 / .085
		-.10	.017 / .088	.021 / .084	.018 / .088	.023 / .083
	η3	.10	.016 / .091	.021 / .086	.018 / .090	.023 / .085
		-.10	.017 / .087	.021 / .083	.018 / .087	.022 / .083

Table A40 Continued

Sample size	The first-order factor that has noninvariant parameters	Relative differences in noninvariant intercepts of the first-order factor	Freely estimating noninvariant intercepts		Constraining noninvariant intercepts to be equal	
			Freely estimating the mean of ξ in G2	Constraining the mean of ξ in G2 to be zero	Freely estimating the mean of ξ in G2	Constraining the mean of ξ in G2 to be zero
600	η1	.10	.009 / .072	.016 / .065	.011 / .071	.019 / .064
		-.10	.009 / .067	.016 / .062	.011 / .067	.017 / .061
	η2	.10	.009 / .071	.016 / .065	.011 / .071	.018 / .064
		-.10	.009 / .068	.016 / .063	.012 / .068	.017 / .062
	η3	.10	.009 / .072	.016 / .065	.011 / .071	.019 / .064
		-.10	.009 / .067	.016 / .062	.011 / .067	.017 / .061

Note: Δκ denotes the latent mean difference of the second-order factor. The subscript refers the group number. ξ denotes the second-order factor. G2 refers Group 2, which was the reference group. For each condition, the first value refers RMSEA and the second one refers SRMR.

APPENDIX B

THE COMPARISON BETWEEN THE RELATIVE BIAS AND ITS ROUGH ESTIMATE

Table B1

Relative Bias of the Latent Mean Difference of the Second-Order Factor and Its Rough Estimate for the Exploration of the Influence of Partial Invariance of Second-Order Factor Loading (Δκ = .00)

N	$\gamma_{2,1(G2)} - \gamma_{2,1(G1)}$	$\gamma_{2,1}$ is constrained to be equal		$\gamma_{2,1}$ is constrained to be equal but $\gamma_{3,1}$ is freely estimated	
		Relative Bias	Rough Estimate	Relative Bias	Rough Estimate
300	.20	-.0005	0	.0001	0
	-.20	-.0005	0	-.0014	0
600	.20	-.0001	0	.-.0001	0
	-.20	.0004	0	-.0004	0

Note: N denotes total sample size. *Δκ denotes the latent mean difference of the second-order factor. N* denotes total sample size. $\gamma_{2,1}$ is the second-order loading relating the 2nd first-order factor to the second-order factor and G2 (G1) denotes group membership.

Table B2

Relative Bias of the Latent Mean Difference of the Second-Order Factor and Its Rough Estimate for the Exploration of the Influence of Partial Invariance of First-Order Factor Loading (Δκ = .00)

Sample size	The number of noninvariant first-order loadings	Relative differences in $\boldsymbol{\lambda}_{6,2(6,2;7,2;10,3;11;3)}$	Constraining $\boldsymbol{\lambda}_{6,2(6,2;7,2;10,3;11;3)}$ to be equal		Constraining $\boldsymbol{\lambda}_{6,2(6,2;7,2;10,3;11;3)}$ to be equal but Freely estimating $\boldsymbol{\lambda}_{7,2(2,1;3,1;14,4;15,4)}$	
			Relative Bias	Rough Estimate	Relative Bias	Rough Estimate
300	1	.20	.0000	0	.0000	0
		-.20	.0000	0	.0000	0
	4	.20	.0001	0	.0001	0
		-.20	.0002	0	.0002	0

Table B2 Continued

Sample size	The number of noninvariant first-order loadings	Relative differences in $\boldsymbol{\lambda}_{6,2(6,2;7,2;10,3;11;3)}$	Constraining $\boldsymbol{\lambda}_{6,2(6,2;7,2;10,3;11;3)}$ to be equal		Constraining $\boldsymbol{\lambda}_{6,2(6,2;7,2;10,3;11;3)}$ to be equal but Freely estimating $\boldsymbol{\lambda}_{7,2(2,1;3,1;14,4;15,4)}$	
			Relative Bias	Rough Estimate	Relative Bias	Rough Estimate
600	1	.20	.0000	0	.0000	0
		-.20	.0000	0	-.0001	0
	4	.20	.0001	0	.0000	0
		-.20	.0000	0	.0001	0

Note: $\Delta\kappa$ denotes the latent mean difference of the second-order factor.

Table B3

Relative Bias of the Latent Mean Difference of the Second-Order Factor and Its Rough Estimate for the Exploration of the Influence of Partial Invariance of Both First- and Second- Order Factor Loading (Δκ = .00)

Sample size	The first-order factor that has noninvariant parameters	Relative differences in noninvariant second-order loadings	Constraining noninvariant loadings to be equal	
			Relative Bias	Rough Estimate
300	η1	.20	-.0003	0
		-.20	.0002	0
	η2	.20	-.0005	0
		-.20	-.0003	0
	η3	.20	.0003	0
		-.20	-.0002	0
600	η1	.20	-.0001	0
		-.20	-.0006	0
	η2	.20	.0000	0
		-.20	-.0003	0
	η3	.20	-.0010	0
		-.20	-.0004	0

Note: Δκ denotes the latent mean difference of the second-order factor.

Table B4

Relative Bias of the Latent Mean Difference of the Second-Order Factor and Its Rough Estimate for the Exploration of the Influence of Partial Invariance of Second-Order Factor Loading (Δκ = .20)

N	$\gamma_{2,1(G2)} - \gamma_{2,1(G1)}$	$\gamma_{2,1}$ is constrained to be equal		$\gamma_{2,1}$ is constrained to be equal but $\gamma_{3,1}$ is freely estimated	
		Relative Bias	Rough Estimate	Relative Bias	Rough Estimate
300	.20	.0042	.0071	.0050	.0095
	-.20	-.0044	-.0100	-.0060	-.0133
600	.20	.0061	.0071	.0079	.0095
	-.20	-.0044	-.0100	-.0062	-.0133

Note: N denotes total sample size. *Δκ denotes the latent mean difference of the second-order factor*. *N* denotes total sample size. $\gamma_{2,1}$ is the second-order loading relating the 2nd first-order factor to the second-order factor and G2 (G1) denotes group membership.

Table B5

Relative Bias of the Latent Mean Difference of the Second-Order Factor and Its Rough Estimate for the Exploration of the Influence of Partial Invariance of First-Order Factor Loading ($\Delta\kappa = .20$)

Sample size	The number of noninvariant first-order loadings	Relative differences in $\boldsymbol{\lambda}_{6,2(6,2;7,2;10,3;11;3)}$	Constraining $\boldsymbol{\lambda}_{6,2(6,2;7,2;10,3;11;3)}$ to be equal		Constraining $\boldsymbol{\lambda}_{6,2(6,2;7,2;10,3;11;3)}$ to be equal but Freely estimating $\boldsymbol{\lambda}_{7,2(2,1;3,1;14,4;15,4)}$	
			Relative Bias	Rough Estimate	Relative Bias	Rough Estimate
300	1	.20	.0010	.0018	.0011	.0019
		-.20	-.0005	-.0025	-.0006	-.0027
	4	.20	.0044	.0071	.0049	.0095
		-.20	-.0033	-.0100	-.0037	-.0133

Table B5 Continued

Sample size	The number of noninvariant first-order loadings	Relative differences in $\boldsymbol{\lambda}_{6,2(6,2;7,2;10,3;11;3)}$	Constraining $\boldsymbol{\lambda}_{6,2(6,2;7,2;10,3;11;3)}$ to be equal		Constraining $\boldsymbol{\lambda}_{6,2(6,2;7,2;10,3;11;3)}$ to be equal but Freely estimating $\boldsymbol{\lambda}_{7,2(2,1;3,1;14,4;15,4)}$	
			Relative Bias	Rough Estimate	Relative Bias	Rough Estimate
600	1	.20	.0009	.0018	.0010	.0019
		-.20	-.0006	-.0025	-.0006	-.0027
	4	.20	.0043	.0071	.0048	.0095
		-.20	-.0031	-.0100	-.0048	-.0133

Note: $\Delta\kappa$ denotes the latent mean difference of the second-order factor.

Table B6

Relative Bias of the Latent Mean Difference of the Second-Order Factor and Its Rough Estimate for the Exploration of the Influence of Partial Invariance of Both First- and Second- Order Factor Loading (Δκ = .20)

Sample size	The first-order factor that has noninvariant parameters	Relative differences in noninvariant second-order loadings	Constraining noninvariant loadings to be equal	
			Relative Bias	Rough Estimate
300	η1	.20	.0015	.0021
		-.20	-.0065	-.0150
	η2	.20	.0038	.0014
		-.20	-.0052	-.0140
	η3	.20	.0040	.0021
		-.20	-.0062	-.0150
600	η1	.20	.0017	.0021
		-.20	-.0065	-.0150
	η2	.20	.0041	.0014
		-.20	-.0047	-.0140
	η3	.20	.0040	.0021
		-.20	-.0066	-.0150

Note: Δκ denotes the latent mean difference of the second-order factor.

Table B7

Relative Bias of the Latent Mean Difference of the Second-Order Factor and Its Rough Estimate for the Exploration of the Influence of Partial Invariance of Second-Order Factor Loading (Δκ = .40)

N	$\gamma_{2,1(G2)} - \gamma_{2,1(G1)}$	$\gamma_{2,1}$ is constrained to be equal		$\gamma_{2,1}$ is constrained to be equal but $\gamma_{3,1}$ is freely estimated	
		Relative Bias	Rough Estimate	Relative Bias	Rough Estimate
300	.20	.0110	.0142	.0140	.0190
	-.20	-.0081	-.0200	-.0104	-.0266
600	.20	.0105	.0142	.0134	.0190
	-.20	-.0080	-.0200	-.0104	-.0266

Note: N denotes total sample size. *Δκ denotes the latent mean difference of the second-order factor. N* denotes total sample size. $\gamma_{2,1}$ is the second-order loading relating the 2nd first-order factor to the second-order factor and G2 (G1) denotes group membership.

Table B8

Relative Bias of the Latent Mean Difference of the Second-Order Factor and Its Rough Estimate for the Exploration of the Influence of Partial Invariance of First-Order Factor Loading (Δκ = .40)

Sample size	The number of noninvariant first-order loadings	Relative differences in $\boldsymbol{\lambda}_{6,2(6,2;7,2;10,3;11;3)}$	Constraining $\boldsymbol{\lambda}_{6,2(6,2;7,2;10,3;11;3)}$ to be equal		Constraining $\boldsymbol{\lambda}_{6,2(6,2;7,2;10,3;11;3)}$ to be equal but Freely estimating $\boldsymbol{\lambda}_{7,2(2,1;3,1;14,4;15,4)}$	
			Relative Bias	Rough Estimate	Relative Bias	Rough Estimate
300	1	.20	.0016	.0036	.0019	.0038
		-.20	-.0009	-.0050	-.0011	-.0054
	4	.20	.0085	.0142	.0094	.0190
		-.20	-.0062	-.0200	-.0066	-.0266

Table B8 Continued

Sample size	The number of noninvariant first-order loadings	Relative differences in $\boldsymbol{\lambda}_{6,2(6,2;7,2;10,3;11;3)}$	Constraining $\boldsymbol{\lambda}_{6,2(6,2;7,2;10,3;11;3)}$ to be equal		Constraining $\boldsymbol{\lambda}_{6,2(6,2;7,2;10,3;11;3)}$ to be equal but Freely estimating $\boldsymbol{\lambda}_{7,2(2,1;3,1;14,4;15,4)}$	
			Relative Bias	Rough Estimate	Relative Bias	Rough Estimate
600	1	.20	.0017	.0036	.0020	.0038
		-.20	-.0011	-.0050	-.0011	-.0054
	4	.20	.0081	.0142	.0085	.0190
		-.20	-.0053	-.0200	-.0054	-.0266

Note: $\Delta\kappa$ denotes the latent mean difference of the second-order factor.

Table B9

Relative Bias of the Latent Mean Difference of the Second-Order Factor and Its Rough Estimate for the Exploration of the Influence of Partial Invariance of Both First- and Second- Order Factor Loading (Δκ = .40)

Sample size	The first-order factor that has noninvariant parameters	Relative differences in noninvariant second-order loadings	Constraining noninvariant loadings to be equal	
			Relative Bias	Rough Estimate
300	η1	.20	.0038	.0042
		-.20	-.0124	-.0300
	η2	.20	.0065	.0028
		-.20	-.0097	-.0280
	η3	.20	.0081	.0042
		-.20	-.0116	-.0300
600	η1	.20	.0038	.0042
		-.20	-.0132	-.0300
	η2	.20	.0060	.0028
		-.20	-.0090	-.0280
	η3	.20	.0074	.0042
		-.20	-.0123	-.0300

Note: Δκ denotes the latent mean difference of the second-order factor.

Table B10

Relative Bias of the Latent Mean Difference of the Second-Order Factor and Its Rough Estimate for the Exploration of the Influence of Partial Invariance of Intercepts of Second-Order Factors (Δκ = .00)

N	$\alpha_{2(G2)}$ -$\alpha_{2(G1)}$	α_2 is constrained to be equal		α_2 is constrained to be equal but α_3 is freely estimated	
		Relative Bias	Rough Estimate	Relative Bias	Rough Estimate
300	.10	.0243	.0417	.0324	.0556
	-.10	-.0243	-.0417	-.0310	-.0556
600	.10	.0246	.0417	.0321	.0556
	-.10	-.0248	-.0417	-.0319	-.0556

Note: N denotes total sample size. *Δκ denotes the latent mean difference of the second-order factor. N* denotes total sample size. α_2 is the intercept of the second-order factor and G2 (G1) denotes group membership.

Table B11

Relative Bias of the Latent Mean Difference of the Second-Order Factor and Its Rough Estimate for the Exploration of the Influence of Partial Invariance of Indicator Intercepts (Δκ = .00)

Sample size	The number of noninvariant first-order loadings	Relative differences in $\boldsymbol{\nu}_{\mathbf{6}(\mathbf{6};\mathbf{7};\mathbf{10};\mathbf{11})}$	Constraining $\boldsymbol{\nu}_{\mathbf{6}(\mathbf{6};\mathbf{7};\mathbf{10};\mathbf{11})}$ to be equal		Constraining $\boldsymbol{\nu}_{\mathbf{6}(\mathbf{6};\mathbf{7};\mathbf{10};\mathbf{11})}$ to be equal but Freely estimating $\boldsymbol{\nu}_{\mathbf{7}(\mathbf{2};\mathbf{3};\mathbf{14};\mathbf{15})}$	
			Relative Bias	Rough Estimate	Relative Bias	Rough Estimate
300	1	.10	.0058	.0174	.0066	.0185
		-.10	-.0058	-.0174	-.0066	-.0185
	4	.10	.0278	.0645	.0300	.0860
		-.10	-.0280	-.0645	-.0305	-.0860

Table B11 Continued

Sample size	The number of noninvariant first-order loadings	Relative differences in $\boldsymbol{\nu}_{\mathbf{6}(\mathbf{6};\mathbf{7};\mathbf{10};\mathbf{11})}$	Constraining $\boldsymbol{\nu}_{\mathbf{6}(\mathbf{6};\mathbf{7};\mathbf{10};\mathbf{11})}$ to be equal		Constraining $\boldsymbol{\nu}_{\mathbf{6}(\mathbf{6};\mathbf{7};\mathbf{10};\mathbf{11})}$ to be equal but Freely estimating $\boldsymbol{\nu}_{\mathbf{7}(\mathbf{2};\mathbf{3};\mathbf{14};\mathbf{15})}$	
			Relative Bias	Rough Estimate	Relative Bias	Rough Estimate
600	1	.10	.0060	.0174	.0068	.0185
		-.10	-.0059	-.0174	-.0066	-.0185
	4	.10	.0282	.0645	.0307	.0860
		-.10	-.0285	-.0645	-.0308	-.0860

Note: $\Delta\kappa$ denotes the latent mean difference of the second-order factor.

Table B12

Relative Bias of the Latent Mean Difference of the Second-Order Factor and Its Rough Estimate for the Exploration of the Influence of Partial Invariance of Intercepts of Both First-Order Factors and Indicators (Δκ = .00)

Sample size	The first-order factor that has noninvariant parameters	Relative differences in noninvariant interceps of the first-order factor	Constraining noninvariant intercepts to be equal	
			Relative Bias	Rough Estimate
300	η1	.10	.0074	.0119
		-.10	-.0409	-.0714
	η2	.10	.0131	.0069
		-.10	-.0361	-.0764
	η3	.10	.0091	.0119
		-.10	-.0403	-.0714
600	η1	.10	.0078	.0119
		-.10	-.0407	-.0714
	η2	.10	.0125	.0069
		-.10	-.0359	-.0764
	η3	.10	.0086	.0119
		-.10	-.0409	-.0714

Note: Δκ denotes the latent mean difference of the second-order factor.

Table B13

Relative Bias of the Latent Mean Difference of the Second-Order Factor and Its Rough Estimate for the Exploration of the Influence of Partial Invariance of Intercepts of Second-Order Factors (Δκ = .20)

N	$\alpha_{2(G2)} - \alpha_{2(G1)}$	α_2 is constrained to be equal		α_2 is constrained to be equal but α_3 is freely estimated	
		Relative Bias	Rough Estimate	Relative Bias	Rough Estimate
300	.10	.0236	.0417	.0299	.0556
	-.10	-.0232	-.0417	-.0308	-.0556
600	.10	.0243	.0417	.0319	.0556
	-.10	-.0240	-.0417	-.0314	-.0556

Note: N denotes total sample size. *Δκ denotes the latent mean difference of the second-order factor. N* denotes total sample size. α_2 is the intercept of the second-order factor and G2 (G1) denotes group membership.

Table B14

Relative Bias of the Latent Mean Difference of the Second-Order Factor and Its Rough Estimate for the Exploration of the Influence of Partial Invariance of Indicator Intercepts (Δκ = .20)

Sample size	The number of noninvariant first-order loadings	Relative differences in $\boldsymbol{\nu}_{\mathbf{6(6;7;10;11)}}$	Constraining $\boldsymbol{\nu}_{\mathbf{6(6;7;10;11)}}$ to be equal		Constraining $\boldsymbol{\nu}_{\mathbf{6(6;7;10;11)}}$ to be equal but Freely estimating $\boldsymbol{\nu}_{\mathbf{7(2;3;14;15)}}$	
			Relative Bias	Rough Estimate	Relative Bias	Rough Estimate
300	1	.10	.0060	.0174	.0068	.0185
		-.10	-.0057	-.0174	-.0065	-.0185
	4	.10	.0280	.0645	.0306	.0860
		-.10	-.0269	-.0645	-.0297	-.0860

Table B14 Continued

Sample size	The number of noninvariant first-order loadings	Relative differences in $\boldsymbol{\nu}_{\mathbf{6(6;7;10;11)}}$	Constraining $\boldsymbol{\nu}_{\mathbf{6(6;7;10;11)}}$ to be equal		Constraining $\boldsymbol{\nu}_{\mathbf{6(6;7;10;11)}}$ to be equal but Freely estimating $\boldsymbol{\nu}_{\mathbf{7(2;3;14;15)}}$	
			Relative Bias	Rough Estimate	Relative Bias	Rough Estimate
600	1	.10	.0060	.0174	.0067	.0185
		-.10	-.0058	-.0174	-.0065	-.0185
	4	.10	.0282	.0645	.0308	.0860
		-.10	-.0275	-.0645	-.0297	-.0860

Note: $\Delta\kappa$ denotes the latent mean difference of the second-order factor.

Table B15

Relative Bias of the Latent Mean Difference of the Second-Order Factor and Its Rough Estimate for the Exploration of the Influence of Partial Invariance of Intercepts of Both First-Order Factors and Indicators (Δκ = .20)

Sample size	The first-order factor that has noninvariant parameters	Relative differences in noninvariant interceps of the first-order factor	Constraining noninvariant intercepts to be equal	
			Relative Bias	Rough Estimate
300	η1	.10	.0071	.0119
		-.10	-.0416	-.0714
	η2	.10	.0123	.0069
		-.10	-.0347	-.0764
	η3	.10	.0083	.0119
		-.10	-.0387	-.0714
600	η1	.10	.0075	.0119
		-.10	-.0409	-.0714
	η2	.10	.0125	.0069
		-.10	-.0351	-.0764
	η3	.10	.0090	.0119
		-.10	-.0399	-.0714

Note: Δκ denotes the latent mean difference of the second-order factor.

Table B16

Relative Bias of the Latent Mean Difference of the Second-Order Factor and Its Rough Estimate for the Exploration of the Influence of Partial Invariance of Intercepts of Second-Order Factors (Δκ = .40)

N	$\alpha_{2(G2)} - \alpha_{2(G1)}$	α_2 is constrained to be equal		α_2 is constrained to be equal but α_3 is freely estimated	
		Relative Bias	Rough Estimate	Relative Bias	Rough Estimate
300	.10	.0244	.0417	.0328	.0556
	-.10	-.0223	-.0417	-.0291	-.0556
600	.10	.0244	.0417	.0325	.0556
	-.10	-.0231	-.0417	-.0297	-.0556

Note: N denotes total sample size. *Δκ denotes the latent mean difference of the second-order factor. N* denotes total sample size. α_2 is the intercept of the second-order factor and G2 (G1) denotes group membership.

Table B17

Relative Bias of the Latent Mean Difference of the Second-Order Factor and Its Rough Estimate for the Exploration of the Influence of Partial Invariance of Indicator Intercepts (Δκ = .40)

Sample size	The number of noninvariant first-order loadings	Relative differences in $\boldsymbol{\nu_{6(6;7;10;11)}}$	Constraining $\boldsymbol{\nu_{6(6;7;10;11)}}$ to be equal		Constraining $\boldsymbol{\nu_{6(6;7;10;11)}}$ to be equal but Freely estimating $\boldsymbol{\nu_{7(2;3;14;15)}}$	
			Relative Bias	Rough Estimate	Relative Bias	Rough Estimate
300	1	.10	.0056	.0174	.0062	.0185
		-.10	-.0055	-.0174	-.0064	-.0185
	4	.10	.0271	.0645	.0300	.0860
		-.10	-.0261	-.0645	-.0286	-.0860

Table B17 Continued

Sample size	The number of noninvariant first-order loadings	Relative differences in $\boldsymbol{\nu}_{\mathbf{6(6;7;10;11)}}$	Constraining $\boldsymbol{\nu}_{\mathbf{6(6;7;10;11)}}$ to be equal		Constraining $\boldsymbol{\nu}_{\mathbf{6(6;7;10;11)}}$ to be equal but Freely estimating $\boldsymbol{\nu}_{\mathbf{7(2;3;14;15)}}$	
			Relative Bias	Rough Estimate	Relative Bias	Rough Estimate
600	1	.10	.0057	.0174	.0065	.0185
		-.10	-.0055	-.0174	-.0062	-.0185
	4	.10	.0274	.0645	.0298	.0860
		-.10	-.0259	-.0645	-.0281	-.0860

Note: $\Delta\kappa$ denotes the latent mean difference of the second-order factor.

Table B18

Relative Bias of the Latent Mean Difference of the Second-Order Factor and Its Rough Estimate for the Exploration of the Influence of Partial Invariance of Intercepts of Both First-Order Factors and Indicators (Δκ = .40)

Sample size	The first-order factor that has noninvariant parameters	Relative differences in noninvariant interceps of the first-order factor	Constraining noninvariant intercepts to be equal	
			Relative Bias	Rough Estimate
300	η1	.10	.0055	.0119
		-.10	-.0405	-.0714
	η2	.10	.0126	.0069
		-.10	-.0334	-.0764
	η3	.10	.0090	.0119
		-.10	-.0372	-.0714
600	η1	.10	.0054	.0119
		-.10	-.0408	-.0714
	η2	.10	.0122	.0069
		-.10	-.0329	-.0764
	η3	.10	.0097	.0119
		-.10	-.0372	-.0714

Note: Δκ denotes the latent mean difference of the second-order factor.