Technology Two Ways:

Modeling Mathematics Teacher Educators'

Use of Technology in the Classroom

by

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A Dissertation Presented in Partial Fulfillment
of the Requirements for the Degree
Doctor of Philosophy

Approved November 2014 by the
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ARIZONA STATE UNIVERSITY

December 2014

ABSTRACT

This study explores teacher educators' personal theories about the instructional practices central to preparing future teachers, how they enact those personal theories in the classroom, how they represent the relationship between content, pedagogy, and technology, and the function of technology in teacher educators' personal theories about the teaching of mathematics and their practices as enacted in the classroom. The conceptual frameworks of knowledge as situated and technology as situated provide a theoretical and analytical lens for examining individual instructor's conceptions and classroom activity as situated in the context of experiences and relationships in the social world. The research design employs a mixed method design to examine data collected from a representative sample of three full-time faculty members teaching methods of teaching mathematics in elementary education at the undergraduate level. Three primary types of data were collected and analyzed: a) structured interviews using the repertory grid technique to model the mathematics education instructors' schemata regarding the teaching of mathematics methods; b) content analysis of classroom observations to develop models that represent the relationship of pedagogy, content, and technology as enacted in the classrooms; and c) brief retrospective protocols after each observed class session to explore the reasoning and individual choices made by an instructor that underlie their teaching decisions in the classroom. Findings reveal that although *digital* technology may not appear to be an essential component of an instructor's toolkit, *technology* can still play an integral role in teaching. This study puts forward the idea of repurposing *as* technology -- the ability to repurpose items as models, tools, and visual representations and integrate them into the curriculum. The instructors themselves became the technology, or the mediational tool, and introduced students to new meanings for "old" cultural artifacts in the classroom. Knowledge about the relationships

between pedagogy, content, and technology and the function of technology in the classroom can be used to inform professional development for teacher educators with the goal of improving teacher preparation in mathematics education.

For Steve, whose love and belief in me never waivers, and for our girls,

Madeleine and Alexandra. I started this journey for myself, but I finished for you.

ACKNOWLEDGEMENTS

My heartfelt thanks and gratitude to all those who generously contributed their time and support as I worked to complete this project.

I am deeply grateful to Jim Middleton for his never-ending enthusiasm and insight. From the time I first walked into his office to describe interests that didn't fit in to any single degree program, he supported my endeavors to cross boundaries and explore questions that fell outside the traditional lines of research in mathematics education. Jim, you helped me think in ways I would not otherwise have known to be possible. I am also sincerely indebted to Barry Sloane, who repeatedly and graciously offered his feedback and time to not only me, but countless other mentees. Barry, you taught me to evaluate research and in doing so made me far better at my own work. And to Ray Buss and Bob Atkinson, my heartfelt thanks for your wonderful collegiality, thoughtful comments and time throughout this process.

A number of faculty and colleagues over the years have assisted me. I owe my thanks to Dan Battey, David Berliner, Mary Lee Smith, and Carole Edelsky for their insight and guidance in undertaking educational research. My thanks also to all my friends and colleagues in the Mary Lou Fulton Teachers College for the years of conversation and kind encouragement to keep moving forward. And of course, my highest regard and gratitude to my participants who graciously allowed me to visit their classrooms and through their passion for their subject made me want to become a mathematics educator.

Above all, I am profoundly grateful to my family. To Tom and Betty, who warmly welcomed me as their daughter and whose love for our girls knows no bounds. To my grandparents, Roberta, Glenn and Gene, who would have been so proud, and Betty, who may have lovingly questioned my sanity but never my ability. To my siblings, Kelly, Vicky,

Ryan, Sarah, Darcy and Joe, whose love, laughter and friendship are something special and unique in this world that few others will ever know. To my sister Darcy, who answered the phone every time I needed her and never failed to tell me that I wasn't alone and that I could finish. And to my parents, Glenn and Jan, my first and best teachers, who have given me more than I can ever thank them for, but whose best gift was never telling me that I couldn't, only that I can.

Finally, to Steve, Maddie and Allie. Je vous aime de tout mon cœur.

This research was supported by a grant from Arizona State University.

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CHAPTER 1

INTRODUCTION AND PURPOSE

For a number of years I provided professional development for higher education faculty across the university, primarily on topics related to instructional strategies and technology integration. Over time, it became evident that the faculty members I worked with were trained to be researchers and experts in their field, but not teachers. They expressed a strong interest in improving their teaching, yet faced numerous constraints related to their professional careers, research agenda, and personal lives.

Although I enjoyed working with faculty across disciplines and degree programs, the work often felt like an uphill battle. So when an opportunity arose to join a college of education, I took the new position with the expectation that working in a college of education would mean working with colleagues who valued the teaching profession and engaged in communal discourse about learning. In many ways, this was true. My colleagues were passionate about learning and about improving the quality of education for all students. And yet, they faced the same challenges as others in higher education: the demands of their research, the pressure of promotion and tenure, the service obligations to the university and their professional community, and more.

In addition to these pressing demands, the preparation of future teachers for the complex task of educating our youth remains a major responsibility of most education faculty. The loud calls for education reform in the K-12 education system fall largely on schools of education to address. New standards focusing on conceptual understanding, inquiry-based learning and real-world application (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010) place increasing requirements on the education system to change how teachers teach and what students learn.

Teacher educators must prepare teachers who can teach to the curriculum standards, integrate technology across the curriculum, meet the needs of diverse student populations, and prepare their students to pass high stakes tests (Cochran-Smith, 2003).

Of the numerous charges posed for 21st century teachers and teacher educators, one that strongly captures my interest is the challenge of integrating technology across the curriculum. The rapid evolution of technology and the increasing complexity that comes with its exploding potential – consider, for example, the difference between learning to operate a TV in the 1940s and learning to operate a smartphone in the 2000s – explains why integration of technology in education continues to receive special attention. Many revised education standards emphasize that teacher candidates must demonstrate the ability to develop learning environments and experiences that effectively integrate technology (International Society for Technology in Education, 2008; National Council for Accreditation of Teacher Education, 2008; National Council of Teachers of Mathematics, 2000). Yet, for a variety of reasons such as teacher capability, technology infrastructure, school culture and organizational constraints, the use of technology continues to fall short of its potential to support learning (Fishman, Marx, Blumenfeld, Krajcik, & Soloway, 2004).

The evolving definition of "technology" also means its enactment changes over time. Over the last century, through the semantic process of contextual specialization, society has redefined "technology" in such a way that commonly accepted tools such as pencils, chalkboards, rulers, films, and other physical, non-digital or "analog" technologies are perceived as distinct from digital resources such as computers, software, or Internet-based content and applications. When Shulman (1986) described what he termed "curricular knowledge," he made no distinction between such tools but included all of them as part of

the toolkit from which an instructor draws those most appropriate to teach particular content at a particular level. This is no longer the case.

Rather than distinguishing between digital and non-digital tools, I believe that pedagogical technologies should be conceived of as a system of knowledge, skills, and organization whose function is defined by the context in which the system is used. In other words, it is the way that a person teaches and how he or she uses artifacts to effect student learning that is significant, not the tools themselves. To explore the relationship between content, pedagogy, and technology, this study examines teacher educators' personal theories about the instructional practices central to preparing future teachers and how they enact those personal theories in the classroom.

Teaching about Teaching

It seems a logical assumption that successful K-12 teachers or education researchers who join the ranks of university faculty will also be successful teacher educators, yet these practices are not the same. An experienced practitioner may not be able to transfer her practical and professional knowledge to the new role of teaching about teaching (T. Russell & Loughran, 2007), and the Ph.D. programs that prepare education researchers seldom provide educational and professional development for their future role as teacher educators. Many novice teacher educators struggle to determine what content and strategies they should teach to future teachers (Berry, 2008), and as a result they likely derive their pedagogy from the instructional practices modeled while students in the K-12 education system (Lortie, 1975), teacher candidates (Bullock, 2007) or graduate students. The lack of well-defined knowledge about a pedagogy of teacher education (Korthagen, 2004) or formal preparation for teacher educators leaves faculty to adopt strategies from their K-12 experiences and their own teacher preparation programs, or rely on university or college-wide resources related to

teaching and learning. However, faculty professional development usually focuses on updating or learning new skills rather than sustained reflection and discussion about education reform and the importance of teacher educators' own practices.

If we agree that an individual's belief system affects his or her teaching practices (Richardson & Placier, 2001), then an exploration of a teacher educator's belief structures about teaching will provide insight about the instructional practices and actions she takes in preparing future teachers. As individuals with primary authority over their own classrooms, teacher educators make decisions about not only *what* they teach – within the scope of the topic at hand – but also *how*. Their beliefs about the value and function of various instructional practices and technologies and confidence in their own knowledge and skills affect the curriculum they design and enact in their classroom. If, in turn, the strategies used in teacher education programs affect future teachers' knowledge, beliefs, and proficiency to teach (Cochran-Smith & Zeichner, 2005), this underscores the central role of teacher educators in education reform.

Teaching that Integrates Technology

Revised standards for the teaching profession by the National Council for Accreditation of Teacher Education (2008), the International Society for Technology in Education (2008) and the National Council of Teachers of Mathematics (2000) emphasize moving beyond the acquisition of technology skills towards the effective integration of technology across the curriculum. Data from the Professional Education Data System 2007-2008 Survey suggest an increased emphasis on the integration of technology by faculty in teacher preparation courses, but also a need for additional data on the extent to which teacher educators integrate technology (Ludwig, Kirshstein, Sidana, Ardila-Rey, & Bae, 2010). Although studies indicate that faculty members recognize the importance of

technology in teacher preparation, this does not necessarily transfer to its strategic integration in teacher education programs and classes (Falba, Strudler, & Boone, 1999; Strudler, Archambault, Bendixen, Anderson, & Weiss, 2003). Obstacles similar to those experienced in K-12 schools, such as lack of time (Wepner, Ziomek, & Tao, 2003), real or perceived lack of equipment, facilities, and resource people (Eifler, Greene, & Carroll, 2001), skill and comfort level using technology (Strudler et al., 2003), and lack of understanding and/or vision on how to integrate technology in meaningful ways (Strudler & Wetzel, 1999), continue to pose challenges to successful integration of technology by education faculty teaching in pre-service programs.

For these and other reasons, the ready access to technology for students and teachers has had little meaningful impact on K-12 or higher education instruction (Cuban, 2001) and discussion about how to use technology to foster deep learning continues to focus on many of the same considerations historically raised about the knowledge and skills needed to teach any content area. One such example relates to the category of pedagogical content knowledge introduced by Shulman (1986), in which he described an over-emphasis in the research literature on procedural elements of instruction and a greater need for questions exploring how teachers decide what content to teach, how they teach it, and how they help students acquire new knowledge. These same questions receive attention in what has been termed Technological Pedagogical Content Knowledge (TPACK) (AACTE Committee on Innovation and Technology, 2008; Mishra & Koehler, 2006), a framework that argues technology is not context-free and using it effectively requires an understanding of the relationship between technology, pedagogy, and content. The emphasis on preparing teachers who can integrate technology across subject areas means that faculty must

understand and demonstrate for students not only the *pedagogy* for teaching content, but also understand and demonstrate the *technology* for teaching content.

Technology Two Ways

Chefs use the expression "two ways" to describe a dish that focuses on one main ingredient, but is prepared in two different styles: artichoke two ways, salmon two ways, kale two ways, etc. The intent is to showcase a chef's expertise, versatility and creativity by preparing a dish not just one way, but two, or even three. Good chefs remain passionate about their profession and pursue learning experiences throughout their career to expand their knowledge and skills with various equipment, techniques, ingredients, and styles. Great chefs, though, also possess the distinguishing characteristic of creativity. Like great teachers, they demonstrate an ability to take ingredients and equipment – content – and techniques and styles – pedagogy – and apply them in new and unexpected, yet effective, ways. They possess a "veritable armamentarium of alternative forms of representation, some of which derive from research whereas others originate in the wisdom of practice" (Shulman, 1986, p. 9).

One ingredient: technology. Two styles: analog and digital. The instructor who understands what makes the "learning of specific topics easy or difficult" (Shulman, 1986, p. 7), who calls on multiple ways of representation and explanation, who draws on previous knowledge, conceptions, or preconceptions of students, who selects the most appropriate materials from the variety available, who creates new ways to use familiar materials – that is a fluent instructor who can use one ingredient multiple ways. Such an instructor can "transform his or her own expertise in the subject matter" (Shulman, 1986, p. 8), as well as how to teach that subject matter, in a way that future teachers can comprehend and apply. While assuredly not familiar with all the strategies and tools available, great teachers stay

current with new knowledge in their field and try new learning experiences to improve their performance and that of their students. Their exploration of new opportunities may lead them to explore emerging technologies, existing technologies that are new to them, tools not designed for educational use, or non-digital materials that can be adapted or repurposed for pedagogical purposes.

Purpose of the Study

Beck and Wynn (1998) describe technology integration as a continuum that extends from a standalone course on technology in education at one end, to integration of technology across all courses and clinical experiences in the program at the other. Rather than exploring the relationship of pedagogy, content, and technology within the context of a course focused on educational technology in the classroom, I have selected to explore technology use in a content methods course. Although it would have been possible to select cases from a range of methods courses such as social studies, science, mathematics and English, I opted to select one content area as the focus of this study: mathematics. Because knowledge, beliefs, and pedagogy vary across disciplines (Shulman, 1986), focusing on mathematics education allows me to better understand the integration of technology knowledge, content knowledge, and pedagogy knowledge as enacted by a representative sample of all full-time teacher educators offering mathematics methods courses in a college of education at a major metropolitan university. The research methods employed in this study could transport to other content areas, content area method courses, and general teacher education courses, such as child and adolescent development, assessment, or clinical experiences, to provide a greater understanding of these practices in teacher education curriculum.

This study does not attempt to directly influence the activities or tools that instructors use in their undergraduate classes on methods of teaching mathematics, but rather explores the instructional practices the instructors believe are most critical to preparing elementary educators to teach mathematics at the K-8 level and how they enact those beliefs in the classroom. What personal theories do mathematics teacher educators hold about the instructional practices necessary for preparing future mathematics teachers and what role does technology play in those beliefs? How does an instructor enact those personal theories in her classroom practices? What does the classroom discourse reveal about the relative emphasis and relationship between pedagogy, content, and technology as enacted by each teacher educator? What reasoning underlies the decisions an instructor makes prior to or during classroom instruction about how to present the pedagogy, content, and technology?

This study contributes knowledge about the role technology plays in teacher educators' belief structures about preparing future teachers and the relative emphasis and relationship of pedagogy, content, and technology as enacted in their classroom instruction. Results from the study provide insight that can help the profession respond to calls to mediate interaction between learners and teachers, among learners, and between learners and tools (Kozma, 1991; Kozma, 1994) or transform activities to influence cognitive, motivational, and social elements of the learning environment via technology mediation (Pea, 1987).

Research Questions

The overarching research questions for this study on instructor conceptions about the teaching of mathematics and the enactment of those beliefs in the context of a pedagogy course on mathematics education include the following:

1. What are teacher educators' personal theories about the instructional strategies most critical for preparing future mathematics teachers?
2. How do teacher educators enact their personal theories in the classroom?
3. What does classroom discourse reveal about the relative emphasis and relationship between pedagogy, content, and technology as enacted by teacher educators?
4. What is the function of technology in teacher educators' personal theories about the teaching of mathematics and their practices as enacted in the classroom?

CHAPTER 2

BACKGROUND LITERATURE AND THEORETICAL FRAMEWORK

In the following sections I outline the importance of faculty engagement in a pedagogy of teacher education and research that furthers the knowledge base on teacher educators' personal theories about teaching and learning and associated instructional practices. In addition, I discuss the conventional use of technology in comparison to the potential transformation of activities mediated with technology. Finally, I outline the theoretical underpinnings of my perspective on cognition as situated in context, as well as technology as situated in context. This review serves to assemble ideas from three main areas of research – teacher preparation, technology integration, and cognitive science – into a cohesive framework for exploring teacher educators' personal theories about the teaching of mathematics, the instructional practices teacher educators use in their undergraduate courses on methods of teaching mathematics, and the function of technology in these teacher educator beliefs and practices.

Developing a Pedagogy of Teacher Education

The changing context of the university environment as well as the ongoing focus on education reform places new challenges on colleges of education and individual faculty members. For faculty entering the profession of teacher education from a research pathway, the educational and professional development provided to graduate students in most Ph.D. programs does not sufficiently prepare them to teach teachers, while faculty moving from classroom teaching face the challenge of sharing their practical experience and successful practices without relying on a "pedagogy-of-presentation" approach (Berry, 2007; Berry, 2008).

In comparison to non-education faculty who primarily teach knowledge and skills related to their field of expertise, it may be more accurate to represent the transition from teacher to teacher educator as a shift in emphasis from teaching content to teaching pedagogy. Faculty members seldom have an occasion to observe or learn about the ways others teach, much less participate in a supportive culture that engages participants in the activities, understandings, and value systems of a social community committed to teaching (Loucks-Horsley, Hewson, Love, & Stiles, 1998) and teaching about teaching. At the same time that a teacher educator works to counter the teaching beliefs and practices internalized by students through their K-12 educational experiences (Lortie, 1975), she must also develop her pedagogy of teacher education, distinct from her pedagogy of K-12 teaching (Murray & Male, 2005).

Although thousands of colleges of education require teacher candidates to participate in clinical experiences under the supervision of classroom teachers who serve as mentors and models of professional practice, education scholars do not have similar opportunities to experience a structured apprenticeship in the practice of teacher education (Wilson, 2005). Results from the Research about Teacher Education (RATE) study showed that while teacher education faculty used a variety of teaching methodologies, lecture and discussion remained the activities most commonly employed in college of education classrooms (Howey, 1989). The Educator Questionnaire included in the Teacher Education and Development Study in Mathematics (TEDS-M) promises to yield more current information about the type of learning activities employed in mathematics, mathematics pedagogy, and general pedagogy courses in teacher education programs, but findings of this type were not included in the relevant published reports (Center for Research about Mathematics and Science Education, 2010; Tatto et al., 2012).

Just as classroom teachers need to participate throughout their career in ongoing professional development to improve their knowledge and practice (Darling-Hammond & Ball, 1998), so too do teacher educators. The "stance of inquiry" that Ball and Cohen (1999) describe as central to the role of the teacher should also be central to the role of the teacher educator (Cochran-Smith, 2003), such that teacher educators involved in the day-to-day practices of preparing teachers can examine and research their own work (Zeichner, 1999) to provide a better understanding of the process by which education faculty become teacher educators, the belief structure that underlies their classroom practices, and the actions they take in helping students learn about teaching.

Learning to Teach with Technology

In support of revised standards that emphasize the development of learning experiences that integrate technology (International Society for Technology in Education, 2008; National Council for Accreditation of Teacher Education, 2008; National Council of Teachers of Mathematics, 2000), many teacher education programs include a course on teaching with technology. The educational technology coursework often emphasizes the use of technology to address real instructional problems and needs, but these courses are often designed and taught by faculty with background and expertise in educational technology. The shift in emphasis from basic technology skills in the 1990s to effective pedagogical use of technology in the early 2000s is now being followed by another shift, this time towards technology integration embedded in content courses, methods courses, general teacher preparation courses, and clinical experiences. In order for technology-pedagogy-content integration to occur across the teacher preparation program, instructors will need to provide students with experiences that engage them in not only authentic problem solving with technology, but in technology-related problem solving authentic to a particular content area

or foundational topic. To provide teacher candidates with models and strategies for integrating technology, pedagogy, and content in ways that transforms student cognition and learning, teacher educators will need to turn a critical eye towards their own instructional practices and beliefs and model a reflective process for their students (Loughran & Berry, 2005).

Given the challenges facing educational reform, many related to the lack of motivation for faculty to alter their teaching practices, the availability of university resources, and the additional "burden" of integrating educational technologies into K-12 and higher education classrooms, it is not surprising that faculty resist external pressures to re-conceptualize or restructure the instructional planning and learning activities of their teacher preparation courses. Designing content-based activities that effectively integrate technology while also meeting the learning objectives for their students poses a particular challenge for faculty, who often feel they have less technology expertise than their students and little incentive to learn. Expressions such as "digital divide," "digital natives," and "digital immigrants" encourage this impression. These phrases lend credence to the ideas that technology is a modern-day phenomenon, that some must work harder than others to use technology, and that technology integration poses an additional challenge for those who might otherwise feel confident in their teaching. However, the technology that students know and the ways they have used it likely does little to prepare them to learn with educational technologies, much less effectively integrate technology in their own teaching.

Many universities offer technology professional development for their faculty, but these opportunities usually focus on standalone technology skills, such as how to use a student response system, and in some cases also incorporate pedagogy, such as how to design student response system questions that assess knowledge from a current class session.

However, the broad-based, short-term workshops do little to address the interdependency of content, technology, and pedagogy in order to help faculty effectively integrate technology. If teacher educators recognize and take advantage of opportunities to reflect on the way they teach and then modify their own pedagogy and practices accordingly, there exists an opportunity to positively influence teacher education and the practices of prospective teachers (Lunenberg, Korthagen, & Swennen, 2007). There is a need for long-term professional development to help teacher educators design meaningful uses of technology to effect student learning, however we must first understand teacher educators' conceptual beliefs about teaching how to teach a particular content area, how they enact those beliefs in their classrooms, and the function of technology in their beliefs and practices.

(Re)conceptualizing Technology Integration

Much of the talk and professional development directed towards teacher educators emphasizes how to use and integrate technology, and less on the opportunities technology provides to positively affect social and cognitive processes. This focus on the procedural aspects of technology promotes a larger misconception of technology as a tool with a pre-defined function rather than a system of knowledge, skills, and organization. Many faculty who integrate technology in their teaching replicate their existing instructional methods, for example by using a learning management system to provide digital copies of their existing course materials, capturing some portion of audio or video instruction, conducting discussions online, or creating experiences in virtual environments that often closely resemble the real world. The conventional use of technology to replicate, not transform, instructional practices (Bonk & Dennen, 2003; Naidu, 2003) likely results from a tendency to use instructional methods that are familiar, albeit delivered via technology, and the perception of technology as a product or tool rather than a process or activity. Such

implementations of instructional media in learning environments reveal an expectation that the integration of technology will improve learning, even though the way in which the material is taught has not changed (Ehrmann, 1999).

The rapid evolution of technology and its increasing complexity explains why some faculty may resist or be unwilling to take on this challenge. And yet many do not resist, but make concerted efforts to integrate technology in ways they believe to be instructive or meaningful for their students, even though their attempts may fall short of the ideals of those in the educational technology field. If teacher education programs are to meet the challenge of preparing candidates who can successfully integrate knowledge of content, knowledge of teaching, and knowledge of technology, the faculty teaching courses in these programs must develop an understanding of the many ways in which technology can be used as well as its inherent and imposed constraints and affordances (Koehler & Mishra, 2008).

Discussions of the constraints and affordances of technologies in learning environments sometimes assume a predetermined impact in the vein of Norman's (1988) work in the field of design, whereby a doorknob affords turning, a pencil affords writing, a video affords watching, and so on. These activities are not guaranteed, however, since a doorknob will not be turned if a person does not want to enter, a pencil will not write if a person has nothing to say, and technology in a classroom will not influence learning unless its use aligns with specified outcomes and learning activities. These activities must be designed in keeping with the ecological interaction of the object and the environment, with the human as part of the environment and the environment as part of the human psychological system (Gibson, 1977). Often, technology is integrated and examined without reflection on its function or its interaction with the environment, and many research studies focus on student achievement or the assessment of learning objectives rather than the

transformation of activities through technology mediation, refinement of a curricular intervention, or development of hypotheses and appropriate measures and methods for use in future research.

The transformation of activities mediated with technology in a particular context provides an opportunity for influencing cognitive, motivational, and social elements of the learning environment (Pea, 1987). The example of model-eliciting activities serves to illustrate this concept. In discussing the role of technology-based tools in mathematics instruction, Lesh, Zawojewski, and Carmona (2003) argue that technology offers more than new topics that must be addressed because of a new technology or a new way to do old activities. In their models and modeling perspective, using technology as a tool to solve a mathematical problem should not mean that the students are engaged with the technology rather than the mathematics, but that the technology facilitates some new way of thinking and helps students develop a construct that has meaning even when the technology is absent (Johnson & Lesh, 2003). While working on a model-eliciting activity students externalize their thinking through representational media, such as language, symbols, diagrams, or tables and graphs generated with a spreadsheet. The construction of these representations induces change in the students' own conceptual systems as well as those of their peers when communicated to each other (Johnson & Lesh, 2003).

Externalization of a student's understandings or models through written communication in a social context provides another opportunity for influencing the cognitive and social elements of student learning (Pea, 1992). Embedding collaborative tools in not just the learning environment, but also in local participants' activity structures, can serve to create a culture of inquiry and collaboration (Hoadley, 2002). While the technology itself may not be easily modified to fit the particular activity structure, students and

instructors can develop practices for externalizing and sharing their models that serve as a mechanism for testing, refining, revising, and extending their ways of thinking. In a similar manner, computer-supported Knowledge-Building Communities (Scardamalia & Bereiter, 1994; Scardamalia & Bereiter, 2006) initiate students in a culture of building knowledge as a community that, together, is more complex than the interaction of the individuals. Although the examples of model-eliciting activities and Knowledge-Building Communities may seem to pose exemplary uses of technology, they also describe meaningful learning environments in which students engage in technology-related problem solving authentic to the contexts of mathematics, science, and others. These models are notable because of their focus on knowledge building rather than knowledge acquisition, and how technology can facilitate these educational ideas.

Knowledge as Situated

The philosophy of constructivism provides insight regarding the ways in which individual learners construct knowledge from their experiences. Piaget formalized this theory over a number of years, refining and revising his view on how individuals internalize knowledge through the processes of assimilation and accommodation. Central to his view of cognitive development was the way in which learners integrate new experiences into existing cognitive structures, thereby creating, changing, and advancing schemas of knowledge (Piaget, 1952). These schemas serve as prototypes that give structure to events, processes or images and represent an individual's way of understanding and remembering knowledge (Bartlett, 1932).

Exploration of the organization and representation of these mental structures and how they influence the acquisition and use of knowledge forms the focus of schema theory (Anderson, Montague, & Spiro, 1977). Semantic networks, originally used in information

processing as a way to represent the semantic relations between concepts, provide one mechanism for visually modeling individual schemata. For example, the representational structures of novices and experts regarding some process or knowledge might be compared to explore the differences in their understanding and application (e.g., Chi, Feltovich, & Glaser, 1981). Similarly, one could examine the degree of similarity between construct systems of two groups, such as teachers and students regarding what they believe makes mathematics intrinsically motivating, or "fun" (Middleton, 1995). Another use includes developing a model of an individual or group's understanding of some concept, such as college calculus students' understanding of the limit concept (Williams, 2001).

Although it can be useful to understand the ways in which learners internalize and represent knowledge, the constructivist focus on "internal mental constructions of the individual" (E. Smith, 1995, p. 23) does not address the social dimensions of learning. Granted, the constructivist approach allows for social interaction with other students and teachers, but the associated analysis focuses on the progression of each individual's understanding of the content. In contrast, situated theory allows a focus on the individual, but places learning in the context of experiences and relationships in the social world (Lave & Wenger, 1991) and allows for an analysis of the simultaneous influence of the social on the individual and vice versa (Cobb & Yackel, 1996). A blending of these two perspectives, then, might acknowledge that learning takes place in context and also address the critique that individuals cannot construct knowledge completely independent of social and communal influences.

The blended perspective described above, which I refer to as situated constructivism, is particularly useful as a framework for exploring how an instructor enacts his or her individual cognitive structure within the classroom context. This particular study does not

examine the activities of the social communities to which the participants belong, but focuses on individuals' beliefs and practices while still acknowledging the broader social and institutional influences. Using a situated constructivist perspective, this study explores not only individual instructors' cognitive schemata regarding the instructional methods they believe to be most critical for preparing undergraduate education students to teach mathematics, but also how the instructors enact those schemata within the ecological context of a classroom.

Personal Construct Theory

One theory that embodies the constructivist representation of an individual's personal system of meaning while also acknowledging the social influences of a person's experiences and interactions in the world is Personal Construct Theory (Kelly, 1955). This theory emphasizes understanding how a person interprets his or her lived experiences and then constructs an individual understanding of those events. In his Fundamental Postulate, Kelly proposed that as individuals we are not shaped directly by experiences, but by how we construe them, with each new experience providing an opportunity to reconsider, or reconstruct, our system of meaning (Kelly, 1955). His depiction of "man-the-scientist" allows each person to develop and hold theories about the world around him that in turn affect how the person perceives and interprets new experiences. Each new experience provides an opportunity for an individual to evaluate the experience using their personal theory and then modify or reconstruct their theory accordingly, a philosophical position Kelly termed "constructive alternativism." As such, it is not the event or experience that changes, but an individual's understanding of the event and their personal construction of its context and significance. A change in constructs may occur at any time, but it is the individual who determines whether their existing models or personal theories explain the

events experienced or whether their present constructs require revision or replacement. To elaborate the fundamental postulate, Kelly developed eleven corollaries, of which five warrant discussion as they relate to this research study.

The Construction Corollary. The first corollary, the Construction Corollary, explains that it is through the replication of events and experiences that a person gives meaning to any one concept. To describe the notion of construing, Kelly explains that a person must first abstract from the flow of all events and experiences those that have some common theme that differentiates them in a significant way from other events or experiences. The replicated events are identified and noted by features that characterize them in some manner and mark their similarity to one another, while distinguishing them in particular ways from other events not in the set. It is these features that Kelly terms a "construct." The structured interviews conducted with each participant in this study aim to elicit a reasonably precise description of the interviewee's constructs regarding the teaching methods she considers to be most critical for preparing undergraduate education students to teach mathematics at the K-8 level. The kinds of constructs offered by each participant during the elicitation process should reveal individual areas of emphasis about how and why they believe particular teaching methods are critical to the learning of mathematics. Constructs not offered or discussed may also provide insight.

The Individuality Corollary. In the Individuality Corollary, Kelly reiterates that because each person differs in how they experience events, they also differ in how they construe events. As a result every individual has a unique construct system. Although one criticism of Personal Construct Theory questions the focus on the individual, rather than his or her social interaction (Holland, 1970), Personal Construct Theory acknowledges the social influence on the individual even though it primarily seeks to understand the individual

system. Kelly explains that no matter how closely associated two people may be, they can never experience a single event the same way. Individuals are shaped by past experiences and the schema they apply to the anticipation of all future events differs as a result. For this reason, two people expressing their constructs related to methods of teaching mathematics will not reveal the same model. Within the context of education, articulation of an individual's model can help others understand the constructs or patterns she uses to anticipate and interpret lived experiences, such as articulating an instructor's personal model about methods of teaching mathematics so that students understand the rationale for how she enacts that model in the classroom.

The Dichotomy Corollary. The Dichotomy Corollary expounds the notion of a construct as a dichotomous structure that contains by definition both a similarity and a contrast that together provide meaning. Here, Kelly introduces the use of a triad of elements to elicit constructs by asking a person to identify an aspect that explains the way in which two items are similar and contrast from the third. Establishing a minimum context of three elements for eliciting a construct improves the precision and detail of otherwise challenging constructs. For example, the meaning intended by a descriptor such as "true" can be most precisely conveyed by identifying its contrast of "imaginary," "inaccurate," "careless," "deceitful," "disloyal," "corrupt" or other, depending on the meaning a person attributes to "true" and the element it is being used to describe. Similarly, a descriptor such as "less risk" provides little information when presented only in comparison to the negative of "more risk" rather than defining risk as the "possibility of technology failure" versus "not requiring specific equipment," thereby reiterating the importance of the context.

The repertory grid technique, initially reported by Kelly and called the Role Construct Repertory Test (Kelly, 1955), is a system for eliciting personal constructs with the

goal of not just understanding, but representing in an individual's own terms, the meaning some idea or object has for him or her. This process allows an investigator to generate an accurate representation of the personal system of meaning an instructor uses to interpret and anticipate the social world, in this case how she defines the teaching methods perceived as the most critical to the preparation of elementary education mathematics teachers.

The Fragmentation Corollary. In describing an individual's personal system as being in a constant state of flux, Kelly recognizes the ongoing evolution of a construction system in response to new experiences. By iteratively testing and retesting hypotheses a person can determine whether new experiences and knowledge necessitate a reconstrual of some portion of his or her construct system, referred to by Kelly as the Experience Corollary. In turn, the Modulation Corollary qualifies this reconstrual by explaining that the variation that can occur depends on the permeability of the related constructs; in order to change, a construct must have some degree of permeability and Kelly does not believe a construct can be completely impermeable although some constructs may change so infrequently as to appear concrete.

The Fragmentation Corollary continues the explanation of the structure of an individual's construction system by qualifying that when a person tests new hypotheses or introduces them into his or her system, he or she may maintain old, incompatible constructs. These old and new constructs co-exist within a larger, superordinate system. For this reason, an individual may demonstrate or express constructs that seem inconsistent or contradictory, but have been resolved in a way that satisfies the individual's own sense of logic. These self-made rules that a person develops in response to situations and experiences constitute an individual theory that allows for the tolerance of periodic or even daily events that would otherwise seem incompatible with a person's construction system. Without recognizing the

reason for these apparently inconsistent models, one may instead perceive individuals as unable to make observable conceptual changes or as maintaining misconceptions despite seemingly obvious evidence to the contrary.

The Sociality Corollary. Within the framework of cognition as situated in activity, context and culture, the Sociality Corollary has particular relevance for education. This corollary explains that it is not by having the same or a similar construct system that one person plays a role in the learning process of another, but by understanding the other person's construct system. The educational applications of understanding students' construct systems should be distinguished from the teaching methods used by an instructor, since an instructor may engage students with the goal of understanding their personal constructs and co-constructing knowledge through any number of instructional methods. However, if an instructor fails to understand the students' knowledge and ways of understanding, he or she may miss or fail to provide opportunities to bring students into a new way of thinking about, understanding, and in this case teaching a particular domain. Researchers, too, need to understand the schemata held by teacher educators regarding the learning and teaching of content. To be meaningful and effective, teacher preparation reform must be a social and co-constructed process and researchers cannot expect teacher educators to engage in this reform unless they first believe that researchers understand and accept the way they construe the complex system of learning and instruction.

Technology as Situated

As a culturally transmitted system, human language and the meanings of words change over time in response to social, cultural and historical shifts in communication. The Greek origin of the word "technology" suggests a meaning related to art, craft, or skill and indicates process or activity (Merriam-Webster, 2003). Historically, technology was

interpreted broadly and associated with topics such as cultural expression, fine arts, and social order, as well as more familiar areas such as medicine, agriculture, and transportation (Carlson, 2005). In nineteenth and twentieth century Western society, "technology" was used in frequent proximity to topics such as "information," "computer," or "industrial." Through the semantic process of contextual specialization, in which the meaning of a word changes in response to frequent co-occurrence with another word (Henning, 2005), people began dropping the descriptors until, for many, "technology" came to be synonymous with hardware or machinery. This shift has resulted in a misconception of technology as an object rather than a system of knowledge, skills, and organization. Integration of technology in education reflects this shift through the emphasis on how to integrate a tool, not the ways in which technology can mediate interaction between learners and teachers, among learners, and between learners and tools (Kozma, 1991; Kozma, 1994) or how the use of media might transform cognition and learning (Pea, 1987).

This conception of technology as situated in context parallels the argument that knowledge cannot be abstracted from the context in which it is developed, but is inherently linked to the activity, context, and culture in which it is learned (Brown, Collins, & Duguid, 1989). Past emphasis on technology as a stand-alone tool with pre-determined constraints and affordances denies the inherent ecological interactions that occur between object, environment, and the human psychological system (Gibson, 1977). More recently, new frameworks such as Technological Pedagogical Content Knowledge have emphasized technology as context-dependent and argued that using it effectively requires an understanding of the relationship between technology, pedagogy, and content (AACTE Committee on Innovation and Technology, 2008; Mishra & Koehler, 2006).

Technological Pedagogical Content Knowledge

One criticism of studies that explore the integration of technology in higher education and education more broadly is the lack of a clearly articulated theoretical or conceptual framework. Many researchers in the field of educational technology pursue lines of research grounded in meaningful frameworks that guide their research design, methods, data collection and analysis and contribute in kind, however many practitioners and faculty members outside the field of education who investigate their own practices fail to clearly situate their research within the existing literature and articulate how their study advances the existing knowledge and theoretical understanding of the field. Although no one framework can or should be used to frame all research related to technology integration in education, recognition of the complex domain formed by the integration of knowledge, design, and technology and the need to bridge education research and practice led Mishra and Koehler to pursue a body of work contributing to theory, methodology, and practice of integrating technology in learning environments (Koehler, Mishra, Hershey, & Peruski, 2004; Koehler & Mishra, 2005; Koehler, Mishra, & Yahya, 2007) and develop a model they call Technological Pedagogical Content Knowledge (Mishra & Koehler, 2006).

Technological Pedagogical Content Knowledge extends the notion of pedagogical content knowledge introduced by Shulman (1986) in his landmark piece that described the intersection of pedagogy and content, previously thought of as two mutually exclusive domains within teacher education. Shulman recognized that a teacher must have knowledge of pedagogy and also knowledge of content, but that simultaneous consideration of the two bodies of knowledge at once should be considered a new type of specific knowledge needed to represent, explain, and transform the subject matter in order to teach it. As an example, the procedural steps that form the instructional process of problem-based learning and

transport across context and content can be categorized as pedagogical knowledge, while the curriculum, in the form of real-world problems, individual studies, and team collaboration embedded within the context of mathematics content knowledge, can be categorized as pedagogical content knowledge. Mishra and Koehler (2006) visually represent this intersection of pedagogy and content as shown in Figure 1.

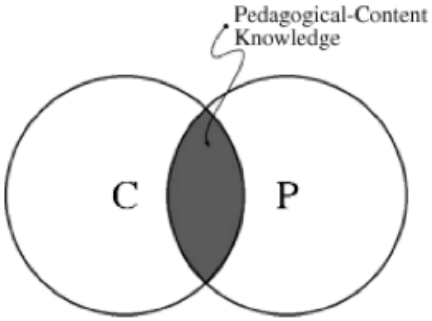


Figure 1. Pedagogical knowledge and content knowledge joined by pedagogical content knowledge (Mishra & Koehler, 2006, p. 1022).

To these two circles, Mishra and Koehler propose the addition of a third, representing technology. Prior to the evolution of computing technology, they explain, the technology used in classrooms was so commonplace that it was not thought of by most as technology at all. Indeed, this perception of textbooks, chalkboards, pencils, manipulatives, and other "non-technologies" continues today. In contrast, the social redefinition over the last ten to fifteen years of "technology" to mean hardware, software, and other similar machinery has resulted in these items becoming a very visible part of the classroom that, while part of the teaching and learning environment, also place new requirements on teachers to integrate them in meaningful ways. The perception of many individuals of technology as a separate set of knowledge and skills that need to be acquired in addition to those related to pedagogy and content means that the third circle representing technology is rendered separate and distinct from the other two (see Figure 2). It seems reasonable to infer

that an instructor's individual conceptions and beliefs about technology broadly, and particular hardware and software more specifically, would strongly affect the extent and way in which she integrates technology in her teaching and student learning. Given the rapid evolution of technology, an instructor who sees only the "how" of technology and not the "why" may focus on the procedural steps of learning how to use a tool or become overwhelmed with the myriad of technologies available and the self-perceived gap in her skill set and decide not to make any attempt to stay current with new tools as they become available.

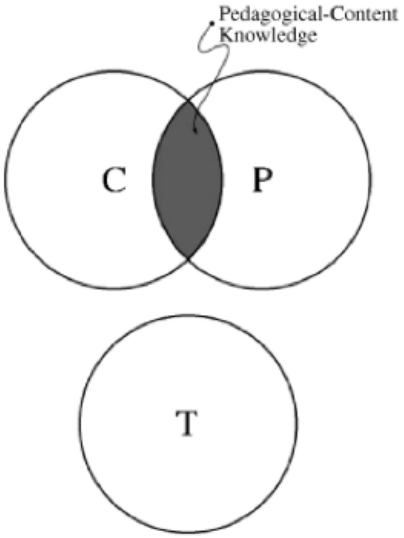


Figure 2. Intersecting domains of pedagogical knowledge and content knowledge, with technology represented as a separate domain of knowledge (Mishra & Koehler, 2006, p. 1024).

In comparison, an instructor who reflects on the way he or she wants to influence student cognition by mediating some learning activity with technology places the learning objective first and then focuses on identifying and using the technology most appropriate for the content and pedagogy. Or, alternatively, she may explore new opportunities for learning made available by a technology and evaluate how she might change her representation of the

content and pedagogy accordingly (Koehler et al., 2011). This refocuses the integration of technology on ways of representing, explaining, and transforming not just content, but pedagogy and content both, in order to mediate interaction among learners or influence cognitive, motivational and social elements of the learning environment (Kozma, 1994; Pea, 1987). In the same way that the procedural steps of problem-based learning intersect with mathematics content knowledge to inform the design and selection of the problem themselves, technology too intersects with pedagogy, intersects with content, and intersects with pedagogy and content. This integrated model proposed by Mishra and Koehler (2006) emphasizes the complex interplay between each of the three components: pedagogy, content, and technology (see Figure 3).

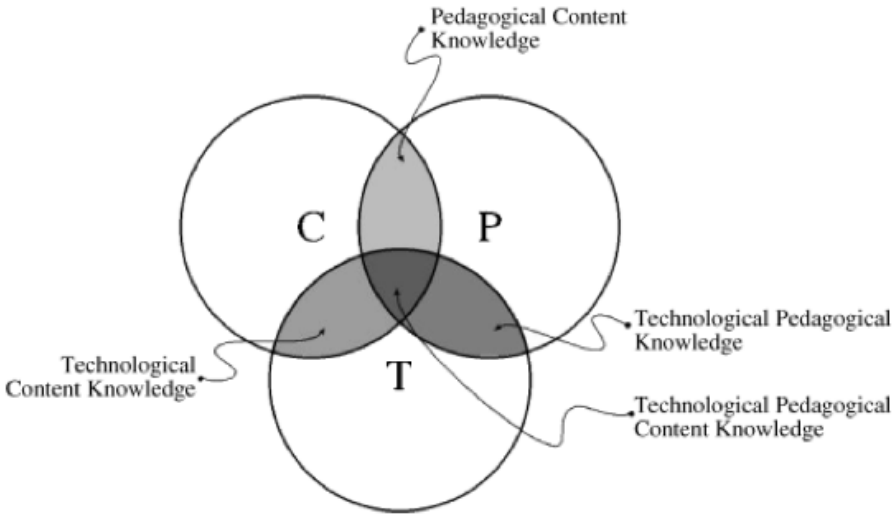


Figure 3. Overlapping domains of pedagogical knowledge, content knowledge and technological knowledge (Mishra & Koehler, 2006, p. 1025).

The TPACK framework represents very clearly the theoretical position that technology should not be considered as context-free and that complex interactions exist between content, pedagogy, and technology. However, we can assume that the relationships illustrated in the diagram are not the same as those conceived by instructors or enacted in

their classrooms, either for deliberate reasons or as a result of influencing variables. A teacher may conceive of the relationship between these three constructs very differently, such as the separation of technology from pedagogy and content as illustrated by Figure 2, or perhaps technology as a tool for rendering graphs of mathematical equations but of little use for facilitating interaction among students. Alternatively, an instructor may conceptualize the relationship of these constructs in a manner similar to Figure 3, but be required to implement a curriculum that restricts them to certain content, pedagogy or technology, have limited technology available in their classroom or school, or need to alter their planned curriculum in response to student demand or need. If we assume that the ratio and overlap of each representative circle is not conceived of in the same way by every individual and is influenced by other factors, Figure 4 illustrates one visual representation of how an individual might conceive of or enact the relationship between these variables. In designing professional development for teacher educators or teacher preparation programs to promote the integration of pedagogy, content, and technology, it would be useful to first understand the way instructors conceive of and enact the relationship of these three constructs and why.

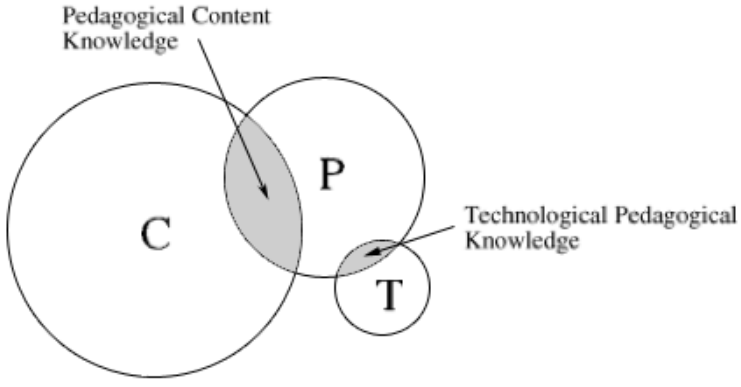


Figure 4. Sample diagram representing a possible rendering of an individual's enactment of the relationship between pedagogy, content, and technology.

Research methods for measuring TPACK. The development and refinement of the TPACK framework emerged from design-based research conducted on a series of "Learning Technology By Design" seminars. In these seminars, students work in teams to develop technology-based solutions to authentic educational problems such as creating a video that explains an educational concept or redesigning a Web site. One such seminar provided the context for a study that explored the development of TPACK through the authentic task of designing an online course (Koehler et al., 2007). In the study, the researchers used quantitative discourse analysis to segment observation field notes into discourse episodes and code the episodes as content, pedagogy, technology, or some combination of the three. The associated analyses revealed that students shifted from discussing content, pedagogy, and technology as three separate topics to discussing them as interconnected constructs.

Mishra and Koehler later proposed discourse analysis as a useful research method for analyzing patterns, characteristics, and relationships in classroom talk across the major categories of content, pedagogy, and technology (Koehler, 2013). This dissertation study employs quantitative content analysis to identify the frequency and duration of classroom discourse related to pedagogy, content, technology, and combinations of the three. The conceptual framework of technology as situated that underlies this study provides a rationale for exploring the three related constructs of technology, pedagogy, and content, rather than technology as a discrete construct separate from the other two domains of knowledge.

CHAPTER 3

RESEARCH DESIGN AND METHODS

This study explored the instructional strategies that three teacher educators believe are most critical to preparing elementary educators to teach mathematics at the K-8 level with the goal of understanding how the instructors conceive of and enact the relationships between pedagogy, content, and technology.

Research Questions

The overarching research questions for this study on instructor conceptions about the teaching of mathematics and the enactment of those beliefs in the context of a pedagogy course on mathematics education include the following:

1. What are teacher educators' personal theories about the instructional strategies most critical for preparing future mathematics teachers?
2. How do teacher educators enact their personal theories in the classroom?
3. What does classroom discourse reveal about the relative emphasis and relationship between pedagogy, content, and technology as enacted by teacher educators?
4. What is the function of technology in teacher educators' personal theories about the teaching of mathematics and their practices as enacted in the classroom?

Research Design

The complex research questions regarding teacher beliefs, teacher practices, classroom discourse, and the role of technology within those beliefs and practices warrant the use of "multiple methodology," or the use of at least one quantitative and one qualitative method in tandem (M. L. Smith, 2006). This study employs a triangulation mixed methods design (Creswell, 2005; Hammersley, 1996), in which quantitative and qualitative data are

collected at the same time and both are equally valued during analysis and interpretation. Such designs endeavor to use the strengths of one method to offset the weaknesses of the other. For example, the challenge of generalizing quantitative data to naturally occurring situations (Neisser, 1976) is offset by the contextual information provided by qualitative data, and the dependence of qualitative studies on the context in which the data is collected is offset by the external validity and generalizability provided by a well-designed quantitative study.

In this study, the use of repertory grids to conduct structured interviews provided a method for quantifying qualitative data, as did the content analysis conducted with transcripts from in-class observations. To provide insight regarding the cognitive processes linking the instructors' beliefs about methods of teaching mathematics and their enactment of content, pedagogical and technological events in the classroom, I conducted post-observation retrospective protocols with each instructor. Table 1 outlines the research questions, data sources, and methods of analysis used in this study.

Table 1
Alignment of Research Questions, Data Sources and Methods of Analysis

	Conceptions	*Enactment*	*Retrospection*
Research question	What are teacher educators' personal theories about the instructional strategies most critical for preparing future mathematics teachers?	How do teacher educators enact their personal theories in the classroom? What does classroom discourse reveal about the relative emphasis and relationship between pedagogy, content, and technology as enacted by teacher educators?	What is the function of technology in teacher educators' personal theories about the teaching of mathematics and their practices as enacted in the classroom?
Data source	Structured interviews: transcripts	Classroom observations: field notes and transcripts	Semi-structured interviews: transcripts
Methods and analysis	Repertory grids: process analysis, eyeball analysis, construct characterization, cluster analysis	Quantitative content analysis: frequency analysis, chi-square test of independence, linear regression	Retrospective protocol: content analysis of verbal reports

Participants

In spring 2012, a research-intensive university in the southwestern United States offered 11 undergraduate sections of methods of teaching mathematics to students at the K-8 level. Nine instructors taught the 11 sections: one faculty adjunct, five clinical instructors having some or no background in mathematics education, and three full-time faculty members in the field of mathematics education. To accommodate students completing a full-year student teaching experience in a K-8 school, all senior-year education coursework was offered in a university classroom in the school districts where students completed their clinical experiences. This meant that in spring 2012, 11 undergraduate sections of methods of teaching mathematics in elementary schools were taught at ten site-based locations around the metropolitan area and the state. The geographic spread of the observation sites as well as the times at which the classes were offered (concurrent or consecutive sections at different sites on Tuesday, Wednesday, and Thursday of each week) meant it was not possible for me to observe all sections of the course.

To address this constraint, after obtaining IRB approval (see Appendix A) I invited the three full-time faculty members in mathematics education to participate in the study (see Appendix B), with the rationale that over the period of their employment at the university those instructors would teach the largest number of students in the undergraduate elementary education teacher preparation program. Two of the instructors each taught one section of the course and one taught three sections; I conducted one interview with each instructor and, for the instructor who taught multiple sections, conducted all observations in one section of the course. This sampling allowed me to explore the beliefs and practices of the full-time faculty most involved in the design of the mathematics methods course and the preparation of undergraduate teacher candidates to teach elementary education mathematics.

In asking the three full-time faculty members teaching during the semester of study to participate, I endeavored to draw a sample that shares salient characteristics with full-time mathematics education faculty at comparable institutions. However, in order to shield the identity of the teacher educator participants, I have chosen to present their background information in an aggregate summary. All three participants completed a bachelor's degree in elementary education, taught elementary education for three to seven years with one of them specializing as a mathematics teacher for an additional three years, then completed a master's degree in elementary education with an emphasis on mathematics education. In addition, they all hold a doctoral degree in mathematics education from a public university classified as "very high research activity" in the Carnegie Classification of Institutions of Higher Education (Carnegie Foundation for the Advancement of Teaching, 2012), with the degrees conferred 15 to 20 years prior to this study. At the university, two of the instructors hold tenured positions and one a non-tenure line position, all focusing on mathematics education at the elementary school level. The two tenured faculty have been awarded grants for mathematics-related research, curriculum development, and teacher or teacher educator professional development totaling close to 5 million dollars over the last 20 years. Professional publications related to mathematics education by the three participants include more than 20 refereed journal articles, 8 book chapters, over 10 published conference proceedings, and more than 60 presentations at conferences held by organizations including the American Education Research Association, the National Council of Teachers of Mathematics, and the School Science and Mathematics Association.

Constraints that became Opportunities

The affiliation agreements between the university and the school district partners where the university offers its site-based teacher preparation courses require the provision of

a classroom meeting certain requirements, including actively maintaining an instructor computer, providing wireless access for students using personal laptops, and installing a SMART Board, projectors, and printer. With eager anticipation, I entered Cassie's classroom in spring 2012 for my first observation of the study. I chose a back corner to set up the video camera where I would have good visibility of the instructor and the students, as well as access to a table of five to six students seated next to me. After setting up my audio and video recorders, I sat down to take some notes on the classroom setting: how many students, the location of the tables, the type of equipment…and realized there was no digital technology available in the classroom other than what the students had brought with them – not even a computer station and overhead projector. Throughout the observation, my mind was turning over the unexpected challenge this would pose for my study. Walking out with the instructor after class, I asked her about the lack of a computer in the classroom, which I understood to be standard equipment the district needed to provide. She said that the previous week, when she had taught the first class, she had found out from the site coordinator that in order to comply with the requirement that a computer be available, an instructor could request that a computer station be delivered to the classroom (or picked up by the instructor at the front office and wheeled to the classroom) and returned afterwards. Rather than pick up and drop off a computer station each week, the instructor elected to teach without the computer station and overhead projector. Had the computer been in the room, she said she would have used it, but since it wasn't central to her teaching and it was inconvenient to get it set up each week, it was easier to modify her instruction and do without it.

Driving back from this first observation, I was very discouraged – how could I conduct my study as planned? Was it fair to conduct the same analysis for all three

participants if one of them did not have the same access to certain equipment in the classroom? If digital technology were really central to the instructor's teaching, wouldn't she have made the extra effort to pick up, set up and return the equipment each day? In my field notes for each of my observations that week, I carefully documented the use of certain tools and techniques (such as Base 10 blocks, marshmallows, and strategy sharing), noting that each instructor used different tools and instructional strategies. I interpreted this as a choice made by each instructor about the instructional strategies central to his or her teaching beliefs. So why, then, did I place more weight on one instructor's choice not to use digital technology even though it was less convenient for her than for the other instructors? Wasn't she making a choice, just as the other two instructors had made a choice about whether to bring certain objects to class, such as manipulatives they owned, created from ordinary objects, or checked out or borrowed from the university or school district resources? Why did I feel that the lack of technology in one classroom derailed my study? Although there was no way to know *how* she would have used a computer station were it located in her classroom, I didn't feel that it would be a fair comparison if I analyzed the data for her class the same way as for the other two instructors. But why not?

The answer was that I had inadvertently prioritized the integration of digital technology over analog technology in the classroom. I fell into the same trap that captures many of us in this modern age – that technology is limited to the set of objects and tools that you plug in, run off of a battery, or access via some device. I had somehow gotten lost and betrayed my own beliefs in technology as a system, rather than a set of tools, and it was thanks to the absence of a computer station in a classroom that I found my way back.

Eloquent definitions of terms such as "cognitively oriented technology innovations" (Fishman et al., 2004), "technological pedagogical content knowledge" (Mishra & Koehler,

2006), and "cognitive technologies" (Pea, 1987) have helped change the way people think about technology's potential and use in education. What if we play with some of these new categories and distinctions in the fashion described by Shulman (2002) and apply them to "normal" classes – not classes focused on integrating technology in K-12 classrooms, or classes where use of technology plays a strategic role, or professional development aimed at increasing the use or changing the use of technology by teachers or teacher educators. What would I learn about the way people teach if I used a framework typically associated with digital technology to explore teaching in "regular" classes? What would I learn about the way instructors integrate technology – analog technology – and the function of that technology in their teaching?

Data Collection

This research study examines a set of data collected over the course of one 15-week academic semester from a representative sample of the three full-time faculty members who served as the instructor of record for five out of 11 sections of methods of teaching mathematics in elementary education at the undergraduate level. In this study, I explored how the three instructors construe the instructional strategies they consider to be central to the teaching of mathematics, and the ways in which they enacted the relationships between content, pedagogy, and technology in their classroom instruction. To model the instructors' beliefs about the teaching methods they feel are most critical to preparing teachers in mathematics education, I employed the repertory grid technique. Content analysis of the classroom discourse revealed the relative emphasis and relationship of pedagogy, content, and technology in each observed undergraduate teacher preparation mathematics methods course. Retrospective protocols conducted after each observed class allowed me to explore

the reasoning and individual choices made by an instructor during "critical incidents" in the classroom.

Structured Interviews to Map Semantic Networks

One to four weeks prior to the first in-class observation of each participant, I engaged in individual interviews to elucidate information regarding the way each instructor gives meaning to, or construes, the teaching methods she considers to be most critical for preparing undergraduate education students to teach mathematics at the K-8 level. Procedures for eliciting repertory grids were adapted from those outlined by Kelly (1955) and the techniques detailed by Jankowicz (2004). The goal of each interview was to obtain a precise description of the interviewee's constructs and values for the selected topic of "methods of teaching mathematics" as described in their own terms.

In selecting a topic for the structured interviews, I initially considered using "technology," "pedagogy," and "content" as elements chosen by me to elicit how and what each instructor thought about the three terms, their relationship and relative importance. However, three elements are too few to elicit a representative list of constructs. Another option, that of using the specific lessons observed as elements in the grid, was also considered but set aside because I did not want to limit the repertory grids to the teaching methods used in the classroom, but instead develop models about the teaching methods each instructor identified as most critical to preparing future mathematics teachers and why. Using "instructional practices for preparing future mathematics teachers" as the topic allowed me to develop an understanding of how and what an instructor construed as the most critical teaching methods and then compare that model to what was observed in the classroom. Asking the participants to provide the elements in the repertory grid ensured the topic was represented from the instructor's point of view. Individual areas of emphasis, such

as teaching methods that emphasized content over pedagogy or technology, emerged accordingly.

Single-person, face-to-face structured interview sessions were scheduled with each participant to develop their repertory grid. The verbal interviews were conducted by me and recorded and later transcribed for reference. Although I wanted to set up a quiet location to minimize interruptions from in-person or phone interruptions, this proved challenging in two of the three interviews. In one case, we had to change the interview location approximately 45 minutes into the 90-minute session; in the other, ongoing interruptions via phone and the arrival of a colleague resulted in a decision to stop for the day and complete the interview a few days later.

Since eliciting a repertory grid is a collaborative process, during the interview I sat side by side with the interviewee so as to discuss and review the grid together as it was developed. Prior to beginning the interview, I reviewed the purpose of the interview as part of a doctoral research study, a brief description of the repertory grid technique, and conditions of confidentiality and anonymity (see Appendix C).

Using a blank grid sheet like the one shown in Figure 5, I filled out the details of the grid as provided by the interviewee. The topic to be addressed during each of the structured interviews was "instructional practices for preparing future mathematics teachers" and the guiding question used to elucidate elements was "what instructional practices do you consider most critical for preparing undergraduate elementary education students to teach mathematics?" Jankowicz (2004) describes four alternatives for determining how to choose elements to include in the repertory grid: the researcher chooses the elements, the interviewee chooses the elements, the researcher and interviewee choose the elements together, or the researcher elicits elements by providing general categories to which the

interviewee responds with a named element. For this study, I allowed the interviewee to choose the elements for discussion since this ensured that the topic was represented from the interviewee's point of view as a teacher educator preparing future elementary education teachers in mathematics education. I did not, however, want the interviewee to omit issues that are central to this research study and the exploration of how each instructor conceives of the relationship between technology, pedagogy, and content. For this reason, at one point during the interview I asked each interviewee to describe a situation that was important to his or her conception of what students need to be developing as future elementary education mathematics teachers that also incorporated technology in some significant or meaningful way. The intention for introducing this question was to activate the consideration of technology by the interviewee while identifying additional elements for later discussion and rating, however the response to this question did not yield meaningful elements that were added to the grids. Through a brief 5-10 minute discussion, I asked each interviewee to identify 7-9 elements that represented the range of instructional practices she wished to discuss. These elements were written down as individual elements across the top of the repertory grid form and also written on individual index cards, for use when eliciting constructs.

Topic:	Instructional Practices for Preparing Future Mathematics Teachers
Elements:	7-9 different instructional practices considered by the teacher educator to be most central to preparing undergraduate elementary education students to teach mathematics
Constructs:	8-12 constructs elicited from the instructor
Ratings:	5-point scale

Emergent Pole (what two elements in the triad have in common)	*Element 1*	*Element 2*	*Element 3*	*Element 4*	*Element 5*	*Element 6*	*Element 7*	*Element 8*	*Element 9*	*Element 10*	*Implicit Pole (what one element in the triad does not share with the other two)*
Construct A1											*Construct A2*
Construct B1											*Construct B2*
Construct C1											*Construct C2*
Construct D1											*Construct D2*
Construct E1											*Construct E2*
Construct F1											*Construct F2*
Construct G1											*Construct G2*
Construct H1											*Construct H2*
Construct I1											*Construct I2*
Construct J1											*Construct J2*
Construct K1											*Construct K2*
Construct L1											*Construct L2*

Figure 5. Empty repertory grid sheet with location of elements and constructs indicated.

The remaining steps in the repertory grid procedure aimed to elucidate how and what the interviewee thinks about the elements. Through a systematic comparison of each of the elements, I intended to find out *how* the interviewee thinks by capturing the constructs she used to describe and differentiate the elements, then *what* the interviewee thinks by analyzing the ratings she assigned to each element using each construct as a rating scale (Jankowicz, 2004).

Working from the list of elements, I selected three of the elements (such as elements 1, 3 and 5) and asked the instructor which two of them were the same in some way and different from the third. I provided the interviewee the index cards on which the elements

were written as a way to help him or her focus on the three elements and the construct that differentiated them. I assured each interviewee that there was no correct answer, but that the goal was to understand how the interviewee construed each element. When the interviewee identified which of the two elements were the same, I asked what it was the two elements had in common that the third element did not. This term or phrase was written as Construct A1 in the emergent pole of the repertory grid form (see Figure 5). I then asked the interviewee to articulate what word or phrase would express a contrast, with the goal of obtaining a bipolar expression that had two clearly contrasting poles, not merely poles in which one term is the logical opposite of the other (such as "applied" and "not applied"). This was written down as Construct A2 in the implicit pole of the repertory grid form and together the pair of words or phrases represented one of the interviewee's constructs. Before continuing, the interviewee and I briefly discussed the construct provided to ensure that both of us had a shared understanding of what the interviewee intended in using that particular term or phrase and that the construct written in the repertory grid form accurately represented the interviewee's meaning.

Next, I presented the construct to the instructor as a rating scale with the term or phrase in the emergent pole defining the "1" end of a 5-point scale and the term or phrase in the implicit pole defining the "5" end of a 5-point scale. A 5-point scale was selected for the purposes of this study so as to allow for a "middle" position when rating each element, as compared to a 4-point or 6-point scale. If a scale with an even number of points had been used, it might have forced a preference for one pole of the construct or another that was not actually representative of the interviewee's position. Starting with the three elements initially related to the construct, I asked the interviewee to rate each element on the scale and wrote the ratings in the grid as they were given. Then, the interviewee was asked to rate each of the

remaining elements using that construct. At the end of this step, the first row of the repertory grid was complete.

To complete the grid, I repeated the series of steps required to elicit a construct, beginning by providing a different triad of elements (such as 2, 4, and 6; 3, 5 and 7; 4, 5 and 9; etc.), then asked the interviewee to rate each of the three elements on the 5-point scale represented by the two poles of the construct and then rate the remaining elements on the scale. The goal was to elicit between 8 and 12 constructs, with the process being repeated until the interviewee could not offer any new constructs that had not already been discussed.

Analysis of the verbal data gathered from the individual interviews using the repertory grids technique was conducted following the analysis methods outlined by Jankowicz (2004). The research emphasis is in developing models that represent the way each individual conceives of the teaching methods they consider to be most critical to preparing teachers in mathematics education. Two types of analysis were conducted: descriptive analysis of the grid contents, and cluster analysis of the grid structure and by extension the relationships between elements and constructs.

Developing a description of each grid formed the first step in the grid analysis. The purpose was to examine the immediately obvious relationships and gain a general understanding of *what* the interviewee thought about the topic (through the constructs elicited) and *how* they thought about the topic (through the ratings of elements on constructs). Three descriptive analysis techniques were used: process analysis, eyeball analysis, and construct characterization (Jankowicz, 2004).

A process analysis was completed shortly after conducting each interview, with the intention of reflecting back on the interview process to note observations about the

interviewee's stance toward the topic. Table 2 provides the list of questions that guided the review of the interview and helped identify any process issues.

Table 2
Questions to Guide Process Analysis (Jankowicz, 2004, p. 78-80)

Grid Component	*Guiding Questions*
Topic	How did the interviewee react to the introduction of the reasons for doing the grid?
Elements	Were all elements that the interviewee proposed used in the grid? If not, which ones were not used and why? Does this provide any information about the way the interviewee perceives the topic?
	If the researcher proposed elements, how did the interviewee respond to the elements that were proposed?
Constructs	How did the interviewee respond when the qualifying phrase was used to help the interviewee address constructs particularly relevant to the topic?
	Was the qualifying phrase useful in helping to elicit constructs? Or was it avoided by the interviewee?
	Which constructs required more thought to generate than others?
	What general impressions were formed about the kinds of constructs being offered during the elicitation process? Do they seem to indicate anything on their own, prior to a formal analysis?
Ratings	Did the ratings procedure appear to be meaningful to the interviewee? Did it make sense to them and seem like a credible interview technique?
	Were there any elements that the interviewee found it especially difficult to rate on particular constructs?
	Did the interviewee make any particular comments during the grid elicitation process that seemed meaningful about the procedure itself?
	Was there at any time a departure from the usual grid elicitation procedure? Was this a satisfying grid interview or not?

The eyeball analysis served to provide a simple description of what the grid presents as a whole. The overview begins by stating the topic of the grid, along with any qualifying phrases that were used during the grid elicitation. The elements are then described, including how many were included in the grid, how they were agreed upon, whether any were chosen by negotiation between the interviewee and myself, and how long it took to arrive at the final list. This is followed by a description of the constructs, including how many were obtained and how long they took to elicit. The constructs used by the interviewee to make sense of the topic are described with emphasis paid to the way the interviewee distinguished the emergent from the implicit pole. The rating scale is then reviewed, along with whether there

are any particularly obvious characteristics about the ratings overall, such as whether the interviewee used mostly the two ends of the scale, provided mostly neutral ratings, or omitted ratings because they felt certain constructs did not apply to certain elements. Then, a review of what the interviewee said about each element is provided by reading through and briefly describing the ratings for each column on each construct. The analysis also describes in particular the ratings given for any elements that were chosen by negotiation between the interviewee and myself. The overview concludes with a summary of the main points and initial observations that I developed.

The analytic technique of characterizing constructs provides a mechanism for evaluating whether some, or all, of the elicited constructs were of a certain type and determining whether this is significant given the aims of this research study and the grid topic. Types of constructs considered included core versus peripheral constructs (to identify constructs central to the interviewee's identity), propositional versus constellatory constructs (to identify constructs that inherently indicate how an element will be rated on other constructs as well), pre-emptive constructs (to identify constructs which preclude the presence of other constructs), and other types as appropriate. The analytic procedure for conducting each construct characterization included identifying constructs of that type (such as core, constellatory, or pre-emptive), assessing how many of the full set of constructs are of that type, determining whether the presence or absence of that type of construct was significant, and assessing how constructs of that type related to other constructs in the set. In some cases, there were no constructs of these types or their presence was not significant given the grid topic, so I have not over-interpreted the constructs at this time.

After completing the descriptive analyses, I conducted a cluster analysis of each grid structure to highlight and explore the relationships between elements and constructs. I opted

to use a divisive model of a cluster analysis rather than a network analysis that would analyze the relationship and distance between nodes within a network. This latter method of analysis emphasizes the strength of the relationship between individual nodes, whereas a cluster analysis brings into focus the similarities between constructs and between elements to highlight patterns and subgroups within the larger dataset. Themes identified through the cluster analysis provided information regarding the personal theory held by each instructor regarding the teaching of mathematics education and also formed the basis for probing questions asked during the retrospective protocols after each classroom observation.

Each grid contained at least six elements and six constructs, meeting the requirements for using a cluster analysis. I used WebGrid 5 (Centre for Person-Computer Studies, 2010), an online grid elicitation and analysis tool, to conduct the cluster analysis of the repertory grids. The steps outlined in Table 3 were applied to each cluster-analyzed grid produced with the WebGrid software package.

Table 3
Steps for Interpreting a Cluster-Analyzed Grid (Jankowicz, 2004, p. 123-125)

Step	*Description*	*Details*
1	Examine the [elements/constructs]	Note which [elements/constructs] have been reordered and are now next to each other.
2	Examine the shape of the [element/construct] dendogram	Note how many major branches the dendogram has, indicating how many distinct clusters of elements exist.
3	Identify [construct/element] similarities and differences	For each dendogram cluster, identify the [constructs/elements] on which these [elements/constructs] receive similar and different ratings.
4	Explore what this means	Reflect on the meaning of the similarities and differences of [element/construct] ratings within each cluster. If the interviewee is available, discuss the possible significance and implications with him or her.
5	Find the highest % similarity score	Find the two adjacent elements with the highest % similarity score, then find the pair of adjacent elements with the second highest % similarity score. Determine whether the second pair forms a separate cluster form the first pair or whether they belong to the same cluster.
6	Examine the remaining scores	Repeat the process of comparing % similarity scores, evaluating clusters, and reflecting on (or discussing with the interviewee) the significance of the clusters.

Note. The procedure was completed to interpret the cluster-analyzed elements first, then the constructs.

Classroom Observations

At the end of each interview, I asked each instructor to review their spring class schedule with me and identify which classes she believed to be the five most important sessions related to methods of teaching mathematics. In identifying these five lessons, I asked the instructors to reflect on which sessions contained the most critical information for a future elementary education teacher when teaching mathematics. The guiding question I used to prompt for this information was, "If a student were to attend only five class sessions this semester, which five would you identify as the most critical for examining methods of teaching mathematics in elementary education?" In asking the instructor to identify the five most critical lessons, I was cautious to not reveal an interest in any one category of technology, pedagogy, or content over another. Table 4 lists the topics of the class sessions observed as described in each instructor's course syllabus.

Table 4
Topics of Classroom Observations

Week	*Cora*	*Jamie*	*Cassie*
2		Problem solving Chapter reading: • Planning the problem-based classroom • Developing early number concepts and number sense	Problem solving NCTM Standards Chapter reading: • Teaching through problem solving • Planning the problem-based classroom
3	Early number sense Developing meaning for operations Chapter reading: • Developing meanings for the operations • Helping children master the basic facts	Early number sense Developing meaning for operations Chapter reading: • Developing meanings for the operations • Helping children master the basic facts	Addition/Subtraction Multiplication/Division CGI Chapter reading: • Developing meanings for the operations • Helping children master the basic facts • Strategies for whole-number computation • Computational estimation with whole numbers

4			Number sense Place value concepts Chapter reading: • Developing early number concepts and number sense • Whole-number place-value development
6	Numeration systems Number theory: Factors, multiples, primes, composite, identity Place value Chapter reading: • Whole-number place-value development • Algebraic thinking: Generalizations, patterns, and functions	Numeration systems Number theory: Factors, multiples, primes, composite, identity Place value Chapter reading: • Whole-number place-value development • Algebraic thinking: Generalizations, patterns, and functions	
8	Number sense, equivalency, and comparison of fractions Chapter reading: • Developing fraction concepts	Number sense, equivalency, and comparison of fractions Chapter reading: • Developing fraction concepts	Fractions Chapter reading: • Developing fraction concepts • Computation with fractions
11	Multiplication and division of rational numbers Chapter reading: • Developing measurement concepts • Geometric thinking and geometric concepts		Decimals and percentages [c] Chapter reading: • Decimal and percentage concepts and decimal computation
12	Data analysis [a] Chapter reading: • Concepts of data analysis • Exploring concepts of probability	Engineering [b] Chapter reading: • Developing concepts of exponents, integers, and real numbers • Proportional reasoning • Exploring concepts of probability	

[a]This observation removed from Cora's dataset. [b]This observation removed from Jamie's dataset. [c]This observation removed from Cassie's dataset.

After obtaining instructor consent in accordance with Institutional Review Board regulations regarding research studies involving human subjects, I conducted observations of five classes for each instructor during the spring semester. Each class session was recorded

with a digital video camera and audio recorder and the tapes were transferred to a secure hard drive after each class session. I took detailed field notes using my laptop during the class sessions, noting events or dialogue of particular interest along with the times when they took place.

Erickson (1986) notes three strengths and two limitations related to use of a recording device. Use of a video recorder allowed me to fill in details there was not time to write down during the observations, but I did not rely only on the recording but sat in the room directly so as to understand contextual information that was not be captured on film. Notes and sketches created while sitting in the classroom were invaluable when dialogue or exchanges captured digitally were unclear. My in-person notes captured some student interaction not visible through the camera lens. Also, since new lines of inquiry and exploration become of interest and relevance partway through the study, I was able to return to the observation videotapes and replay the footage, noting different activities and interactions than were initially captured in my field notes.

Coding and analysis of the data gathered from the classroom observations were conducted using content analysis (Berelson, 1971) with the goal of developing models that represent the relationship of pedagogy, content, and technology as enacted by the instructor and students in each undergraduate mathematics methods course observed. I employed quantitative content analysis, rather than qualitative or a combination of the two, to objectively and systematically identify the frequency and duration of classroom discourse related to pedagogy, content, technology, and combinations of the three. The analysis employed a procedure similar to that of Constant Comparative Analysis (Glaser, 1965), but followed the specific steps outlined by Riffe, Lacy, & Fico (1998).

The first step required drawing representative samples of content. I observed and collected data on five lessons identified by each instructor as the most important sessions related to methods of teaching mathematics. After completing the observations, I removed one observation from each instructor's dataset, the reason being that one instructor asked a guest lecturer to conduct two-thirds of the lesson observed and another instructor organized a lesson so that six student groups presented activities on a series of assigned topics. The sessions removed were the last observed for each instructor, so I removed the last observation for the third instructor as well. Therefore, observation data from four lessons for each instructor were analyzed. These lessons represent almost one-fourth (12 of 45 hours) of the in-class instruction required of a three-credit course – likely more by the time an instructor reviewed the course syllabus and schedule at the beginning of the semester, administered an in-class final at the end of the semester, and canceled one class to accommodate the spring break schedule of the local school district where the classes were conducted.

Using the audio and video recordings as my primary data sources, I created transcripts of the classroom discourse from the observed lessons for each course and then expanded the transcripts to include observable actions, figures, or models from the video footage and my field notes. These transcripts served as the representative samples of content for analysis. Within the transcripts of each the four lessons observed for the three instructors, I identified the major discourse episodes, or "coherent sequences of sentences of a discourse, linguistically marked for beginning and/or end, and further defined in terms of some kind of 'thematic unity'" (van Dijk, 1981).

To analyze and categorize the discourse episodes, I followed the approach outlined by Miles and Huberman (1994) to develop an initial list of *a priori* codes (see Table 5) drawn

from the TPACK framework (Mishra & Koehler, 2006), with the goal of identifying the focus of discourse by the instructor and students. As articulated in the TPACK framework, it is assumed that discourse episodes may reflect one element in isolation (technology, pedagogy, *or* content), or more than one element (technology *and* pedagogy, pedagogy *and* content, technology *and* content, or technology *and* pedagogy *and* content).

Mishra and Koehler's descriptions of the bodies of knowledge encapsulate a broad sense of "technology": older and newer, analog and digital. However, if I coded using these same components I would lose the ability to categorize and explore use of analog and digital technology separately. Establishing two sub-codes for each component – technology-digital, defined as the set of devices based on the binary system such as computers, online videos, websites, and software, and technology-analog, defined very broadly as any non-digital media, object or device such as pencil and paper, VCRs, textbooks, manipulatives, games, and visual representations – enabled me to analyze them separately and then together as needed. The use of sub-codes allowed me to identify cases where instructors repurposed non-digital technologies as educational technologies and explore the function that technology served in their instruction. Accordingly, episodes emerged where the instructor did not use digital technology, but still used "technology" in a way that demonstrated strategic integration of technology and content, technology and pedagogy, or technology and pedagogy and content.

To code off-topic discourse episodes, I used two additional categories not related to the TPACK framework: logistics and social (Koehler et al., 2007).

Table 5
List of a priori *Codes from Definitions of TPACK Elements (Mishra & Koehler, 2006, p. 1026-1030)*

Descriptive Label for Codes	*Code*	*Code Definitions*
Content	C	Talk about the subject matter to be learned or taught
Pedagogy	P	Talk about techniques, processes, practices, and methods of teaching and learning
Technology (Analog)	T-A	Talk about "standard technologies, such as books, chalk and blackboard"
Technology (Digital)	T-D	Talk about "more advanced technologies, such as the Internet and digital video"
Pedagogy and Content	PC	Talk about "pedagogy that is applicable to the teaching of specific content"
Technology (Analog) and Content	T-A C	Talk related to the way in which analog "technology and content are reciprocally related"
Technology (Digital) and Content	T-D C	Talk related to the way in which digital "technology and content are reciprocally related"
Technology (Analog) and Pedagogy	T-A P	Talk that demonstrates "knowledge of the existence, components, and capabilities of various [analog] technologies as they are used in teaching and learning settings, and conversely, knowing how teaching might change as the result of using particular technologies"
Technology (Digital) and Pedagogy	T-D P	Talk that demonstrates "knowledge of the existence, components, and capabilities of various [digital] technologies as they are used in teaching and learning settings, and conversely, knowing how teaching might change as the result of using particular technologies"
Technology (Analog) and Pedagogy and Content	T-A P C	Talk that demonstrates "thoughtful interweaving of three key sources of knowledge: [analog] technology, pedagogy, and content… to develop appropriate, context-specific strategies and representations"
Technology (Digital) and Pedagogy and Content	T-D P C	Talk that demonstrates "thoughtful interweaving of three key sources of knowledge: [digital] technology, pedagogy, and content… to develop appropriate, context-specific strategies and representations"
Logistics	L	Classroom logistics and management,
Social	S	Social dialog among students and/or the instructor, not related to course content

To aid me in applying the codes consistently across data and over time, I developed and refined clear operational definitions along with examples from the discourse episodes as the study proceeded. Table 6 provides the operational definition of each code along with an example excerpted from the classroom observation transcripts.

Table 6
Operational Definitions of Codes and Examples from Classroom Observation Transcripts

Code	*Operational Definition*	*Example*
C	Talk about mathematics as a subject matter, as content to be learned. Talk by the instructor or teacher candidates about mathematics content – either their own mathematics knowledge and skills or that of their future students.	Sarah: It's not a fraction. … Vicky: Huh? Sarah: It's not thirds, though. Ryan: Yeah, it's not thirds. Vicky: One, two, three? Sarah: But they're not equal. Vicky: Does the fraction have to be equal parts? Sarah: Yeah.
P	Talk by either the instructor or teacher candidates about teaching that is not related to the teaching of mathematics, such as classroom management, being prepared, assessment, or teacher identity. The discussion may focus on pedagogy in either the university or K-12 classroom.	Kristina: You can question on all levels of Blooms, basically according to how in-depth you want your answer to be? Instructor: Ok and what is Blooms? What do you mean by Blooms? Kristina: The taxonomy of knowing knowledge and analyzing and being able to note, like for me it means like knowing something fully and completely, being able to make questions of your own about something.
T-A	Talk about non-digital technologies not specifically related to mathematics content or the teaching of mathematics, such as writing implements, textbooks, or paper.	Instructor: All right. If you turn [the fraction circle piece] the other way, Kevin and Megan, you're going to be able to also see the names of the fractional parts that you have there.
T-D	Talk about digital technologies not specifically related to mathematics content or the teaching of mathematics. Examples include laptops, how to set up or use equipment such as a document camera or SMART Board, or navigate a website.	Diane: I just had a question. I tried to open the PowerPoint from last week, and I couldn't figure out what program to open it with. Because it wouldn't open with Microsoft PowerPoint. Instructor: And that's what I saved it as. Diane: Do you use a Mac? Instructor: Yes. Diane: I'm having the same problem with a different professor. So maybe it's ASU? Diane: Yeah, it could be. But – Instructor: Let me go in and save it as an older version of PowerPoint and see if that helps. I think it's the PTTX, or something. It's made – I'll try an older version and see.

Code	*Operational Definition*	*Example*
		Diane: Okay; thank you.
PC	Talk about the teaching of mathematics or activities to help the teacher candidates teach mathematics, either by modeling instructional strategies or having them experience activities designed to help them understand how students construct mathematics knowledge, acquire mathematics skills, and develop habits of mind towards learning mathematics. Examples include talk about the appropriate age to introduce certain concepts and activities, strategies for assessing student mathematical understanding, and the function some technique serves in the teaching of mathematics.	Instructor: So what did we learn from this? What have you learned from this problem solving experience so far? Karen: There are multiple strategies… Instructor: Multiple strategies to solve the problem. Yes, very good. What else? Jeanine: Don't underestimate the students… Instructor: Don't underestimate the power of the students' thinking. So don't go in thinking, "Oh, my kids are 7 years old. They are not going to be able to do it." Give them a chance. How are you going to find out unless you give them a problem to solve? What else have you learned? Yes, Shelly? Shelly: Problem solving on its own offers a learning experience. You don't have to be directly, like, direct instruction. Instructor: Right. The kids were working on their own. The teacher was a facilitator and that's a very important point again, because if you go in and you tell them what to do, they depend on you and when they are on their own, they don't feel that they can do it.
T-A C	Talk, use or modeling of analog technology that facilitates the learning of mathematics content. Examples include the use of a everyday objects such as paper, cups, or popsicle sticks, that are non-specific to mathematics education, or mathematics manipulatives, which are specific to the teaching of mathematics. Includes use of visual models, such as drawings or physical artifacts, to help students develop their understanding or externalize their mathematical thinking.	Instructor: You've probably done a lot of stuff with place value, but one thing that --kids have to understand you can take a bunch of single things and group them. So you could give kids a handful of…poker chips – Traci: Pencils – Instructor: Pencils – Kyle: You talking about gambling, man? Instructor: Cotton swabs, anything. And you can say, "Well, how many groups of 5 can you get out of this?" and they could count out groups of 5. You could say, then ultimately, you build up to, "How many groups of 10?" And then the whole idea, well, 12 means 1 group of 10 and 2 left over, that sort of thing. So you really need to build place value concepts before kids start doing this kind of stuff. Okay.
T-D C	Talk, use or modeling of digital technology that facilitates the learning of mathematics content. Examples include the use of a SMART Board, that is non-specific to mathematics education, or virtual manipulatives, which are specific to the teaching of mathematics. Includes use of digital models or tools that allow students to interact with mathematics content in new ways to help	Sarah: I started doing fractions about a week and a half ago, our sixth graders and fifth graders, and Smart Exchange, SMART Board web site has like really good premade lessons, and the kids can actually go up there and interact with it. They can go and move the pieces of the pie and put them all in like – it makes a noise when you get the fraction right. Instructor: Okay, so if we Google "SMART Board activities, fractions" then – Sarah: Well, it should be on the SMART Board program. Instructor: Okay, great.

Code	*Operational Definition*	*Example*
	students develop their understanding or externalize their mathematical thinking.	
T-A P	Talk about analog technology that relates to teaching (either teacher candidates or K-12 students). This may include talk about how to use an analog technology in an instructional context (such as a whiteboard, that is non-specific to mathematics, or a manipulative, that is more specific to the teaching of mathematics), explaining why use of a particular analog technology (such as a whiteboard) would be appropriate, or having some general knowledge of an analog technology and its pedagogical use.	Instructor: A lot of students have their packet there and they take notes on the pages and they tell me at the end that that helps them with studying with the final...for the final, okay?
T-D P	Talk about digital technology that relates to teaching (either teacher candidates or K-12 students). This may include talk about how to use a digital technology for instructional purposes (such as a discussion board for students to communicate), or explaining why use of a particular digital technology (such as a video or website) would be appropriate.	Instructor: I put these examples up there for you to see visually. Did it help for some of you to be able to see the visuals? Some of you it didn't, and I made both of them. Some of the Lecturers that are using these PowerPoints said that they found it helpful to make a copy and put it on the board so the pre-service teachers could see it also. So again, some of the students might need the visuals, some might not. But if you found it helpful, then you know that the students will find that helpful too and see it.
T-A P C	Talk about analog technology that relates to the teaching of mathematics (either to the teacher candidates or the future K-12 students). Examples include explaining how and why use of a particular analog technology (such as a whiteboard, that is non-specific to mathematics, or Unifix cubes, which are specific to the teaching of mathematics) would be appropriate to the teaching of mathematics, discussing when its appropriate to use one technology versus another, and completing classroom activities for the purposes of modeling the integration of technology,	Instructor: And what you want them to do is first start using the beans. And you as a teacher want to see which of your students are using the manipulatives and which ones don't need them. And that way you'll see what level your students are at. Always have the manipulatives – the beans and the counters – available to them so they can start forming these on their own. Because again – yeah? Betty: I used the counters yesterday for a child who was doing addition. He was having trouble counting onward, from highest numbers. So I got the counters out for him and he was able to then do the problem with the counters. So I found that that student needs the counters to be able to do the addition problems. So I was excited because I got to see where he was at.

Code	*Operational Definition*	*Example*
	pedagogy, and content.	
T-D P C	Talk about digital technology that relates to the teaching of mathematics (either to the teacher candidates or the future K-12 students). Examples include explaining how and why the use of a particular digital technology (such as Excel, that is non-specific to mathematics education, or virtual manipulatives, which are specific to the teaching of mathematics) would be appropriate for the teaching of mathematics, discussing when its appropriate to use one technology versus another, and completing classroom activities for the purposes of modeling the integration of technology, pedagogy, and content.	Instructor: So the other two assignments that you have to do. We briefly talked about virtual manipulatives when we were doing our [inaudible] presentations, and we said that there is a national library of virtual manipulatives that was put together by the state university of Utah, and you can go visit that library, and when you go there, you'll notice that there are a lot of manipulatives. Some of them, we have already mentioned in class, the Geoboards, the tangrams, the Base 10 blocks, you're going to find these pattern blocks that we used last week for geometry and we're using this week for fractions, plus many, many more that we haven't brought in this class. Okay? So for this particular assignment, I would like you to go and visit the library, and you are going to see that there are the manipulatives, there are different grade levels, you're going to find lesson plans there, you're going to find activities. So play with this. Spend some time there. Find one manipulative that maybe you find interesting, that maybe you saw me using in class and you're wondering how can I use this on the computer? How can I do maybe a different kind of activity with a virtual manipulative? So I want you to find one manipulative, work with it for some time, become comfortable. See how you can use it in your own classroom, and then basically what you're going to turn in is an evaluation of that manipulative. How was it? Was it interesting? Was it an interactive one? Were you getting feedback as you were working with it? Was it easy to use, or did it make you feel frustrated? What kind of activities can you do with it? What grade level is it appropriate for? Okay? So I put a bunch of questions there that I would like you to have in mind as you work with that manipulative. But, I don't want it to give me plain answers to those questions. For example, "Yes, it was user-friendly." "No, it wasn't interactive." Give me a well-written evaluation. Would you recommend it? Why? Why not? Okay? So put some thought into your paper.
L	Classroom logistics and management, such as taking attendance, reviewing assignment due dates, required components and how to submit, preparing for or cleaning up an in-class activity, or redirecting student discussion or attention if off-topic	Instructor: Any questions? All right, one person from each table please make sure that all of your sticks are in the plastic bag and that it's closed and please bring it to the front table. Then I would like you to please make sure that your Unifix cubes are put together in groups of ten and then when you have them in groups of ten, bring them…I'm going to put it on the table there, the box, and make sure that you put everything in this box.
S	Social dialog among students and/or the instructor, not related to course content	[Students laughing.] Instructor: What? Susan: My drink almost fell, like last week. It didn't. Instructor: Shoot. They're going to start charging us for the paper towels in here.

In addition, codes and their definitions were reviewed on an ongoing basis to determine if the way in which they were interpreted or applied to discourse episodes changed over time given increasing familiarity with the data or in light of new insights (Miles & Huberman, 1994).

To quantify the duration of each episode to which a particular code was assigned, I used the number of characters in the episode transcript as a proxy for classroom time. For example, a comment by an instructor to "arrange the blocks on your table in three groups of three" received a frequency count of 57, whereas a longer episode consisting of several paragraphs of discourse received a higher frequency count based on the number of characters included. A linear regression analysis evaluating the strength of the relationship between the number of seconds in three 15-minute segments from the beginning, middle and end of each observed session and the number of characters in the corresponding transcript excerpts revealed the number of seconds to be a highly significant predictor of the number of characters ($\beta = .93$, $p < 0.01$), accounting for 87% of the variance in the number of characters and thus indicating this proxy accurately represents the duration of each episode.

All observations contained some episodes of varying duration for which transcripts could not be generated and coded, such as individual work time or group discussion during which the student discourse was not audible. To account for any such segments lasting more than 10 seconds in duration, I first calculated the average number of characters per second for each class using three 15-minute segments from the beginning, middle and end of each session and the number of characters in the corresponding transcript excerpts. I then determined the number of seconds of each non-transcribed episode and multiplied the

duration by the average number of characters per second for that class to calculate the episode's character length. Coding of these segments could not be as detailed as that of episodes for which I had transcripts, but this allowed me to account for and code all class time.

The data collected and coded in the quantitative content analysis were analyzed to identify patterns, characteristics, and relationships in the distribution of talk across the seven primary categories related to the TPACK framework. In order to draw inferences about the relative emphasis on the identified categories, a quantitative analysis of the frequency and duration of each category related to the TPACK framework represents the talk that occurred in the classroom discourse in the four sessions analyzed for each participant. Statistical analyses of the data determined whether there were significant differences in the pattern of categories across sessions in a course, and across instructors and also explored the strength of the relationship between each instructor's conceptual model and enactment in the classroom.

To visually represent the relationship of the classroom talk, I created "area-proportional" Venn diagrams, in which the size and overlaps of the circles correspond to the frequency of categories assigned to each discourse episode. It should be noted that when using Venn diagrams to represent the intersection of the three variables of technology, pedagogy, and content, the use of circles did not permit depiction of an accurate area-proportional diagram. Therefore, I provide alternate depictions using Euler diagrams generated using the euler*APE* open source software (Micallef & Rodgers, 2014). In this way, I visually represent the relative emphasis and overlap of classroom talk as it relates to pedagogy, content, and technology, as well as similarities and differences across instructors. These diagrams differ for each instructor observed and provide insight into the way

technology is introduced to future teachers in their courses on methods of teaching mathematics at the K-8 level.

Retrospective Protocols

Directly after each observed class session, I conducted a brief, 5 to 15-minute retrospective protocol with the instructor to gather information regarding the cognitive processes associated with critical incidents (Flanagan, 1954) that suggested a decision point related to an element or construct elicited during the repertory grid interview. The retrospective protocols were conducted following the procedures outlined by Ericsson and Simon (1993). The goal of each retrospective protocol was to gather contextual information regarding the lesson as enacted by the instructor and why, during a specific incident that occurred during the class, the instructor responded in a certain way or how that incident demonstrated a belief or way of thinking evinced by the instructor during the structured interview that elicited the repertory grid.

The verbalization process described by Ericsson and Simon (1993) uses as its framework information-processing theory, which assumes that humans store information in multiple types of memories, each of which has different storage capacities, organizations, and methods of access (H. A. Simon, 1979). Information stored in short-term memory, meaning information that was recently acquired or accessed from long-term memory, can be accessed directly for the purposes of generating a verbal report, whereas information in long-term memory that has not been recently accessed or attended to first requires the cognitive task of moving it to short-term memory before it can be produced as part of a verbal report. Not all information in short-term memory may be moved to long-term memory, so to elicit information from a participant about a recent process or cognitive activity the report should be generated as soon as possible before the information is discarded or moved into long-

term memory and perhaps grouped with other similar processes and generalized as part of a larger cognitive process.

While conducting the classroom observations, I used a laptop for field notes to write down events that occurred during class, make quick observations, and flag particular comments made by the instructor and/or students that I wanted to examine in closer detail when reviewing the video/audio recording and transcripts. The use of a video/audio recording device allowed me to take these field notes and also concentrate on identifying critical incidents that would be used as context for the probing questions during the retrospective protocol. While observing each class, I paid particular attention to observable actions or comments made by the instructor or the students that demonstrated a significant decision about the instructional practices or activities in the classroom. My hope was that the resulting consequence of the incident would make it clear that the instructor altered, or decided not to alter, in some way the planned classroom activity or discourse in response to the incident. However, it became clear that most classroom activities were planned and structured in such a way that limited the opportunity for critical incidents of an unplanned nature. For this reason, during the retrospective protocol I usually asked the instructor to describe how specific classroom events represented her conception of a specific element as being representative of one or both poles of a specific construct.

During each observation, I selected one to three critical incidents to focus on during the verbal report conducted with the instructor directly after the class. In the retrospective protocols conducted for this study, the probing question focused on the reason, motive, or cognitive process related to an overt event that occurred during the class session. Noting key details about the incident, such as a specific comment made by a student and the instructor's response, or vice versa, helped the instructor recall the incident, his or her cognitive process

when responding, and information about the specific event rather than generalizations about the class or class sessions as a whole. This allowed the instructor to respond to the probing question with a good amount of detail. Data from the retrospective protocols were used to provide insight regarding the cognitive processes related to enactment of content, pedagogical and technological events by the instructor in the classroom and also relationships to the elements and constructs identified in the repertory grid analysis.

Elements and constructs elicited from the repertory grids were used to develop probing questions that, in combination with the context of a particular critical incident, provided information about the instantiation of the instructor's planned activities. This served as a member check for the results of the content analysis and also provided a link between the instructor's conception of the instructional strategies she believed to be most critical to the teaching of mathematics and the enactment of those conceptions in practice. Probing questions generated from critical incidents (Z) in the classroom as they related to constructs (X) about methods of teaching mathematics (Y) included: How did your perspective on Y as X influence how you structured the activity of Z? How was your decision to do Z related to your description of X as Y? How might your beliefs about X as Y have influenced what you tried in class today when you Z? When Z happened, how did your response reflect your perspective of X?

I was cautious about asking questions that appeared judgmental, such as "why did the lesson go like that" or "why did you think doing this was a good idea," as the intention of the verbal reports was not to ask instructors to defend or justify their actions or decisions, which might lessen rapport or perhaps cause them to provide an answer that concealed their thought process or rationale.

Analysis of the retrospective protocols followed the methods outlined by Ericsson and Simon (1993). Within the transcripts of each verbal protocol conducted with the instructors, I segmented the major assertions or propositions. Each segment was evaluated against three tests of validity to determine whether the verbal report was produced by the same cognitive process as the original critical incident being discussed: relevance to the critical incident or activity being discussed; consistency with other segments from the same verbal report, indicating pertinence of the verbalizations in response to the probing questions and the observed class session; and commitment to memory, in that if the instructor was able to respond to the prompt then the original critical incident was attended to as well and therefore identified by the instructor as having some cognitive significance. If the segment of the verbal report met these three criteria, the segment was used as evidence for understanding the motives and reasons of the cognitive process related to the critical incident.

Once evaluated as a valid segment, I analyzed the statements made by each instructor across all of their verbal reports to identify patterns in the reasoning and rationale for how they enacted planned activities, develop theories about the instructional decisions they made, and explore linkages between the elements and constructs generated in the repertory grid analysis and the content analysis of the classroom observations.

CHAPTER 4

RESULTS

In this chapter I present the data analysis and results as three cases, one for each participant, organized by the three categories in which I have grouped my four research questions: conceptions, enactment and retrospection. Structured interviews and repertory grids, along with content analysis conducted with transcripts from in-class observations, provided an opportunity for quantifying qualitative data that were equally valued during analysis and interpretation. The retrospective protocol conducted with each instructor after each observed class session provided insight regarding the cognitive processes related to enactment of content, pedagogical, and technological events in the classroom.

Cora

Conceptions

Descriptive analysis of the structured interview process. The repertory grid elicited with all three participants addressed the topic of instructional practices for preparing future teachers to teach mathematics at the K-8 level. To begin the element elicitation process, I read the following topic and asked Cora to brainstorm verbally in response to the topic: "Think about a situation from your regular teaching practice that is important to your conception of what your students need develop as future elementary education mathematics teachers. This may be an education situation or instructional strategy or practice. Please describe that situation for me using a single term or short phrase." Partway through our initial discussion about the elements, I asked a follow-up prompt intended to activate the consideration of technology: "Describe for me another situation from your regular teaching practice that achieves the same purpose, this time one that incorporates technology in some significant or meaningful way."

The initial discussion with Cora elicited 16 elements. It is important to acknowledge that although I endeavored to objectively note the elements as each participant talked, by nature of the fact that I wrote down the elements as they vocalized their thoughts about the topic, there was some subjective selectivity in the process. For this reason, I reviewed my notes with each participant to finalize the list of elements before we proceeded to the construct elicitation process.

During our review to determine which elements should be used in the final list, Cora consolidated some that were redundant, such as "personal experiences" and "connected experiences" and added the element "student re/grouping." In response to the follow-up prompt, Cora proposed one element: providing technology mathematics material as resources. Earlier, she had also supplied the element "providing links/URLs to fill in the gaps and tools to help them grow." However, probing for elements related to technology use felt like it imposed too much on the natural flow of discussion about the topic and prompted elements that were more representative of my interests than her understanding and conceptions of the topic. When we refined the final list of elements, Cora decided to eliminate the two related to technology, as they seemed forced in response to the follow-up prompt. Our final list included nine elements.

After entering the final list of elements on the grid, I wrote each element on a notecard so we could move the elements around on the table as we talked. Initially, I used preselected combinations of elements (such as 1, 3, 5; 2, 4, 6; 3, 4, 8; etc.). After eliciting and rating three pairs of constructs, I began to look at the elements that had similar ratings on two or more pairs and used them in a triad so Cora would need to identify a construct that distinguished them in some way.

When eliciting constructs, I first used a phrase similar to the following: "Which two elements are the same, and which one is different?" However, the construct elicitation process was particularly difficult with Cora, in part because she focused on a comment I had made during the element elicitation process about how her discussion indicated an emphasis on knowing her students and personalizing her instruction based on who her students are. Mentioning this, which I intended as a way of validating my impressions with Cora, inadvertently focused her attention on those themes.

This influenced this construct elicitation in three ways: a) Cora always paired the two elements in the triad that she thought most closely related to the themes of knowing your students and customizing curriculum for the needs of students, and as a result b) Cora kept suggesting constructs that related to those two themes and overlapped with previously-discussed constructs or c) Cora had a difficult time generating a term or phrase to represent the implicit pole – what one element in the triad did not share with the other two – because she could not verbalize what the third item did not share with the other two. We often had to talk abstractly about what would be on the other end of a continuum represented by the emergent pole, which was difficult for Cora to articulate since we were talking about abstract terms rather than using all three elements in the triad to facilitate the discussion. Partway through the construct elicitation process, I provided Cora an example, using "lecture," "discussion" and "group work" as sample elements and saying that a pair of constructs that could be used to describe what two have in common and one does not would be "student-centered" and "teacher-centered." This example clarified the process for Cora.

The rating procedure itself made sense to Cora, although she gave multiple ".5" responses, such as 2.5, 4.5, and 1.5. I noted these on the grid, but when I asked her to use either one number or the other, such as 2 or 3 but not 2.5, she modified her ratings. At

times, Cora tried to rate the elements in terms of how much they contributed or related to those themes, rather than where they fell in terms of representing one pole of the construct or the other.

Although Cora seemed to understand the grid topic and my interest in her instructional practices, I was initially discouraged during the interview because the types of elements Cora proposed did not align with the types of practices I had expected her to suggest as elements in the grid. I had conducted two practice interviews to familiarize myself with the grid elicitation process and those interviews had yielded instructional practices such as discussion, lecture, group work, problem solving and modeling. After completing the interview sessions and having some time to reflect on the grids, I see that that allowing the participants to respond to the grid topic as they understood it yielded more information about what each instructor places value on in her classroom.

Descriptive analysis of the repertory grid. This section reviews what Cora said about each element by reading through and briefly describing her ratings for each column on each construct. Figure 6 shows the repertory grid elicited with Cora, with the nine elements displayed along the bottom and the eight construct pairs on the left and right of the grid. The grid follows a general convention that the left-hand end of the construct (the emergent pole that describes what two elements in the triad have in common) defines the "1" end of a 5-point scale, and the right-hand end of the construct defines the "5" end of the 5-point scale.

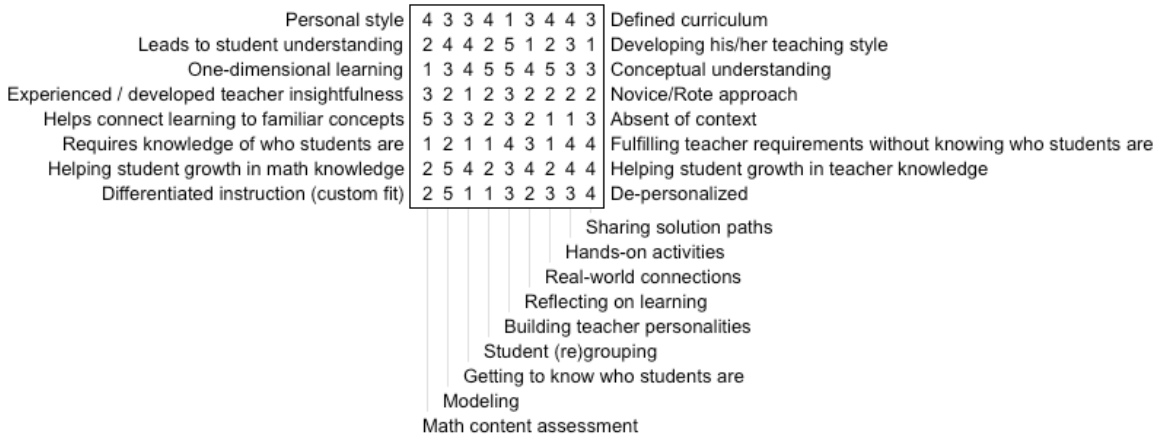


Figure 6. Repertory grid elicited with Cora.

Cora views mathematics content assessment in the way she defined it (a pre-assessment to help her gauge her students' mathematics knowledge) as being more determined by a defined curriculum than by personal style. She uses mathematics content assessment as a strategy that leads more toward student understanding than helping students develop their teaching style. As construed by Cora, mathematics content assessment is an activity that represents one-dimensional learning, not conceptual understanding. It is a strategy that she construes as being used equally by those with experienced/developed teacher insightfulness as well as those who have a novice/rote approach. As defined by Cora, mathematics content assessment is absent of context and not used to help students connect learning to familiar concepts. Using information from the mathematics content assessment requires knowledge of who the students are, rather than allowing them to fulfill their requirements as a teacher without knowing who their students are. It helps students grow in mathematics content knowledge more so than grow in teacher knowledge. Information from the mathematics content assessment allows an instructor to differentiate instruction to fit the needs of students, rather than de-personalizing the instruction.

Modeling is seen as an activity that equally represents personal style and a defined curriculum. Cora views modeling as focused more on developing students' teaching style than leading to student understanding, and relies more on experienced/developed teacher insightfulness than a novice/rote approach. She believes modeling focuses entirely on helping student growth in teacher knowledge, rather than mathematics knowledge. She does not see modeling as a strategy that represents one-dimensional learning more or less than conceptual understanding, nor does she see it as a strategy that is more or less defined as one that helps students connect learning to familiar concepts than one that is absent of context. Cora believes that modeling requires some knowledge of who students are, yet describes modeling itself as definitively de-personalized rather than being differentiated for each student.

Cora believes that instructional activities that help her get to know who students are require experienced/developed teacher insightfulness, require knowledge of who the students are, and represent differentiated instruction. She perceives these types of activities as more focused on the development of an instructor's teaching style and as more of an aid in student growth in teacher knowledge. She also states that it is more representative of conceptual learning than one-dimensional learning. She describes the practice of getting to know who students are as equally related to personal style and a defined curriculum, and does not see it as a strategy that is more or less defined as one that helps students connect learning to familiar concepts than one that is absent of context.

Grouping and regrouping students is seen as a practice that definitely requires knowledge of who students are and demonstrates instruction that differentiates based on the students. Cora also strongly believes the strategy helps develop conceptual understanding rather than one-dimensional learning. She sees (re)grouping as a strategy more focused on

helping student growth in mathematics knowledge and leading to student understanding, as opposed to helping student growth in teacher knowledge and developing student teaching style. Cora sees the practice of regrouping students as one that demonstrates experienced/developed teacher insightfulness more than a novice/rote approach, however she also rates it as more demonstrative of a defined curriculum than personal style. Grouping and regrouping students is a technique she defines more so as one that helps students connect learning to familiar concepts than one that is absent of context.

Cora views the strategy of building teacher personalities as one that represents personal style rather than a defined curriculum, and one that works towards conceptual understanding rather than one-dimensional learning. She describes the practice of building teacher personality as one that helps students develop their teaching style, but as one that is not more focused on helping student growth in teacher knowledge than on helping student growth in mathematics knowledge. She also indicated that it is more focused on fulfilling teacher requirements without knowing who students are than on requiring knowledge of who students are. Cora felt that the practice of building teacher personalities is one that does not represent experienced/developed teacher insightfulness more so than a novice/rote approach, nor does it represent differentiated instruction more so than depersonalized instruction. She does not define it more or less as a strategy that helps students connect learning to familiar concepts than one that is absent of context.

When describing the practice of reflecting on learning, Cora identified it as firmly leading to student understanding as opposed to developing his/her teaching style, however she said it is more about helping student growth in teacher knowledge than helping student growth in mathematics knowledge. She indicated that having students reflect on learning demonstrates a practice that is more differentiated for students than de-personalized, but

does not necessarily require the teacher to have knowledge of who the students are. Cora views the strategy of reflecting on learning as one that does not represent personal style any more or less than a defined curriculum, although she sees it as being more representative of experienced/developed teacher insightfulness than a novice/rote approach. She also describes it as being more about conceptual learning than one-dimensional learning and more about helping students connect learning to familiar concepts than being absent of context.

Activities that make real-world connections are seen as activities that focus more on helping student growth in mathematics knowledge than growth in teacher knowledge and more on leading to student understanding than developing teaching style. Cora definitively describes them as related to conceptual understanding and helping connect learning to familiar concepts. She feels they require knowledge of who students are and more experienced/developed teacher insightfulness, but describes them as more related to defined curriculum than personal style. She does not see activities that make real-world connections as more or less representative of differentiated or depersonalized instruction.

Cora sees hands-on activities as equally leading to student understanding and the development of teaching style, but also describes hands-on activities as being more about helping student growth in teacher knowledge than growth in mathematics knowledge. She does not see hands-on activities as a strategy that represents one-dimensional learning more or less than conceptual understanding. She firmly believes that hands-on activities helps connect learning to familiar concepts and that these activities require more experienced/developed teacher insightfulness. However, she says that they are more representative of defined curriculum than personal style and that they can likely be used to

fulfill teacher requirements without knowing who the students are. She describes them as not more or less representative of differentiated instruction than depersonalized instruction.

Cora views sharing solution paths as firmly leading to student understanding, but also more related to helping student growth in teacher knowledge rather than mathematics knowledge. She does not see this practice as representing one-dimensional learning more or less than conceptual understanding, nor does she define it more or less as a strategy that helps students connect learning to familiar concepts than one that is absent of context. She feels it requires more experienced/developed teacher insightfulness, but that it is not more or less reflective of personal style than a defined curriculum. She describes sharing solution paths as one that can be used to fulfill teacher requirements without knowing who students are and that is more depersonalized than differentiated for each student.

Construct characterization. Two themes recur in the constructs: a) customizing or differentiating instruction to the need of students, and b) helping the students develop as future teachers. Many of the constructs address growth through the use of terms such as "novice," "experienced," and "developed," and address their approach as teachers through the use of terms such as "personality," "differentiated," "rote," and "style." Even though Cora talked frequently about developing students' skills and knowledge as teachers, her emphasis seemed to be on developing a personal style and being able to deliver more than a provided curriculum and respond to student need and did not extend to a discussion of "teacher identity." It is possible that although these themes are important to Cora as a teacher educator, her focus during the grid elicitation on the themes of pedagogy and students as future teachers explains the focus of the constructs on these themes as well. Even so, the constructs as described by Cora are thoughtful and deliberate, if sometimes repetitive.

Cluster analysis. Figure 7 shows the cluster analysis of the repertory grid elicited with Cora. I first interpret the cluster-analyzed elements, then the constructs.

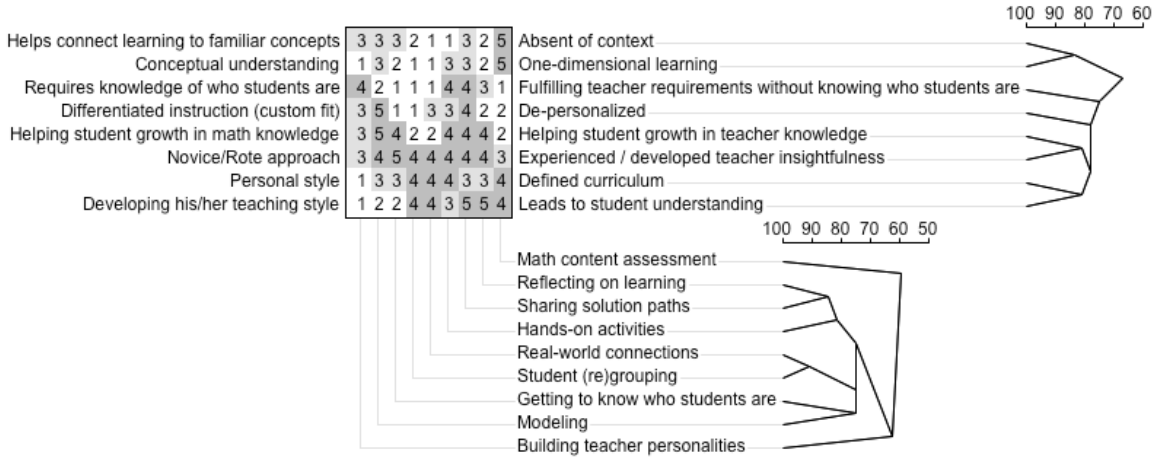


Figure 7. Cluster analysis of the repertory grid elicited with Cora.

Element dendogram. The element dendogram has two main clusters, one of which has two sub-clusters, and two elements each having their own branches separate from all others. The sub-cluster of "reflecting on learning" + "sharing solution paths" and the single branch of "hands-on activities" make up one cluster. The sub-clusters of "real-world connections" + "student (re)grouping" along with "getting to know who students are" + "modeling" make up the second cluster. "Math content assessment" makes up its own branch, as does "building teacher personalities."

The primary distinction between the two main clusters is the more reflective and perhaps collaborative/cooperative nature of the activities in one cluster ("reflecting on learning" and "sharing solution paths") versus the more active nature of activities in the other cluster ("real-world connections," "student (re)grouping," "getting to know who students are," and "modeling"). The anomaly appears to be "hands-on activities," which is rated more similarly to "sharing solution paths" than "real-world connections." This may be

because students engage in hands-on activities in the university classroom and then discuss with each other and the instructor their results.

To a certain degree, one construct that has its own branch, "building teacher personalities" is shaped by the other elements since the other activities/elements are intended to help students develop as elementary education mathematics teachers. The other construct that has its own branch, "math content assessment," primarily describes an assessment Cora gives once at the beginning of the semester to learn more about her students' mathematics knowledge. Although considered important by her, this element did not rate similarly to any of the other elements.

Table 7 displays the similarity scores of elements on Cora's repertory grid that exceed a match of 80%.

Table 7
Matches of 80% and Higher between Elements in Cora's Repertory Grid

Elements	*Match %*
Pair 1	90.6
Student (re)grouping	
Real-world connections	
Pair 2	84.4
Reflecting on learning	
Sharing solution paths	
Pair 3	81.2
Hands-on activities	
Sharing solution paths	

"Student (re)grouping" and "Real-world connections" show the highest percent similarity, with 90.6% similarity in their ratings. This pair forms its own cluster from those with the next highest % similarity score. The reason for the similarity between these two elements does not seem readily apparent, but they are perceived by Cora to be more about leading to student understanding (than developing the students' teacher style) and also used by a

teacher with a more experienced/developed teacher insightfulness (as opposed to a novice). They also require some knowledge of who students are (to group and re-group students and also to make real-world connections appropriate that are meaningful and relevant for the students). Both are perceived by Cora to be more relevant to helping the university students develop their mathematics knowledge more so than their teacher knowledge.

"Reflecting on learning" and "sharing solution paths" share the next highest similarity score, at 84.4%. Cora perceives both of these elements as strongly leading to student understanding and demonstrating experienced/developed teacher insightfulness. This is likely because the process of sharing solution paths with other students in the classroom requires students to verbalize the way they arrived at the solution and why they solved the problem the way they did. This requires the students to reflect on their learning and verbally articulate those reflections.

The final similarity score interpreted is that of "hands-on activities" and "sharing solution paths," at 81.2%. These activities are perceived by Cora to demonstrate an experienced/developed teacher insightfulness, but also be more about helping student growth in teacher knowledge (rather than mathematics knowledge, which may be because the interviewee was thinking of these as being modeled for students as instructional activities rather than using them to help improve the mathematics knowledge of the university students).

Construct dendogram. The construct dendogram has two main branches, one of which comprises "helps connect learning to familiar concepts" and "conceptual understanding," and another broad branch comprising the remaining six constructs. The second main branch comprises four sub-clusters: two of which comprise two constructs and two of which comprise one construct each.

Table 8 displays the similarity scores of constructs on Cora's repertory grid that exceed a match of 80%.

Table 8
Matches of 80% and Higher between Constructs in Cora's Repertory Grid

Constructs	*Match %*
Pair 1	83.3
One-dimensional learning – Conceptual understanding	
Helps connect learning to familiar concepts – Absent of context	
Pair 2	80.6
Experienced/developed teacher insightfulness – Novice/rote approach	
Helping student growth in math knowledge – Helping student growth in teacher knowledge	
Pair 3	80.6
Personal style – Defined curriculum	
Leads to student understanding – Developing his/her teaching style	

The ratings of the elements on "one-dimensional learning – conceptual understanding" and "helps connect learning to familiar concepts – absent of context" have the highest similarity score (83.3% match), with five elements receiving the same rating, two being rated one point different, and two being rated two points different. This is likely because instructional activities that Cora perceives as helping connect learning to familiar concepts, rather than being absent of context, are perceived to have similar characteristics to activities designed to help build conceptual understanding. For example, an instructional activity that Cora perceives to be absent of context (such as following a formula or rule) is likely also perceived as developing one-dimensional learning, rather than a deeper conceptual understanding.

The ratings of many elements on "experienced/developed teacher insightfulness – novice/rote approach" and "helping student grow in math knowledge – helping students grow in teacher knowledge" are similar (80.6% match), with four elements receiving the same rating, three being one point different, and two being two points different. This is likely because of perceived similarities in the implicit pole of these two constructs, which

both address teacher knowledge and experience. For example, an activity Cora perceives as helping student growth in teacher knowledge is likely also perceived as demonstrating characteristics of an activity that would be known to a teacher with more experienced/developed teacher insightfulness.

The ratings of many elements on "personal style – defined curriculum" and "leads to student understanding – developing his/her teaching style" are also similar (80.6% match), with four elements receiving the same rating, three being one point different, and two being two points different. This is likely because of perceived similarities in the explicit pole of these two constructs, both of which address a teacher's style. For example, an activity the instructor perceives as having characteristics that demonstrate a teacher's personal style more than a defined curriculum may seem more representative of an activity that is about developing a student's teaching style than about instructional or mathematics knowledge (i.e., leading to student understanding).

The branch comprising six constructs indicates that Cora thinks a great deal about the development of individuals as teachers and the importance of teacher knowledge – not just in mathematics content knowledge but also in pedagogy knowledge (and knowledge of pedagogy specific to teaching mathematics) – and identity. This underscores the importance Cora places on getting to know who students are and using that knowledge when teaching. The six constructs clustered together address this in some way, while the other two constructs clustered together primarily address student mathematics learning – building conceptual understanding and teaching mathematics in context.

The reversal of three of the constructs makes it easier to see Cora's focus on teacher pedagogy knowledge and teacher identity. If only one pole is examined, for example, "personal style" and "developing his/her teaching style," many of the constructs may seem

very similar and have strong overlap. To understand how Cora conceptualizes each of these constructs differently, it is important to look at the opposing pole so as to understand what the initial pole is being compared *to*. Even so, it is easy to see the two main themes of teacher pedagogy knowledge and teacher content knowledge that are present in all of the constructs.

Summary. Two primary themes can be identified in the repertory grid developed with Cora: a) pedagogy, and b) the students as future teachers.

The elements represent a range of activities, with some being broader than others and likely enacted via additional activities or strategies not specified during the interview. For example, the element "modeling" represents demonstrating a behavior or strategy to be imitated by the students, but during the interview Cora did not specify how she might enact the element "building teacher personalities." In other cases, Cora indicated she had a particular activity in mind when proposing an element, as with "math content assessment," which she described as an assessment she gives students at the beginning of the semester to assess their mathematics knowledge.

Due to Cora's focus on the two themes mentioned previously, some of the constructs appear to overlap, such as "leads to student understanding – developing his/her teaching style" and "helping student growth in math knowledge – helping student growth in teacher knowledge," which both have poles related to knowledge on one side and pedagogy on the other.

Some ratings of elements on different constructs seem contradictory. For example, the ratings of "modeling" on "requires knowledge of who students are – fulfilling teacher requirements without knowing who students are" and "differentiated instruction – de-personalize" indicate that Cora construes this element as requiring some knowledge of who

students are, yet also as definitively de-personalized rather than being differentiated for each student.

The repertory grid represents an instructor who cares about who her students are and their responsibility as future mathematics teachers, but who sometimes represent her conceptions about the topic inconsistently, perhaps because she focused not on the larger schema but on each item one-by-one.

Enactment

To explore how Cora enacted her personal theories about the instructional strategies most critical for preparing undergraduate education students to teach mathematics at the K-8 level, I conducted three analyses of the observation data: a) quantitative analyses of the frequency and duration of the episodes assigned to each category related to the TPACK framework, b) statistical analyses of the data to determine whether there were significant differences in the pattern of categories across sessions in a course, and c) analyses of each participant's repertory grid elements to explore connections between their personal theories and their enactment in the classroom.

Distribution of classroom discourse across TPACK categories. This section analyzes the frequency and duration of the episodes assigned to each of the TPACK categories in the four sessions taught by Cora. Of the total class time included in Cora's dataset, 16.53% was coded as Logistics (15.29%) and Social (1.24%). These codes are not displayed in Table 9 and are not included in the associated analyses, as they are not relevant to the research questions explored.

Table 9 displays the percentage of the episodes assigned to the seven TPACK categories in all sessions as well as each individual session. Categories for which I established

the sub-codes of technology-analog and technology-digital display the percentage for each sub-code as well as the total for the category.

Table 9
Percentage of Classroom Discourse Assigned to Each Coding Category in Four Sessions Taught by Cora

	C	*P*	*T*		*PC*	*TC*		*TP*		*TPC*	
			T-A	*T-D*		*T-A C*	*T-D C*	*T-A P*	*T-D P*	*T-A P C*	*T-D P C*
All sessions	5.98	4.46	1.25		27.19	27.13		0.13		17.33	
			0.22	1.03		27.00	0.13	0.00	0.13	17.33	0.00
Session 1	5.29	6.77	1.75		26.68	28.83		0.00		11.82	
			0.66	1.09		28.83	0.00	0.00	0.00	11.82	0.00
Session 2	1.80	2.33	1.07		39.61	16.59		0.00		26.54	
			0.13	0.93		16.59	0.00	0.00	0.00	26.54	0.00
Session 3	10.27	4.91	0.72		16.12	35.04		0.54		19.83	
			0.00	0.72		34.52	0.52	0.00	0.54	19.83	0.00
Session 4	6.98	3.38	1.44		25.69	28.44		0.00		10.40	
			0.00	1.44		28.44	0.00	0.00	0.00	10.40	0.00

To visually represent the relationship of the talk in Cora's class, Figure 8 shows an area-proportional Euler diagram in which the size and overlap of the circles correspond to the relative emphasis and overlap of classroom talk in all sessions as it related to pedagogy, content, and technology.

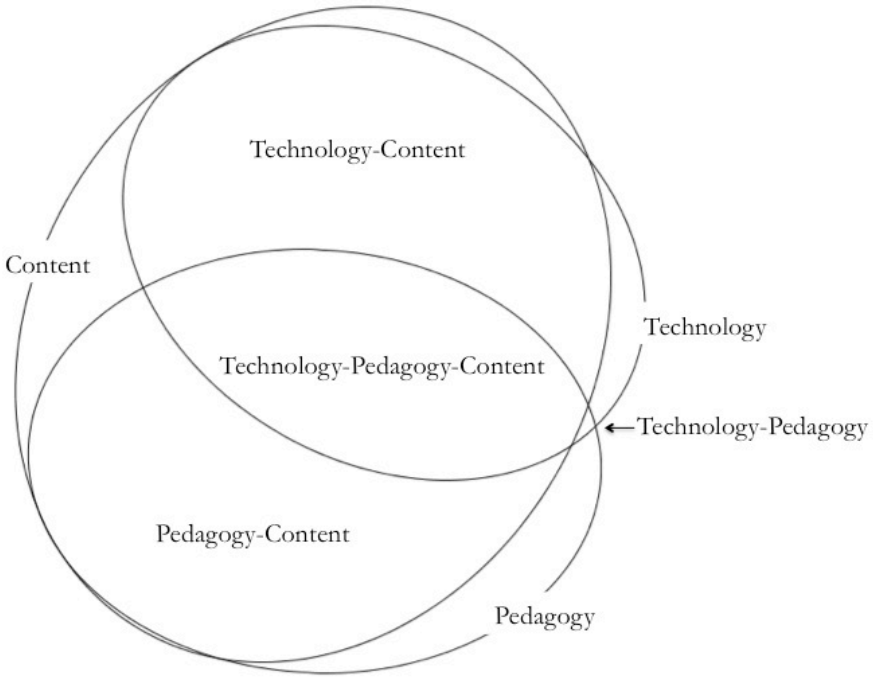


Figure 8. Area-proportional Euler diagram representing the discourse across four sessions of Cora's class.

Examination of the tabular data and visual representation reveals trends about the distribution of talk in Cora's classes. The course selected as the focus of this study addresses methods of teaching mathematics, not mathematics content or methods of teaching alone, so we expect to see the highest percentage of talk in the categories of pedagogy-content and technology-pedagogy-content. The category of pedagogy-content (27.19%) received the most emphasis, followed closely by technology-content (27.13%), indicating that over half of the classroom discourse focused on pedagogy-content and technology-content, but not the interweaving of the three components together. The high percentage of talk about technology-content also reveals Cora's focus on providing her students with an extensive portfolio of activities, games, problems, worksheets and other materials for learning

mathematics. This observed emphasis aligns with a statement Cora made during the structured interview about how important it is to her that the students gain experience with activities used to teach mathematics to elementary education students, because:

> They're the ones who are going to take what's being taught in the class and unless they're comfortable doing this kind of stuff, then it's just going to be that we're doing all these "funsie" activities and then it's gone. And I'm doing that for a purpose, and the purpose is for them to use stuff that they're comfortable with so they can start developing their own teaching styles.

Although Cora discussed the pedagogy of teaching mathematics and reviewed with students a number of activities for each mathematics topic she covered in class, talk about technology-pedagogy-content, such as why a particular manipulative, model or representation would be more appropriate to teach a certain topic, was less frequent (17.33%).

The next-highest percentage of talk occurred in the category of content (5.98%). This does not come as a surprise as the second-most likely topic of discussion after pedagogy and content is content alone, in the form of instructor or student talk about the mathematics knowledge and skills necessary to understand and then teach the mathematics to elementary education students. Of interest is that the percentage of talk coded as technology-content was much higher than that coded as content alone, indicating that when the instructor and students discussed mathematics content it was more often with reference to an analog or digital technology, such as manipulatives, visual models, etc.

The low percentages of talk focused on pedagogy (4.46%) and technology-pedagogy (0.13%) demonstrates that the instructor and students rarely focused on issues of pedagogy alone, likely because the integrated nature of pedagogy-content activities might highlight gaps in content knowledge that need to be discussed, but rarely gaps in pedagogy knowledge alone. The types of activities in which students engaged would be absent of meaning without

the mathematical context. For example, when working to complete a small group activity and build models of three-dimensional shapes using marshmallows and toothpicks, the instructor and students talked about the mathematics content, technology-content, and technology-pedagogy-content, but not any isolated concepts of pedagogy alone. Talk related to the integration of technology and pedagogy was almost non-existent across all classes observed.

Content

Cora: What makes shapes alike and different can be determined by geometrical properties. For example, shapes have sides that are parallel, perpendicular, and neither, or may have line symmetry, rotation symmetry or neither.

Technology-Content

Cora: Now you can work on the vertexes. What's a clue that you can help the students, class, as they're building these? What do the marshmallows represent?

Linda: Vertexes.

Cora: Yeah. They represent the vertexes.

Technology-Pedagogy-Content

Cora: After you've made these, you can have the students play guessing games with these to decide which one it is by asking questions. Right? You can have them guess what the shapes are by having them identify, ask different questions about it….

You can have—listen. You can have all your shapes once you've created them, the ones who didn't eat the marshmallows of, and you could play a guessing game. I could say, "There only is one base. The bottom base is called a polygon." You could take out some of the terminology to have the students guess which shape is sitting on your desk. For example, what are some of the properties of this [pyramid]? What's the base on it?…

So again, once they've created these, start looking at the properties by either guessing or doing what we just did. You're reinforcing the vocabulary where it's not isolated from the actual shape. What happens a lot of times with geometry is everything is taught separate. Here you're

seeing an example of a chunking activity where you're talking about all the properties at once, and you're quizzing the students on all that.

Similarly, very little classroom talk focused on technology alone (1.25%). In fact, technology-only was one of the two categories in which the digital technology sub-code occurred more frequently than analog technology. The talk in this category focused almost exclusively on difficulties with digital technology in the classroom.

Example 1

Cora: I can write on this [SMART Board], right?

Diane: Yeah.

Kristina: With these markers.

Cora: Okay, with these markers…. I don't know how to erase on this thing.

Kristina: Pick up the eraser –

Cora: Oh. Yeah. Okay.

Example 2

Cora: Okay, problem-solve, what you going to do if you don't have a scanner and need one?

Adele: You can take a high-resolution photo, and email it to yourself and then upload it.

Cora: That's great, what else?

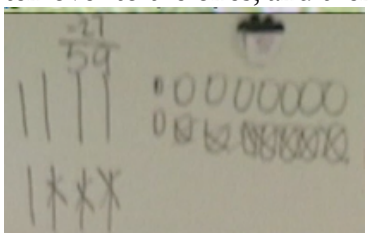
Example 3

Cora: What did I do here? It's not moving. Did it crash and burn on me? Oh! PowerPoint activation failed. Oh here it is, okay.

It is important to note that the categories of technology-content and technology-pedagogy-content show a high percentage only because of the sub-codes used to distinguish and capture talk that relates to analog technology. Take the following excerpt of talk, in which the student references her visual diagram representing base-10 blocks, as an example:

Cora: Who did this one? Tell us what you did here.

Shelly: I crossed out two tens first and then crossed out another ten and moved ten over to the ones, and then crossed out the seven.



Cora: Okay. So you actually put your borrowing there so that it's more explicit.

Shelly: Um-hum.

Cora: And did this help solve the problem with the borrowing?

Shelly: Yes.

Since the technology referenced is a drawing representing base-10 blocks, were it not for the sub-code established to capture instances of classroom discourse that relate to the way in which analog technology and content are reciprocally related, this episode would be coded as content only, not technology-content. Neither the drawing nor the blocks can be qualified as digital technology, yet in terms of representing a model, they do qualify as an analog technology. Reassignment of all technology-analog content codes to content, technology-analog pedagogy to pedagogy, technology-analog pedagogy content codes to pedagogy content and technology-analog codes to logistics would suggest that the discourse in Cora's classroom rarely, if ever, focused on the integration of technology with content and/or pedagogy. In comparison to the distribution shown in Table 9 and Figure 8, the distribution might then look like that shown in Table 10 and Figure 9.

Table 10
Percentage of Classroom Discourse in Cora's Class if Codes were Re-Assigned

	C	*P*	*T*	*PC*	*TC*	*TP*	*TPC*
All sessions	32.98	4.46	1.03	44.51	0.13	0.13	0.00

The categorization and inclusion of analog technology use in the classroom demonstrates that a large, otherwise-unrecognized portion of the classroom discourse addresses the thoughtful integration of technology, pedagogy, and/or content.

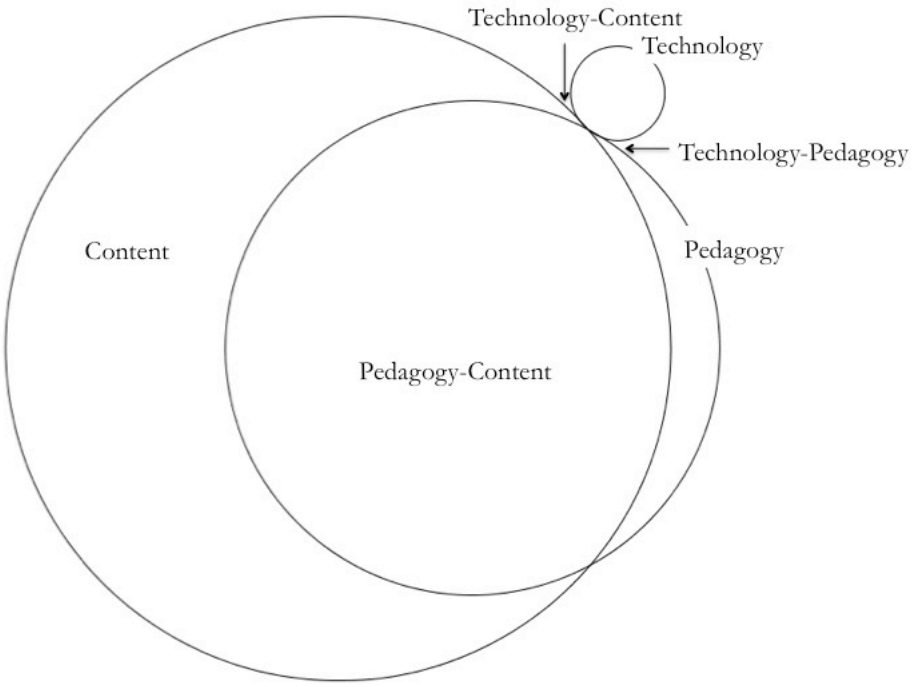


Figure 9. Area-proportional Euler diagram representing the discourse across four sessions of Cora's class if the technology-analog sub-codes were re-assigned.

Test of independence. To examine whether there were significant differences in the pattern of categories across sessions taught by Cora, I performed a chi-square test of independence. The relation between these variables was significant X^2 (18) = 37.32, p <.05, indicating heterogeneity in the distribution of the TPACK categories among observed class sessions. This suggests that the percentage of classroom discourse relating to each of the TPACK categories varied significantly from session to session.

Connections between personal theories and enactment in the classroom. In this study, the repertory grid elicited with each participant serves to represent the teaching methods she considers to be most critical for preparing undergraduate education students to

teach mathematics at the K-8 level, as well as the meaning or value the participant assigns to those teaching methods. To explore the relationship between each participant's personal theories, as represented by the repertory grid, and their enactment in the classroom, I coded the repertory grid elements using the same *a priori* codes defined for evaluating the classroom observations.

Cora's repertory grid included nine elements. Using the interview transcript, I reviewed Cora's definition of each grid element and then applied the code most appropriate given her definition. For example, Cora defined the element "math content assessment" as "a math assessment," the purpose of which was to "assess [the students'] level in mathematics and understanding." I assigned this element a code of content. Modeling, which I assigned a code of pedagogy-content, Cora defined as:

> Modeling for [the students] what they need to do for their students, so I kind of practice what I preach. That's important. They need to see what I'm – they need to see those strategies or practices in my teaching…. In each of the content areas that they have to know how to teach, they're actually doing activities that they would be using with their students. And then I have them do it with each other and then stop and analyze what helped them connect to it and what didn't.

From the classroom observations, it is evident that during some episodes Cora's enactment of strategies such as modeling, sharing solution paths, or hands-on activities demonstrated the integration of content and technology or the interweaving of pedagogy, content, and technology. However, since the use of technology in the classroom could not be assumed based on the definitions provided in the structured interview, I restricted the coding of elements to P, C or PC.

After coding each element, I calculated the frequency of each code assigned to the nine repertory grid elements. Content was assigned to one of nine elements, pedagogy to two elements, and pedagogy-content to five elements. Next, I used linear regression to

evaluate the strength of the relationship between Cora's conception of the instructional strategies central to the preparation of future elementary education mathematics teachers and their enactment in the observed sessions. Due to the small dataset, the distribution of codes assigned to the elements elicited in Cora's structured interview were not a statistically significant predictor of the distribution of codes assigned to the classroom discourse in the observed sessions ($\beta = .97$, $p = 0.16$), yet even so Cora's personal theories accounted for an astonishing 94% of the variance in their observed enactment in the classroom.

Retrospection

After each classroom observation, I conducted a brief retrospective protocol with Cora to discuss a selected incident from the observed class with respect to elicited repertory grid elements and constructs. The themes that emerged from these discussions align with those from the repertory grid and content analyses indicating Cora's focus on preparing her students to be future elementary education mathematics teachers, in particular by providing them with a toolkit of activities they can draw on to teach various mathematics topics at different grade levels.

In each of the post-observation interviews, Cora reiterated how important it was to her to be responsive to student need when contacted via email, phone, or in person for suggestions of activities to use in student teaching classrooms when addressing a particular mathematics topic. Based on the communication she had with students during the week, Cora selected from her materials specific activities she felt would be most useful to cover in class. In this way, all students experienced two or three activities in detail but Cora could also work with the smaller subset of students who had asked about a specific topic and appropriate instructional strategies. She continued to "fill [her] Blackboard shells with all of

these resources" and the PowerPoint slides she reviewed in class contained overviews of about 25 activities each session that she could draw on as needed. This flexibility to adapt to student need allowed Cora to expose students to activities they might not observe in their mentor teachers' "old, outdated bag of tricks," however it also reflected a broad, shotgun approach that detracted from a focused discussion of how the activities facilitated students' cognitive development, how to transition from the concrete to abstract, or how to build on previous knowledge to teach new concepts.

Data from the retrospective protocols also highlighted Cora's purpose in presenting classroom activities to address the construct "helping student growth in math knowledge – helping student growth in teacher knowledge." She acknowledged that some might perceive the in-class activities, such as building geometric shapes with marshmallows and toothpicks, as "busy work" but explained:

> If [the students] are viewing it as busy work then they're missing the whole reason why they're doing these activities. They're doing the activities because they probably never have been taught mathematics in this way and in order for them to teach in this way they need to be familiar with how these concepts are taught and how you learn it as a learner.

Cora also described the activities as serving to "fill in the gaps" and "reinforce the concepts" for students who would benefit from a review of the mathematical content itself. But while she succeeded in providing the students with an instructor toolkit, the time-on-task sometimes required more effort by the students to cut and label paper fraction strips or draw illustrations or diagrams of problems than discuss how to select key mathematics activities they could use again and again with variations to develop strong understanding of the related concepts and skills and develop a curriculum to guide students through a particular mathematics topic over the course of a few weeks or months.

Finally, Cora also emphasized the importance to her of making mathematics "fun" and "engaging" so the elementary education students would not grow up hating mathematics. Certainly, the activities she shared with her students present an alternate way to teach and learn mathematics differently from what many of them experienced when first learning the concepts and skills themselves. On an ongoing basis, Cora asked students "Are you having fun?" and "Are you enjoying this?" and explained that if they were enjoying it, "well, then your students will, too." From the level of engagement by the students in working on the activities, their focus in completing them as described, and participation in the classroom discussion, as well as the frequency with which students asked Cora before, during or after class about suggestions for classroom activities, it was apparent that students valued her attentiveness to their perceived immediate and future needs as elementary education mathematics teachers.

Jamie

Conceptions

Descriptive analysis of the structured interview process. As with Cassie and Cora, the repertory grid addressed the topic of instructional practices for preparing future teachers to teach mathematics at the K-8 level. To begin the element elicitation process, I read the specified topic and asked Jamie to brainstorm verbally in response.

During the element elicitation phase with Jamie I felt some anxiety about the types of elements she proposed because the types did not align with the types of practices I had originally envisioned as elements in the grid. For example, I had envisioned practices such as discussion, lecture, group work, problem solving or modeling, whereas Jamie proposed elements such as "building on prior knowledge" and "putting math in context." However, as with Cora, while talking to Jamie during the element elicitation process, I realized that had I

provided sample elements and constructs I would have learned less about how Jamie herself conceived of and understood the topic.

In retrospect, it would have been appropriate to use a "laddering down" technique for eliciting elements by asking "what sort of activity or strategy do you use when you are trying to break misconceptions about what teaching math means," or "can you suggest a particular way that you build on prior knowledge." Even so, I believe that allowing Jamie to respond to the grid topic as she understood it yielded useful information about how she understood the topic, not only by the constructs elicited but also the elements she proposed.

The initial discussion with Jamie elicited nine elements. During our review of the elements, Jamie rephrased two that she had stated in a negative way, including "breaking misconceptions," so they would be described in a more positive manner, in this case as "developing understanding." One element listed in my initial notes, "models/manipulatives," was never discussed further and was eliminated in favor of "develop versatility with different models/algorithms related to the operations." Jamie used the term "models" much more frequently than "manipulatives" (mentioned only once) and indicated she believed the use of manipulatives was one way to help students develop models and versatility with algorithms or operations.

As with Cora, I asked Jamie a follow-up prompt intended to activate the consideration of technology. In response to this prompt, Jamie stated that she "uses the document camera to share models and things that can be represented" but that her classes are "pretty low tech actually." Again, probing for elements related to technology use felt like it imposed too much on the natural flow of discussion about the topic and prompted elements that were more relevant or meaningful to me than to Jamie. This element was not included in our final list, which comprised seven elements.

After entering the final list of elements on the grid, I wrote each element on a notecard so we could move the elements around on the table as we talked. Initially, I used preselected combinations of elements (such as 1, 3, 5; 2, 4, 6; 3, 4, 8; etc.). After eliciting and rating three pairs of constructs, I began to look at the elements that had similar ratings on two or more pairs and used them in a triad so Jamie would need to identify a construct that distinguished them in some way.

When eliciting constructs, I used a phrase similar to the following: "Which two elements are the same, and which one is different?" Elicitation of the first three constructs was challenging for two primary reasons. First, it took some time to help the participant arrive at a good understanding of the construct elicitation process and what types of traits or characteristics would work as constructs. Initially, Jamie began by describing one element and what it represented, then describing the other two. This appeared to be a way for her to think through the qualities or characteristics of each element, however she was not able to summarize in a brief word or phrase what trait or characteristic was shared by two elements and distinguished them from the third element. After about 10 minutes of discussion trying to elicit the first construct, I provided Jamie the same example I had provided Cora, and this sufficiently clarified the construct elicitation process so that we could continue.

Second, the elements elicited were abstract rather than specific examples of instructional strategies and in some cases could themselves represent a trait or quality of some instructional activities, rather than an activity itself. For example, one could construe the two elements of "breaking misconceptions about what *teaching* math means" and "breaking misconceptions about what *doing* math means" as two poles of a construct: "teaching math" versus "doing math." This is supported by the first construct that was developed: "how you teach" versus "what you teach."

As we progressed through the construct elicitation process, later constructs also took Jamie a long time to propose because she struggled to identify a construct that had not already been used or find another way of distinguishing the elements or representing their qualities. At the end of the interview, I used an open-ended question to ask Jamie if there was another construct "that we haven't covered that you would want represented in terms of the ways you think about these activities, strategies or practices?" Although the participant talked and reflected on this for several minutes, in the end she did not add another construct.

The rating procedure itself made sense to Jamie and she quickly rated the elements on each construct. The ratings process for each construct usually took one to two minutes, compared to a range of two to twelve minutes to elicit each construct. Jamie spoke each rating confidently and moved through them at the same rate throughout the interview.

Towards the end of the interview, Jamie voiced that she would have used different elements if listing them again. This highlights the importance of clarifying the grid topic, while still allowing an interviewee to select elements herself so that the elements are not the same across participants and reveal information about how each individual thinks about the topic.

Jamie: Really I would have picked different instructional strategies (laughing) –

Meredith: Looking back on it?

Jamie: There's so much overlap….

Jamie: You know what I, if I think in general about the classroom, I try to get students involved working on problems, talking to each other, doing small groups, answering questions, presenting, coming up to the board. But when I think about these things, I'm not sure exactly how that ties in there. I mean I'm not sure…it's teacher-led versus student-led or student-engaged, student listening or whatever. These practices that somehow don't seem to…

Jamie: I was just thinking if you had asked me this like a day from now, if I'd have completely different, or a lot of variation really even in the instructional strategies.

Meredith: Because you went through the interview or just because –

Jamie: Well the interview would have something to do with it, it isn't like I'm going to rethink the way I teach but just now that you've been asking me these questions and it's in my brain so I might wake up tonight and say, "well I should have said this or could have said something else I do in my classroom."

Even so, the interview was meaningful and the ratings of elements on constructs appear to be consistent and accurately represent the way Jamie conceives this topic.

Descriptive analysis of the repertory grid. This section reviews what Jamie said about each element by reading through and briefly describing her ratings for each column on each construct. Figure 10 shows the repertory grid elicited with Jamie, with the seven elements displayed along the bottom and the seven construct pairs on the left and right of the grid. The grid follows a general convention that the left-hand end of the construct (the emergent pole that describes what two elements in the triad have in common) define the "1" end of a 5-point scale, and the right-hand end of the construct defines the "5" end of the 5-point scale.

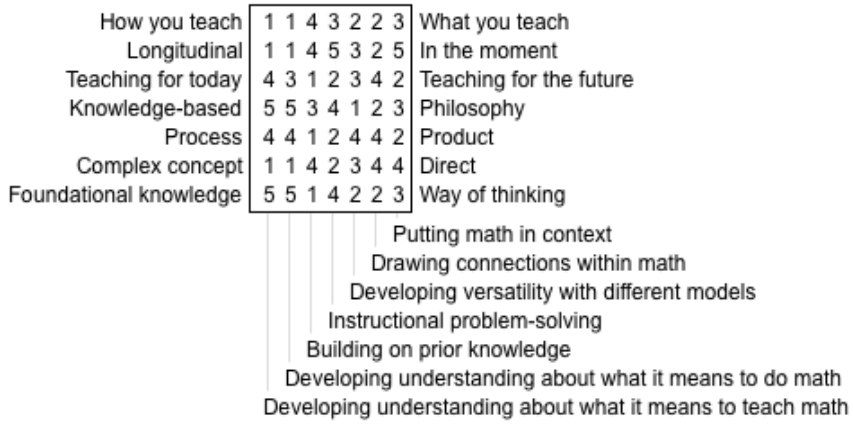


Figure 10. Repertory grid elicited with Jamie.

Two elements warrant specific explanation so as to understand how Jamie defined the phrases provided. Jamie defined "building on prior knowledge" as presenting mathematics problems in a context that will be familiar to students, for example, when presenting word problems. When adding "putting math in context" as an element, she described this as asking the teacher candidates, as opposed to the instructor, to write their own problems as a way to create situation problems their students can work on without having to know the specific mathematics terms associated with what they are being asked to do. One example would be to write a problem that asks students to find the greatest common factor without using the term "greatest common factor" in the problem itself. With these definitions provided, I continue to the descriptive analysis of Jamie's repertory grid.

Jamie perceives developing an understanding of what teaching mathematics means as something that focuses on how you teach, not what you teach. Similarly, she views this as a concept or skill that will be relevant or meaningful for understanding concepts or skills in the future, not about teaching a specific concept or skill for use in the moment only. As construed by Jamie, the development of an understanding of what teaching mathematics means is more about developing knowledge and skills the students will use for a long time into the future – the next day, month, week or year – not short term. Because this is about developing an understanding of what teaching means, she sees this as philosophical, not procedural or simply conveying knowledge of teaching strategies. She identified this as being more about the product than the process, but I believe this is because she qualified product as what is created and process as how the product is created. So the product would be the understanding of what teaching mathematics means and the process would be the way she and the students go about developing that understanding, and she is qualifying the development of that understanding as a noun (product) rather than a verb (process). Jamie

does construe this as a complex concept, not something easy to understand, and also as being a way of thinking, not simply foundational knowledge to be gained.

Jamie perceives developing an understanding of what doing mathematics means very similarly. She perceives this as something that focuses on how you teach, not what you teach, likely because she sees this as something more than just teaching mathematics, but about helping students develop a more meaningful understanding of what it means to do mathematics and move beyond their existing conceptions. Similarly, she views the development of this understanding as something that will be relevant or meaningful for looking at mathematics over time, not about teaching a specific concept or skill for use in the moment only. However, Jamie described the development of an understanding of what doing mathematics means as being equally focused on what students are doing today as well as how that will be used tomorrow or the next day or next year. As construed by Jamie, the development of an understanding of what doing mathematics means is philosophical, not procedural or simply conveying mathematics content knowledge. She identified this as being more about the product than the process and again, I believe this is because she qualified product as what is created and process as how the product is created. So the product would be the understanding of what doing mathematics means and the process would be the way she and the students go about developing that understanding. She construes the development of an understanding of what it means to do mathematics as a complex concept, not something easy to understand, and also as being a way of thinking, not simply foundational knowledge to be gained.

Jamie defined "building on prior knowledge" as putting things in context, for example through the use of word problems. This is to be distinguished from building on prior mathematics content knowledge, which she defined as "drawing connections within

math" and discussed as a separate element. In this context, as construed by Jamie, building on prior knowledge is more about what you teach than how you teach. It is also more about teaching a specific concept or skill at a given time, not necessarily the long-term teaching of mathematics. Similarly, she construes this as teaching for today, not teaching for the future. With respect to whether building on prior knowledge is more philosophical or more knowledge-based, she construes this as being equally about both. Building on prior knowledge to help students gain some mathematics knowledge is about the process, or the way she teaches, rather than the end product of the mathematics knowledge gained by the students. She perceives this strategy as more direct/easy than complex, most likely because she perceives it as a specific strategy she uses (in terms of presenting a mathematics problem in context for students). Jamie also perceives this as being more about helping students gain foundational knowledge (knowledge of mathematics content) than about helping them develop a new way of thinking about mathematics.

Instructional problem solving (asking the right questions / posing the right problems) is perceived by Jamie to be equally about how you teach and what you teach. However, Jamie perceives this as a strategy that is used to teach a specific concept or skill at a given time, not the long-term teaching of mathematics. Similarly, she construes this more about teaching for today than about teaching for the future. With respect to whether instructional problem solving is more philosophical or more knowledge-based, Jamie construes it as being more about philosophy and the approach for teaching mathematics. I believe this is because she sees problem solving as a way to present an alternative model rather than a traditional algorithm. Using instructional problem solving to help students gain mathematics knowledge is more about the process, or the way she teaches, than the end product of the mathematics knowledge gained by the students. She perceives this strategy as

being more complex than easy/direct, most likely because she sees this as a way to challenge students and require them to do more than solve an algorithm presented a more traditional way. She also perceives this as being more about helping students develop a new way of thinking about mathematics than just gaining foundational knowledge (knowledge of mathematics content).

Helping students develop versatility with different models or algorithms related to mathematics operations is construed by Jamie as being more about how you teach than what you teach. She sees this as being equally about the longitudinal teaching of mathematics as well as the immediate need to teach a specific skill or knowledge. Similarly, she perceives this as being equally about teaching for the future and teaching for today. However, Jamie sees this as being focused on knowledge acquisition, not a philosophy about teaching mathematics. For her, developing versatility with different models is more about the end product of teaching the alternative model and the mathematics knowledge gained by students than the process or way she teaches. She perceives this strategy as being both direct (in terms of teaching an alternate algorithm) and complex (in terms of teaching students more than one algorithm and helping them understand that more than one algorithm exists and can be appropriate). For Jamie, this is more about helping students gain foundational knowledge than develop a new way of thinking. This aligns with her characterization of this element as being more about the knowledge base than philosophy.

When describing the practice of drawing connections within mathematics across grade levels and content material, Jamie identified this as being more about how you teach than what you teach. She perceives this as being more about the longitudinal teaching of mathematics than about addressing a short-term need to teach a specific skill or knowledge. Similarly, she construes this as more about teaching for the future than teaching for today.

With respect to whether drawing connections within mathematics is more philosophical or knowledge-based, Jamie construes it as being more about the mathematics knowledge gained than the philosophy and approach for teaching mathematics. This is likely because she sees it as a strategy to help students build on mathematics content knowledge they previously learned, but not as a philosophy in terms of what it means to do mathematics. For her, drawing connections within mathematics is more about the end product of the mathematics knowledge gained by students than the process, or the way she teaches. Jamie perceives this strategy as being more direct, rather than complex, and perceives this as being more about helping students gain foundational knowledge than developing a new way of thinking.

Jamie defined "putting math in context" as asking students to write their own problems as a way to create situation problems that their students can work on without necessarily having to know the specific mathematics terms associated with what they are being asked to do (for example, find the greatest common factor). This is to be distinguished from building on prior knowledge, which Jamie defined as her, the instructor, presenting problems in a context that will be meaningful and relevant for her students. In this context, as construed by Jamie, putting mathematics in context is equally about helping the students learn how to teach and also demonstrating that they understand what it is they are teaching. She perceives this as being about teaching a specific concept or skill at a given time, not the long-term teaching of mathematics. Similarly, Jamie construes this as more about teaching for today than teaching for the future. With respect to whether putting mathematics in context is more philosophical or more knowledge-based, she construes this as being equally about both. Asking her students to put mathematics in context and create problems they can use in their classroom is more about the process, or the way they will teach, than the product of the mathematics knowledge itself. Jamie perceives this strategy as more direct/easy than

complex, most likely because she perceives it as a specific strategy she wants the students to use (in terms of creating a mathematics problem for their students). She does perceive this as being equally about helping her students gain foundational knowledge (knowledge of mathematics content) and about helping them develop a new way of thinking about mathematics, since in asking them to write a problem she wants them to understand both the mathematics knowledge and also how presenting mathematics in a way other than a traditional algorithm can help their students do the mathematics without getting stuck on whether they understand the specific mathematics terms that underlie the problem.

Construct characterization. The elements in Jamie's grid can be characterized as abstract rather than specific examples of instructional strategies, which made the constructs difficult to elicit. As a result, Jamie suggested similar constructs during our discussion and at one time stated, "these ideas are all so related to one extent or another, it's hard to see them as opposing."

In comparison to most repertory grids, I believe that the elements rather than the constructs in Jamie's grid best represent her personal values and beliefs regarding the teaching of mathematics. Still, the constructs elicited are meaningful and represent her conceptions of what it means to do mathematics and to teach mathematics. Throughout the interview, Jamie provided detailed examples of what she meant by the constructs or her reason for suggesting them.

The constructs focused on qualities that fell into two main groups: a) how you teach versus what you teach, process versus product, teaching versus learning, minds versus hands; and b) teaching for today versus teaching for tomorrow, this is where you are versus this is where you are going, immediate versus big picture, longitudinal versus immediate.

Cluster analysis. Figure 11 shows the cluster analysis of the repertory grid elicited with Jamie. I first interpret the cluster-analyzed elements, then the constructs.

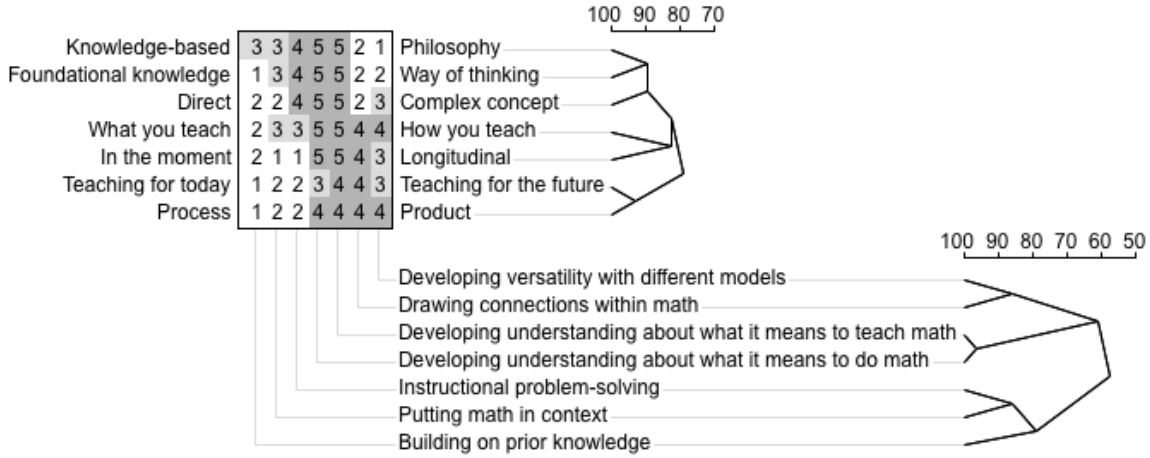


Figure 11. Cluster analysis of the repertory grid elicited with Jamie.

Element dendogram. The element dendogram has two main structures, one of which has two sub-clusters and the other has a triad sub-cluster. The sub-clusters of "developing versatility with different models" + "drawing connections within math" and "developing an understanding about what it means to teach math" + "developing an understanding about what it means to do math" make up one cluster. However, these two sub-clusters have such a low similarity score with one another that I interpret them as two separate clusters. The triad sub-cluster has one sub-cluster of "instructional problem solving" + "putting math in context" and a third element of "building on prior knowledge. The three elements in this triad sub-cluster appear to share meaning.

Table 11 displays the similarity scores of elements on Jamie's repertory grid that exceed a match of 80%.

Table 11
Matches of 80% and Higher between Elements in Jamie's Repertory Grid

Elements	*Match %*
Pair 1	96.4
Developing understanding of what it means to teach math	
Developing understanding about what it means to do math	
Pair 2	85.7
Developing versatility with different models	
Drawing connections within math	
Pair 3	85.7
Instructional problem solving	
Putting math in context	

"Developing an understanding about what it means to *teach* math" and "developing an understanding about what it means to *do* math" show the highest percent similarity, with 96.4% similarity in their ratings. The shared meaning between these two elements likely has to do with Jamie construing both as being more philosophical than knowledge-based, more longitudinal than short-term, and more about how people teach than what they teach. While she makes a distinction between "doing" and "teaching" mathematics, the way that she construes both of those elements is very similar.

"Developing versatility with different models" and "drawing connections within math" share the next highest similarity score, at 85.7%, likely because Jamie sees one as a natural evolution of the other. As she stated during the interview, "[kindergartners] can divide those carrot sticks up but eventually they're going to start talking about the formal operation of division and how you operate things." In this way, Jamie implies that an instructor needs to draw connections between the models students develop when learning early mathematics in order to help them learn advanced mathematics, or help them relate early models they develop to advanced concepts (such as using student understanding of how to find the area of a rectangle to explain the squaring of the sum of two numbers).

The final similarity score interpreted is that of "instructional problem solving" and "putting math in context," which also have an 85.7% similarity in their ratings. The primary difference between these two elements is who serves as the main actor. In the first, the *instructor* develops instructional problems ("a word problem, something in context") and poses them to students to solve. In the second, the *students* write or create problems they can use in their classroom. The reason these two elements are also clustered with "building on prior knowledge" is because in developing problems, Jamie believes it is important to provide those problems in a context that is meaningful – couched in a situation that is familiar to students – so they build knowledge based on what they already understand. However, in referencing "prior knowledge," Jamie does not refer to prior knowledge of mathematics, but rather prior knowledge with respect to terms or situations that are familiar to the students given their age and likely life experiences.

Construct dendogram. The construct dendogram has three main branches, one of which comprises three constructs: "knowledge-based – philosophy," "foundational knowledge – way of thinking," and "direct – complex concept." At first look, the other two main branches seem odd in the way they are paired. It seems that elements should have been rated more similarly on "what you teach – how you teach" and "process – product" and that the second construct should have been reversed in the cluster analysis, and that elements should have been rated more similarly on "in the moment – longitudinal" and "teaching for today – teaching for the future." Instead, "what you teach – how you teach" is paired with "in the moment – longitudinal," and "teaching for today – teaching for the future" is paired with "process – product." Also, the "process – product" construct was not reversed so that its polarity is the same as "what you teach – how you teach," which I would have expected given the apparent similarity between these two constructs.

Table 12 displays the similarity scores of constructs on Jamie's repertory grid that exceed a match of 80%.

Table 12
Matches of 80% and Higher between Constructs in Jamie's Repertory Grid

Constructs	*Match %*
Pair 1	92.9
Teaching for today – Teaching for the future	
Process – Product	
Pair 2	89.3
Knowledge-based – Philosophy	
Foundational knowledge – way of thinking	
Pair 3	82.1
How you teach – What you teach	
Longitudinal – In the moment	
Pair 4	82.1
How you teach – What you teach	
Complex concept – Direct	

The ratings of elements on "teaching for today – teaching for the future" and "process – product" are the most similar (92.9%), with five elements receiving the same rating on the two constructs and two being two points different. While it might at first seem odd that when Jamie construes an activity as being about teaching for today rather than teaching for the future she also considers it to be more about process than product, it seems this is because Jamie sees teaching instructional strategies that are about the process by which one teaches (such as putting mathematics in context and building on prior knowledge) as also being about teaching specific knowledge or skills on a particular day. In contrast, she construes strategies that are about a product or ability she wants students to develop (such as versatility with different models) also as abilities that she believes will benefit their mathematics learning over time (in the future).

Similarly, activities that Jamie perceives as being more about how you teach than what you teach are also perceived as strategies that are longitudinal in nature – not just about

teaching some piece of content ("what") being taught that day/moment, but some method of teaching ("how") that will be useful for students in the future and over time. Hence, the construct pairs of "how you teach – what you teach" and "longitudinal – in the moment," as well as "how you teach – what you teach" and "complex concept - direct" shared an 82.1% similarity in their ratings. One or two key differences in ratings between "direct – complex concept" and "in the moment – longitudinal" dropped the overall similarity score for these three constructs together. While three elements share the same rating on all three constructs, ratings on two elements differ by two or more points.

The final construct similarity score discussed is that of "knowledge-based – philosophy" and "foundational knowledge – way of thinking," with a similarity match of 89.3%. These two constructs form a cluster in the dendogram with "direct – complex concept," whose lowest similarity score appears to be the same as the other two constructs: approximately 89.3%. This suggests that when Jamie thinks of an activity as being more philosophical than knowledge-based and more about a way of thinking than about foundational knowledge, she also thinks of it as being more of a complex concept than a direct instructional strategy.

Summary. The repertory grid represents an instructor who thinks about the teaching of mathematics in terms of the conceptual understanding and beliefs she wants to convey to her students, not specific instructional strategies. Jamie emphasizes helping students develop a conceptual understanding of what it means to do mathematics and teach mathematics, more so than conveying or modeling specific instructional techniques they could use in their classroom. Perhaps Jamie believes that in helping her students develop this conceptual understanding they will in turn be better elementary education mathematics teachers because they will have a stronger grasp of the mathematics they are teaching and

more developed philosophy about what it means to do and teach mathematics. Jamie's focus, in terms of the examples she gave and the classroom situations she described, such as the distributive property and multiplying two binomials, appears to be the later grades in elementary education. Although it is clear she cares about the topic of instructional practices for preparing future elementary education teachers to teach mathematics, I did not gain from her a clear sense of what her beliefs about what it means to do or teach mathematics would look like in practice.

Enactment

To explore how Jamie enacted her personal theories about the instructional strategies most critical for preparing undergraduate education students to teach mathematics at the K-8 level, I conducted the same three analyses of the observation data as for Cora. These included the following: a) quantitative analyses of the frequency and duration of the episodes assigned to each category related to the TPACK framework, b) statistical analyses of the data to determine whether there were significant differences in the pattern of categories across sessions in a course, and c) analyses of her repertory grid elements and constructs to explore connections between her personal theories and their enactment in the classroom.

Distribution of classroom discourse across TPACK categories. This section analyzes the frequency and duration of the episodes assigned to each of the TPACK categories in the four sessions taught by Jamie. Of the total class time included in Jamie's dataset, 27.21% was coded as Logistics (13.44%) and Social (13.77%). These codes are not displayed in

Table 13 and are not included in the associated analyses, as they are not relevant to the research questions explored.

Table 13 displays the percentage of the episodes assigned to the seven TPACK categories in all sessions as well as each individual session. Categories for which I established the sub-codes of technology-analog and technology-digital display the percentage for each sub-code as well as the total for the category.

Table 13
Percentage of Classroom Discourse Assigned to Each Coding Category in Four Sessions taught by Jamie

	C	*P*	*T*		*PC*	*TC*		*TP*		*TPC*	
			T-A	*T-D*		*T-A C*	*T-D C*	*T-A P*	*T-D P*	*T-A P C*	*T-D P C*
All sessions	38.38	7.18	1.62		16.75	3.60		0.00		5.26	
			0.00	1.62		3.56	0.04	0.00	0.00	5.26	0.00
Session 1	39.09	6.97	0.73		22.97	5.80		0.00		1.18	
			0.00	0.73		5.80	0.00	0.00	0.00	1.18	0.00
Session 2	40.74	0.00	1.75		23.43	2.80		0.00		2.90	
			0.00	1.75		2.65	0.14	0.00	0.00	2.90	0.00
Session 3	30.13	0.50	1.80		13.81	5.83		0.00		17.96	
			0.00	1.80		5.83	0.00	0.00	0.00	17.96	0.00
Session 4	43.01	20.72	2.26		6.35	0.00		0.00		0.00	
			0.00	2.26		0.00	0.00	0.00	0.00	0.00	0.00

To visually represent the relationship of the talk in Jamie's class, Figure 12 shows an area-proportional Euler diagram in which the size and overlap of the circles correspond to the relative emphasis and overlap of classroom talk in all sessions as it related to pedagogy, content, and technology.

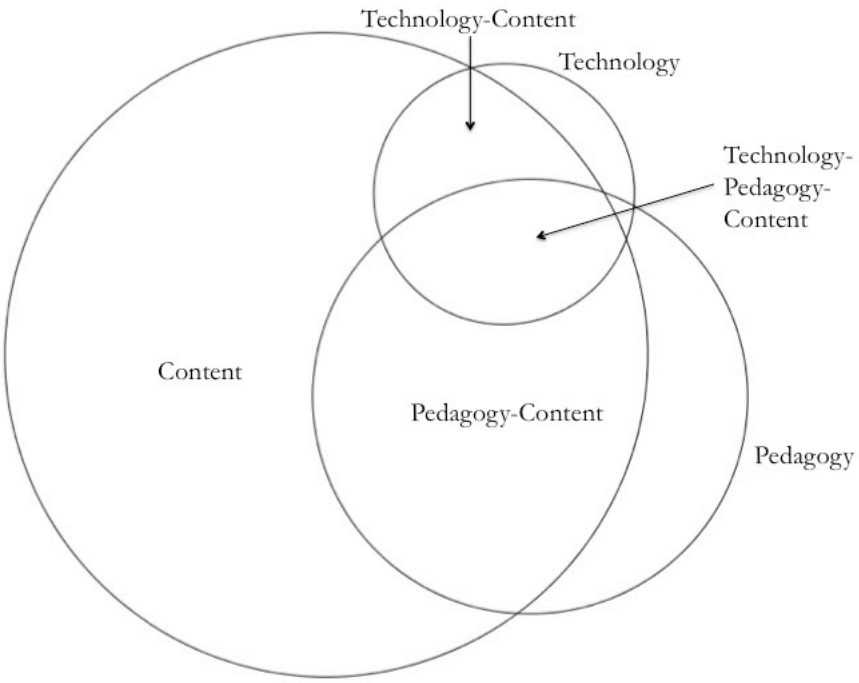


Figure 12. Area-proportional Euler diagram representing the discourse across four sessions of Jamie's class.

Examination of the tabular data and visual representation reveals trends about the distribution of talk in Jamie's classes. Because the course selected as the focus of this study addresses methods of teaching mathematics, we would expect to see the highest percentage of talk in the categories of pedagogy-content and technology-pedagogy-content. However, the content category (38.38%) receives the most emphasis in Jamie's class, far exceeding the percentage distribution in other categories. In addition, the percentage of talk coded as technology-content (3.60%) was very low, indicating that when Jamie and students discussed mathematics content it was rarely with reference to an analog or digital technology. In the class sessions observed, problems were most frequently presented to students as word problems distributed on paper or displayed on PowerPoint. Since Jamie rarely distributed manipulatives or required students to use visual or physical models, the students usually

solved the problems on paper using written symbols, as in the excerpt below in which Jamie assigns a mathematics problem related to problem solving for the students to work on:

> We need to do another math break. We're going to be doing some math in our math break.
>
> (On PowerPoint): [Jamie] has two paper shredders. One will shred a truckload of paper in 4 hours. The other will shred a truckload of paper in 2 hours. How long will it take to shred a truckload of paper with both shredders working?
>
> And this is not one of those quick ones – I'm not going to give you two minutes. I want you to take some time on this. Think about it. See if you can come up with maybe –try some diagrams, some – whatever you want to do to try to make sense of this. I encourage you to work with groups.

On the occasions when Jamie asked students to solve problems in class and suggested they use a variety of strategies, such as manipulatives, pictures, written symbols, oral language or real-world situations, the students most often elected to use written symbols in the form of mathematic or algebraic equations. After asking student volunteers to share their solutions at the front of class, Jamie would model her solution, sometimes using a picture in combination with written symbols.

> Jamie: Okay, I'd like…would somebody like to come up to the front and explain how they did it and give us a reason?
>
> [Student presents solution on the board.]
>
> Julie: When I watched her do it, I was like, "Why didn't I just turn this into an algebra problem all the way along?" I was trying to draw pictures and do all these things, but when I watched you write, "One equals four hours," I was like, "4X + 2X =" you know? Like in my head, I was like, "Oh, that makes so much more sense….
>
> Jamie: Okay. Does somebody have another way to think about it or another way to display it?
>
> Susan: I just put mine with variables, and didn't set it up that way, and solved it.
>
> Jamie: Well, do you want to show us? Because I'm not sure exactly what that means…. Let's look at this - another way to do the same problem.
>
> [Student presents solution on the board.]

Jamie: Did anybody else do it – solve it – using equations or algebra?

[Student presents solution on the board.]

Jamie: I want to talk about how you – I just want to show a way you could do it using a diagram – using a – I want to draw a rectangle and…just because of the representations, I want to bring in another one.

[Jamie presents solution on the board.]

The next-highest percentage of talk occurred in the pedagogy-content category (16.75%), but this category received less than half the time allocated to content alone. During the math breaks that allowed students to review mathematics content related to a particular topic such as problem solving or number theory, the solution sharing that followed usually focused more on how to solve the problem than on discussing topics such as how to assess student understanding based on the solution strategies shared or the benefits of having students share different strategies. In the 15-minute long solution sharing session from which the excerpts below are taken, almost 60% of the talk focused on content or technology-content, with only 30% focused on pedagogy-content. Of that, students initiated and facilitated over 80% of the pedagogy-content discussion.

Jessica: So I have a question for you – if we're messing up this way, would you guide us in the direction to get us on the right path, because – [laughter] – like how would you guide, like let's say I do have a student who stood up and did what I did. How would you guide them to get them on the right path, since they were wrong?

John: So, as teachers, why don't we start off like that – with this – to make the connection right away for the kids? Because I feel like if we would have started with the visual right away with the words, it would have been –

Jessica: He wanted us to find –

John: – But the kids aren't there yet, so how do you make them there?

Jamie: – Well, the idea though, is –

John: But I'm just saying that like, kids make mistakes too, but like if kids make the mistake, how will they know that that's a mistake?

Jamie: Right. What I'm saying is, if you were doing this in your class, the buildup to this over – you know, over a long period of time would have been working with fractions using diagrams, which we haven't done. We haven't done the diagrams and fractions chapter. So, I was relying on things you knew prior to this, but I didn't try to activate them because I don't know what you know. We've got that kind of adult situation. Did you have a comment to make?

Sharon: Well, I just have a comment because people are talking about why don't we just show them this so they don't make a mistake. Well, there was a problem like this in the book where they showed – they said, "We presented it to a bunch of teachers and here is four different ways that all the teachers solved it." Well, most of them were wrong, but because I read through all the different ways the teachers solved it, even though they were wrong, their presentation of it is what helps me realize what the right answer was. So, sometimes, when you see the different versions of it, even if most of them are wrong, it helps you start to recognize why it's wrong and why the right answer is right. So, it was kind of – it kind of helps to make those mistakes. Even if you think it's right and present your mistake to the whole class, that's what kind of teaches you not to make that mistake again.

Jessica: I love that connection. It's just that for me, in the special ed world, that's just too much. We're talking meltdowns, shutdown. Okay, that's not right. That's not right. That's not right. You know what I mean --

Sharon: – Well, and –

Jessica: – and that's why I'm just trying to figure out the balance.

Sharon: – if you have a certain child that you know can't handle the mistakes, then maybe that's when, before they get there, before they present to the class, you go around and you guide them. Like he was saying, in a more simple class, he would have guided you a little bit more than letting us go.

Jamie: Okay, that's good. And I – and that whole thing about you can learn from seeing mistakes and that – especially when you start verbalizing what you've done, a lot of people then start thinking, "Well, wait a minute. Somehow – how do I make sense of this?" You know, the logical way –

John: – Like the first explanation, sometimes, I'm like, "Oh, yeah, that's totally right." But then, the second person explained it different, and I was like, "Wait. That sounds right, too. So, which one is right?" And it made me think about analyzing, "Which one is really right and why?"

The percentage of talk focused on pedagogy (7.18%) indicates that Jamie and the students focused on issues of pedagogy almost half as often as they did pedagogy and content together (16.75%). This figure, along with the low percentages of talk focused on technology-pedagogy-content (5.26%) and technology-pedagogy (0%), indicate that in Jamie's class, the discussion usually focused on the discrete topics of content, pedagogy, and technology, and less often on the reciprocal relationship of two or more categories.

As with Cora's class, we see that the only pair of sub-codes in which talk about digital technology occurred more frequently than talk about analog technology was for the category of technology alone. The digital technology talk in Jamie's classroom was rather evenly divided between difficulties with digital technology, such as the document camera not working, and the instructor sharing technology tools with the students, as in the excerpt below.

Olivia: Yeah, you need the Smart Tools on. You can't erase.

Jamie: I'm not using SMART Board technology. This is the felt pen in PowerPoint. This is PowerPoint.

Olivia: Ohhh! I didn't know that PowerPoint had that option.

Jamie: Oh yeah, way down at the bottom [points to icon in lower left corner of the PowerPoint slide]. See way down at the bottom you can pick a felt pen and then you can use your cursor and draw. In this case you can draw on the SMART Board.

Olivia: That's so cool.

Violet: That is awesome! Snaps to Dr. Jamie!

Jamie: I've been using this for the past few weeks. I thought you guys knew I was doing this.

Violet: No. We didn't know you could do that.

Jamie: And then you can even save it if you want to, or when you close the PowerPoint you can say not to save it and it will be just a regular PowerPoint….

Olivia: I've never met someone who's better with technology than me who was like older than me. Normally, like older people are like not technologically –

Jamie: Yeah I know.

Olivia: So you are impressive.

Similarly, we note that the categories of technology-content and technology-pedagogy-content only receive focus because of the sub-codes used to distinguish and capture talk that relates to analog technology. Reassigning the technology-content codes to content and technology-pedagogy-content codes to pedagogy-content would again suggest that the discourse in Jamie's classroom rarely, if ever, focused on the integration of technology with content and/or pedagogy. In comparison to the distribution shown in Table 13 and Figure 12, the distribution show in Table 14 and Figure 13 highlight the isolated nature of most talk in the classroom when analog technology is not taken into consideration.

Table 14
Percentage of Classroom Discourse in Jamie's Class if Codes were Re-Assigned

	C	*P*	*T*	*PC*	*TC*	*TP*	*TPC*
All sessions	41.94	7.18	1.62	22.00	0.04	0.00	0.00

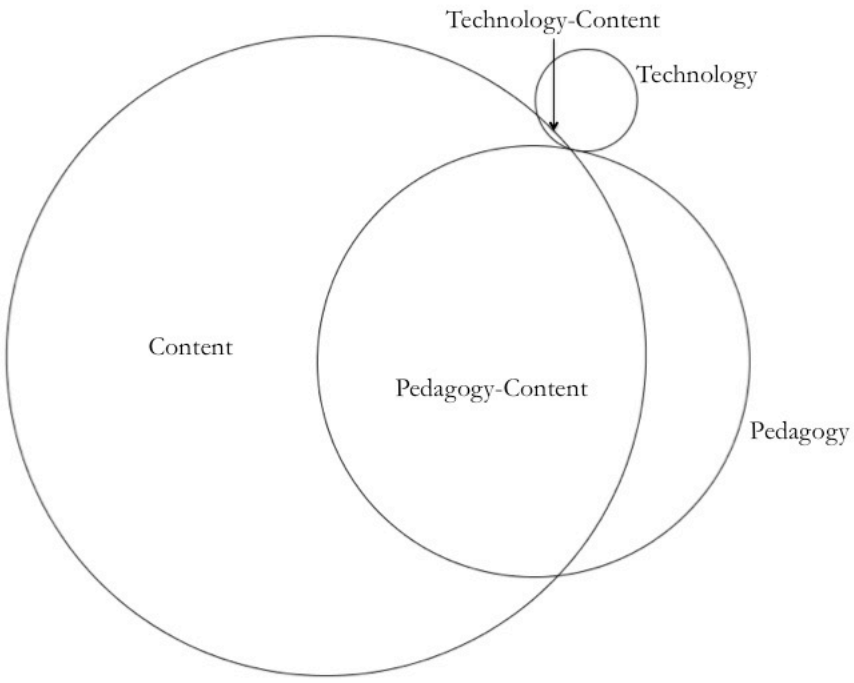


Figure 13. Area-proportional Euler diagram representing the discourse across four sessions of Jamie's class if the technology-analog sub-codes were re-assigned.

Test of independence. To examine whether there were significant differences in the pattern of categories across sessions taught by Jamie, I performed a chi-square test of independence. The relation between these variables was significant X^2 (15) = 100.34, p <.05, indicating heterogeneity in the distribution of the TPACK categories among observed class sessions. This suggests that the percentage of classroom discourse relating to each of the TPACK categories varied significantly from session to session or topic to topic.

Connections between personal theories and enactment in the classroom. Jamie's repertory grid included seven elements. Using the interview transcript, I reviewed Jamie's definition of each grid element and then applied the code most appropriate given her definition. For example, Jamie provided the following definition of "developing the teacher candidates' understanding of what it means to do math":

> I think one thing teachers need to learn, math teachers, is you don't teach math by showing kids how to do math and one thing I do with them is – well I do it at varying levels, but I will ask them "do you think a kindergartener can divide?" and they'll say "no." You know you can write a division problem or whatever on the board and it's like well no they don't get that until 3rd or 4th grade. And then I'll say, "well, what if you gave a kindergartener like nine carrot sticks and you said you want to divide those equally with three people – do you think they could figure out how much each person should get" and they say "yes." And I said "well, kindergarteners can divide, they just can't do it in this formal way that so many teachers think that's the goal of math – is I have to teach them this formal technique or this formal procedure."

This element, as with several others in Jamie's grid, could be coded as either Content or Pedagogy-Content, depending on whether Jamie intended her definition to illustrate the mathematical knowledge of a kindergartner, or elucidate for her students how the ability of young children to solve mathematical problems depends on the way a teacher presents the problem. In these cases, I cross-checked the code I applied to the element with the rating Jamie assigned the element on the construct "how you teach – what you teach." If Jamie gave the element a rating that indicated she construed it as being strongly about what you teach, that indicated a code of Content would be more appropriate than Pedagogy or Pedagogy-Content.

As with the elements in Cora's grid, the use of technology in the classroom could not be assumed based on the definitions provided in the structured interview, so I restricted the coding of elements to P, C or PC. The following transcript excerpt demonstrates one such definition, in which technology is referenced, but not as an essential component of the element:

> Meredith: Okay, so "develop versatility with different models?"
>
> Jamie: Yeah, and I'm talking operations here, operations with different numbers, with different models and different algorithms. Some of it's procedural, which would be algorithms, and some of it's just models and different ways to think about it to come up with an answer.

Meredith: Okay, so are you grouping their models, when you talk about algorithms/formulas, with manipulatives? Or do you think that those are two types of separate activities?

Jamie: Well it's all part of the same continuum. You know, like the kids in kindergarten, they can divide those carrot sticks up but eventually they're going to start talking about the formal operation of division and how you operate things so it's all connected.

After coding each element, I calculated the frequency of each code assigned to the seven repertory grid elements. Content was assigned to three of seven elements, pedagogy to zero elements, and pedagogy-content to four elements.

Next, I used linear regression to evaluate the strength of the relationship between Jamie's conception of the instructional strategies central to the preparation of future elementary education mathematics teachers and their enactment in the observed sessions. Again, due to the small dataset, the distribution of codes assigned to the elements elicited in Jamie's structured interview were not a statistically significant predictor of the distribution of codes assigned to the classroom discourse in the observed sessions ($\beta = .55$, $p = 0.63$), yet Jamie's personal theories still accounted for 31% of the variance in their observed enactment in the classroom.

As previously mentioned, depending on the specific context and function intended by Jamie, several of the elements in Jamie's repertory grid as well as many episodes in the classroom observations could be coded as either content or pedagogy-content. This likely influenced the strength of the statistical association measured between her personal theories and their enactment in the classroom, since the function of certain activities may have been construed one way by me as an observer and another way by Jamie, thereby potentially conflating whether an element or observed episode should be coded as content or pedagogy-

content. For example, during the structured interview Jamie described word problems as a strategy for "putting math in context," explaining that:

> One of the things I love to do is write problems. We wrote out these clones of problems in middle school and that's one of things I really love to do – I don't know, I get a pleasure out of that – and I like to challenge my students to be able to come up with those questions. Like if you're talking about greatest common factor, how can you put that in some kind of context instead of just giving them two numbers and saying what's the greatest common factor? Because if you can put it in some kind of situational problem then they can work on it without even knowing necessarily what that terms means – they can be doing the math without them knowing it. So I try to give them opportunities to try and write the problems, or create problems, that they can use in their classroom.

Here, Jamie articulates her belief that if teachers pose real-world problems that are familiar and contextual, rather than abstract or presented as a formal procedure, students will be better able to figure out ways to solve the problem. But when referencing "real-world problems" she continually equates this with "word problems," as when she describes the repertory grid element "building on knowledge" not as building on prior mathematics knowledge or familiar contexts, but writing word problems using scenarios that would be familiar to students. This association comes up again when she talks to the teacher candidates about how to support students' development of learning:

Jamie: There's two ways to support the development of learning [the meaning of operations] to kids…. Anybody know?

Megan: Manipulatives?

Jamie: Manipulatives. Okay, so some kind of model…. And the other one?

Kathy: Real life?

Gary: Concrete examples.

Jamie: Real life, what? What out of real life?

Kevin: Problems.

Jamie: Problems, okay. So word problems and models. So that's it. You don't start with a sheet of paper that says 5 plus 2, and you say, "5 plus 2 equals 7." Because kids don't know what addition is about. So word problems and models.

Jamie enacts her association of real-world problems with text-based word problems in classroom activities that ask students to classify word problem types or write word problems of their own. She describes these exercises as a way for the students to demonstrate their understanding of common addition and subtraction situations. But she does not directly discuss with the teacher candidates why they should present students with different types of addition and subtraction problems, distinguish between the way an adult would solve the problems mathematically versus a child using direct modeling, or how they can help students move from concrete to abstract problem-solving strategies. For these reasons, I coded many classroom episodes about operations and problem solving as content, not pedagogy-content.

Similarly, Jamie's example of asking a kindergartner to divide nine carrots between three children assumes the teacher provided the child some manipulative to directly model and solve the problem. During our interview, Jamie described manipulatives as part of the "models" continuum, yet during the classes observed, the instructional problems Jamie posed and those she asked students to create were almost always text-based. She rarely provided manipulatives for the students to work with or required them to create visual models of their work. Perhaps she thought the undergraduate students would not use manipulatives if they were provided, that algebraic models would interest them more, or that they would benefit more from review and discussion of alternate ways to solve mathematical problems. Whatever the reason, the mathematical models Jamie discussed in class usually took the form of alternate formula-based or text-based algorithms for solving a problem, not visual or physical models that could be used to explain or derive the abstract algorithm.

Jamie likely intended the sharing of these alternate algorithms as an enactment of her personal theories about the teaching of mathematics and the development of versatility with different models or algorithms, but she did not always articulate the relevance of the alternate models for teaching elementary education students.

Many classroom episodes demonstrated enactment of elements on Jamie's grid such as "instructional problem solving," "draw connections within math across grade levels / content material" and "develop understanding of what it means to do math," which I coded as pedagogy-content based on Jamie's definitions during the structured interview. However, unless Jamie elucidated the pedagogical relevance during class, the instructional modeling and classroom discourse appeared to focus on the mathematics content rather than the methods for *teaching* the content. For this reason, I coded many classroom episodes as content, rather than pedagogy-content. This had the effect of decreasing the measured strength of the relationship between Jamie's personal theories as represented by the repertory grid and their enactment in the classroom observations.

Retrospection

As with Cora, I conducted a brief retrospective protocol with Jamie after each observed class to discuss a selected incident from the observed class with respect to elicited repertory grid elements and constructs. The themes that emerged from these discussions align with those from the repertory grid and content analyses indicating Jamie's focus on what it means to do mathematics and teach mathematics, but less on specific instructional strategies the teacher candidates could use in their classrooms.

Guided by the type of activities often completed by the students in class, such as solving problems, then sharing and discussing the solutions, the post-observation interviews with Jamie focused on mathematics content-oriented elements from the repertory grid, such

as "developing versatility with different models" and "drawing connections within math." Our discussion of these elements further clarified how Jamie defined these elements and also underscored her emphasis and interest in developing her students' conceptual understanding of doing and teaching mathematics.

In one of the early interviews, Jamie articulated her interest in modeling for the students the idea of working on a problem, sharing solutions, and then having a "grand discussion." Indeed, the "math breaks" she interspersed throughout the class sessions provided students an opportunity to work on a problem related to that session's topic, such as problem solving, fractions, or number theory. After the students worked individually or in groups, Jamie asked for volunteers to come to the board so as to allow students to see and discuss various solutions. I noted that the strategies shared almost always used written symbols in the form of numbers, letters, and mathematic algorithms and a particular instance of this became the focus of one of our post-observation discussions. Why students in this course did not often use visual models or representations cannot be explained definitively, but Jamie recognized this as well, stating that:

> The only models that were being used were symbols of math, no one really used any models, and that's why I went up and tried to – cause somebody said that they tried drawing all these pictures and nothing worked – so I went up to show that model to them.

One possible explanation is that students responded to the strategies emphasized in class and consciously or subconsciously emulated what Jamie modeled. Although Jamie discussed the use of manipulatives during her lectures, when she presented problems for the students to work on she only occasionally suggested students consider physical or visual models as a way to solve the problem. During only one of the observed class sessions were physical manipulatives provided to the students and used as the basis for an in-class activity. Since

Jamie did not identify and call on specific students to share strategies she observed them using as she circled the room during the individual or group work time, she did not know ahead of time which strategies would be shared on the board and discussed by the class. As a result, visual models that Jamie shared usually came at the end of the discussion, thereby implying an optional or alternate strategy, rather than a recommended one. Not being encouraged or required to explore other models allowed the students to use the ones most familiar to them from their own mathematics preparation, rather than promoting physical or visual representations of the problem.

A related possibility is that Jamie's definition of "versatility with different models" may focus more on versatility with different algorithms as opposed to a variety of representations to support relational understanding. This is not to suggest that Jamie prioritized instrumental over relational understanding (Skemp, 2006), but in reviewing types of algorithms, the classroom activities emphasized identification of problem types and practice of invented, developmental, or alternative algorithms over how to use them as a teacher to build on students' problem-solving strategies or identify their misconceptions. Jamie explained that she has the students practice these as a way to…

> …demystify the standard algorithm….If you go into classrooms, the kids have these steps to follow and they teach…steps step-by-step, so it's like this magical thing. And it's the *only* magical thing when you do it, so I like to show them that there are other ways to do it…And then I try to explain why it is an algorithm and why it will work all the time.

For the adult students already familiar with concepts of place value, borrowing, and our culture's standard subtraction algorithm, the alternative algorithm of "equal additions subtraction" provided Jamie an opportunity to discuss higher level concepts of "why it is an algorithm and why it will work all the time." But for young students whose understanding of place value and exchanging is not well-developed, emphasizing alternative algorithms may

encourage application without understanding. Not discussing this with the teacher candidates directly may have inadvertently emphasized the algorithms themselves rather than the prerequisite knowledge and understanding required of students and the appropriate way to introduce them in a mathematics curriculum. Even so, Jamie's obvious interest and passion for sharing problems with interesting mathematical structure and highlighting links between mathematics concepts across grades, such as the standard vertical model of multiplication as applied to multiplying binomials in algebra, engaged many of the students in class and exposed them to concepts and connections they may not have seen previously.

Cassie

Conceptions

Descriptive analysis of the structured interview process. As with Cora and Jamie, the repertory grid addressed the topic of instructional practices for preparing future teachers to teach mathematics at the K-8 level. To begin the element elicitation process, I read the specified topic and asked Cassie to brainstorm verbally in response. To clarify, Cassie asked me a question:

> The first thing that came to my mind is that I want my students to be knowledgeable about what they will be teaching, but I achieve that by doing a lot of other different things, so I'm assuming you want the other different things that I do with them?

After I responded in the affirmative, Cassie quickly and assuredly proposed eight elements one after the other, providing short descriptions as she listed each item.

As with Cora and Jamie, I asked Cassie a follow-up prompt intended to activate the consideration of technology. In response to this prompt, Cassie proposed one additional element, "use online resources," which she described as including virtual manipulatives, Blackboard, Tk20 and communication tools. Since Cassie provided examples when suggesting this element, her response seemed meaningful to her and we did not remove it

when we reviewed the final list. Her ratings on this item are interesting to note, though, and will be discussed in the next section.

During our review of the elements, I asked Cassie to look at the list to determine if she would like to add any additional elements. She reflected and added one element: have high expectations. We included all 10 of the original elements in Cassie's final list and did not modify any from her original wording.

After entering the final list of elements on the grid, I wrote each element on a notecard so we could move the elements around on the table as we talked. Initially, I used preselected combinations of elements (such as 1, 3, 5; 2, 4, 6; 3, 4, 8; etc.). After eliciting and rating four pairs of constructs, I began to look at the elements that had similar ratings on two or more pairs and used them in a triad so Cassie would need to identify a construct that distinguished them in some way.

When eliciting constructs, I asked Cassie to identify the two practices that she thought shared some quality and the one that she thought was different in some way. Throughout the process, Cassie responded to all comments or redirections in a positive manner and seemed to easily understand what I was asking or why I was suggesting that we needed to discuss the construct or rating further. She also understood the intention of the two poles as ends of a continuum rather than logical opposites. For example, when proposing the fifth construct, she initially described the common quality as "using technology" and the distinguishing characteristic as "not using technology." When I asked her if there was a way we could define the two elements other than "not technology" she said "not" at the same time and laughed and replied, "yeah, well, let's see." She decided to propose a different construct, but understood the goal of describing the two poles using different terms rather than logical opposites.

While the construct elicitation process seemed easy for Cassie and she proposed constructs rather quickly, one construct took more time and thought to generate than the others: students play a passive role – students play an active role. Cassie's concern was that "passive" not be construed as negative or suggest that the students were not engaged during certain activities. She addressed this concern during the rating process, by electing not to assign any "1" ratings, which would suggest that during some activities students took a completely passive role.

As with Cora and Jamie, at the end of the interview I asked Jamie if there was any other construct we had not yet discussed that she would like to add to the grid. Cassie talked about this for several minutes, spending two to three times longer on this discussion than for any previous construct and using a triad of elements to contextualize the discussion, but in the end did not add another construct.

The rating procedure itself made sense to Cassie, although she spent more time than the other two participants explaining each rating and how she defined its position with respect to the two poles. Throughout the interview process, Cassie made reference to the use of the interview data for my research study and the importance of the interview being conducted in a way that was useful and meaningful for me. For example, while rating an element and using the rating of "3" for the fifth time out of six total for that construct, she evinced concern that she was "not doing this correctly." During the ratings for the next construct, she commented that she was "going to say something that's going to ruin your results," indicating an understanding that the data would be analyzed quantitatively and expressing concern that her ratings might have a negative impact on the data analysis.

> As we finished the interview, Cassie commented on the interview process:
> It is tiring. It's more tiring than teaching, honestly. I found this more tiring than teaching for three hours. Because maybe it makes you think of some things you do

> all the time but you don't really think about the foundation behind it, why you do something.

We discussed this briefly and attributed this to the mental activity that is required by the grid elicitation process. After teaching the course on mathematics methods for K-8 classrooms multiple times each semester for many years, Cassie likely does not have to spend as much cognitive effort on designing her lessons and selecting her instructional strategies each time she teaches the course. In contrast, participating in the grid elicitation process to describe the qualities and constructs of activities she uses in her classroom requires focused cognitive effort to describe her values and beliefs as an instructor.

Descriptive analysis of the repertory grid. This section reviews what Cassie said about each element by reading through and briefly describing her ratings for each column on each construct. Figure 14 shows the repertory grid elicited with Cassie, with the 10 elements displayed along the bottom and the seven construct pairs on the left and right of the grid. The grid follows a general convention that the left-hand end of the construct (the emergent pole that describes what two elements in the triad have in common) define the "1" end of a 5-point scale, and the right-hand end of the construct defines the "5" end of the 5-point scale.

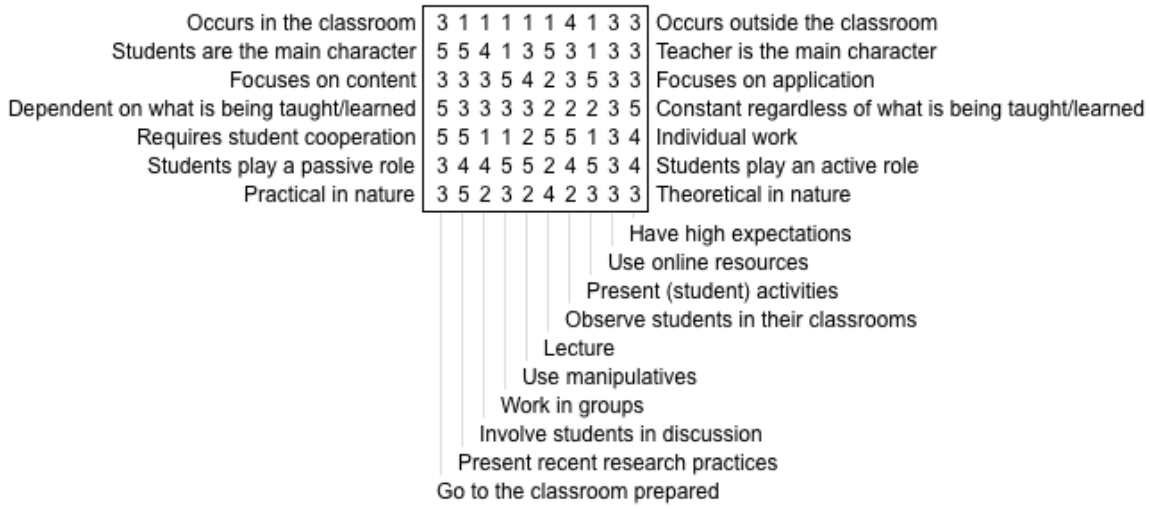


Figure 14. Repertory grid elicited with Cassie.

The elements Cassie proposed range from the abstract (go to the classroom prepared and have high expectations) to concrete (lecture and involve the students in discussion), from general practices (involve the students in discussion, work in groups, lecture, and use online resources) to those specific to teacher education (observe students in their classrooms and present recent research practices) to those specific to mathematics education (use manipulatives), from frequently used (go to the classroom prepared, lecture, and involve them in discussion) to less frequent (have students present activities and observe students in their classrooms).

Cassie views going to the classroom prepared as something that occurs both inside and outside the classroom – outside because she prepares beforehand and inside because she goes to the classroom in advance of each session to set up necessary materials and configure the classroom. She sees the teacher as the main character for this activity and models this as a trait for her students to emulate when they become teachers. Because the teacher is the main character in this practice, the work is individual. Going to the classroom prepared represents an activity that focuses equally on content and application, and on theory and

practice. It is an activity in which Cassie is actively engaged as the one preparing for the class, but she also requires students to prepare by reading or completing the activities assigned.

Presenting recent research practices is seen as an activity that occurs in the classroom and in which the instructor is the main character. The research practices are often presented as content but are meant to be applied, so this activity received a rating of "3" on the content-application construct. Since research practices are presented with some frequency but are still dependent on what is being taught/learned, the practice received a rating of "3" on the dependent-constant construct as well. Cassie presents the research practices on her own as an individual, but believes students are pretty actively engaged when she presents the research practices. She defined this practice as purely theoretical in nature.

Cassie views including students in discussion as something that occurs in the classroom and requires a great deal of student cooperation and active engagement. Discussion applies equally to material that focuses on content and application and is slightly more practical in nature than theoretical. She identified the teacher as being more of a main character when involving students in discussion, possibly because the teacher is the one actively facilitating the discussion among students. Since student discussion occurs with frequency but is still being dependent on what is being taught/learned, the practice received a rating of "3" on the dependent-constant construct.

Working in groups is a practice that always occurs in the classroom, has the students as the main characters, and requires a high level of student cooperation. Group work always focuses on application of some material and requires students to be actively engaged. Since group work occurs with frequency but is still being dependent on what is being

taught/learned, Cassie gave the practice a rating of "3" on the dependent-constant construct. She sees group work as equally theoretical and practical in nature.

The use of manipulatives always occurs in the classroom and requires involvement of both the instructor and students. The emphasis is primarily on application and students are actively engaged when using manipulatives. Usually student cooperation is required and manipulative activities are usually practical in nature. Since the use of manipulatives occurs with frequency but is still being dependent on what is being taught/learned, Cassie gave the practice a rating of "3" on the dependent-constant construct.

Lecture always occurs in the classroom and the instructor is the main character. As such, this is individual work and students are somewhat passive in this activity. Lectures usually focus on content and information that is more theoretical in nature. When Cassie uses lecture depends on what is being taught.

Cassie observes students in their own classrooms outside of the regular class time and believes that the instructor and the students play an equal role in this activity. Cassie's student observations focus equally on the students' content knowledge and also their application of concepts and skills. When she conducts the observations depends largely on what is being taught/learned in that student's class. The work is individual, in that it is being performed individually by the instructor doing the observation and also by the student as a teacher in his/her own classroom. The student is actively engaged in that teaching process and the instructor is actively engaged in observing the student. Since the purpose of the observation is to see the student actively teaching, Cassie sees the observation as primarily practical in nature.

Student presentations of activities occur in the classroom and require a high level of student cooperation, especially since the students are the main characters. The presentations

focus on the application of some material and require students to actively demonstrate some activity. At what time student presentations are conducted depends largely on what is being taught/learned in the class. Cassie expects students to demonstrate an understanding of both the theoretical and practical nature of the activity they present.

Cassie rated "use online resources" as a 3 on every construct in the grid, likely because the element was broad in scope and its characterization depends on what type of online resources are considered and how they are used. Cassie believes the use of online resources occurs both inside and outside the classroom, requires both the teacher and the students to participate, and sometimes requires students to use the resources individually and sometimes cooperatively. Online resources might focus on either application or content and be both theoretical and practical in nature. Since the use of online resources occurs with frequency but is still being dependent on what is being taught/learned, she gave the practice a rating of "3" on the dependent-constant construct. Students may sometimes be actively engaged when using online resources, or more passive if they are being demonstrated or the students are not required to engage with the resources.

Cassie establishes high expectations for her students through her out-of-class materials and also when working with the students in class. Also, she has high expectations of the student work whether conducted in class or out of class. She sees herself, the instructor, as a main character in setting the expectations and the students as main characters since they are the focus of her expectations. High expectations are upheld whether the student work focuses on application or content, or is theoretical or practical in nature. Cassie's high expectations for students are constant and apply to each individual. She expects them to actively engage in the class to meet her expectations.

Construct characterization. The constructs proposed often use terms or phrases that reflect familiar categorizations or descriptors of learning activities, such as "active/passive," "learner-centered/teacher-centered," "content/application," and "theory/practice." While these terms may be familiar to a general audience, that does not preclude them from being personally meaningful and relevant in terms of representing Cassie's characterization of the teaching methods she considers to be most central to preparing teachers in mathematics education.

Of the seven constructs Cassie provided, I identified five as core and two as peripheral, with the two constructs I identified as peripheral being the first two elicited: occurs in the classroom – occurs outside the classroom, and students are the main character – teacher is the main character. It is possible that Cassie began the construct elicitation process by suggesting traits more descriptive in nature that did not express her more personal, fundamental values.

I consider these constructs to be peripheral for two reasons. First, because they describe traits that tend to represent one pole (in the classroom), the other pole (outside of the classroom), or a neutral position (equally occurring in class and out of class), and secondly because Cassie's ratings reflect this polarity by having the fewest number of mid-level ratings (i.e., 2 or 4). Only one out of ten elements received a 2 or 4 rating on each of these two constructs. A third construct, requires student cooperation – individual work, might also appear peripheral but based on the type of elements included in Cassie's grid, such as involving students in discussion and working in groups, I believe this construct reflects one of Cassie's core values that students work in cooperation in the classroom.

Similarly, I identified the same two constructs as propositional because these two constructs describe basic traits of the set of elements, such as occurring in the classroom or

outside of the classroom, and do not appear to be as personally meaningful in terms of an element's qualities or characteristics.

I characterized two of the non-propositional constructs as constellatory, or "those which imply the position of an element on other constructs very strongly indeed" (Jankowicz, 2004, p. 85): focuses on content – focuses on application, and students play a passive role – students play an active role. Knowing that an element focuses more on application likely indicates that it will also be characterized as an instructional practice that requires students to take an active role. The cluster analysis, discussed in the next section, validates this characterization.

As previously stated, Cassie was thoughtful and deliberate in the choice of terms and phrases she used to describe element characteristics. For this reason, I do not believe the non-propositional constructs are stereotyped, although Cassie sometimes employed terms generally familiar in teacher education, such as passive, active, practical, and theoretical. Cassie was less prone to providing long phrases or explanations to describe the construct, but thoughtfully and carefully selected one word or a short phrase for the pole of each construct.

Cluster analysis. Figure 15 shows the cluster analysis of the repertory grid elicited with Cassie. I first interpret the cluster-analyzed elements, then the constructs.

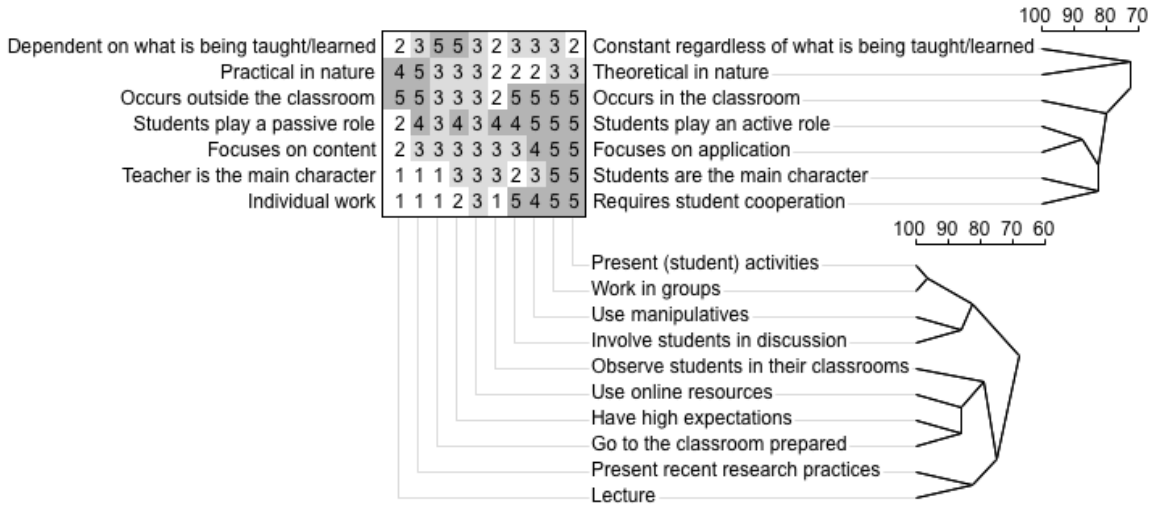


Figure 15. Cluster analysis of the repertory grid elicited with Cassie.

Element dendogram. The element dendogram has two main structures, each of which has two sub-clusters. The sub-clusters of "present (students) activities" + "work in groups" and "use manipulatives" + "involve students in discussion" make up one cluster. The sub-clusters of "present recent research practices" + "lecture" and "have high expectations" + "go to the classroom prepared" along with "use online resources" and "observe students in their classrooms" make up the second cluster. The primary distinction between the two main clusters is the role of the student versus the instructor in those activities. Although both the students and instructor play a role in most of the activities, activities in which student play a more active role and work with each other are clustered together in one group.

Table 15 displays the similarity scores of elements on Cassie's repertory grid that exceed a match of 80%.

Table 15
Matches of 80% and Higher between Elements in Cassie's Repertory Grid

Elements	*Match %*
Pair 1	96.4
Work in groups	
Present (students) activities	
Pair 2	85.7
Use online resources	
Have high expectations	
Pair 3	85.7
Involve students in discussion	
Use manipulatives	
Pair 4	85.7
Go to the classroom prepared	
Have high expectations	
Pair 5	82.1
Work in groups	
Use manipulatives	
Pair 6	82.1
Present recent research practices	
Lecture	

"Work in groups" and "present (students) activities" show the highest percent similarity, with 96.4% similarity in their ratings. "Involve students in discussion" and "use manipulatives" show an 85.7% similarity, as do "go to the classroom prepared" and "have high expectations." "Present recent research practices" matches 82.1% with "lecture." Because "work in groups" and "present (students) activities" share so many ratings, the strong similarity between "work in groups" and "use manipulatives" is not as evident on the dendogram, however these two elements show an 82.1% similarity. This similarity is expected, since students often work in groups when using the manipulatives.

The sub-clusters that have the highest similarity score share ratings for the reason that one activity often partially encapsulates or describes the method of delivery of the other. For example, Cassie described the student presentations of activities as work they complete in groups, therefore the "present (students) activities" element shares many ratings with the

"work in groups" element. Similarly, when students "use manipulatives," Cassie often has them "work in groups" and then "involves students in discussion" with each other and herself during and after the activity. Also, Cassie often "presents recent research practices" as part of the "lecture" component of the class, so these elements again receive similar ratings on many of the constructs. Although the element "go to the classroom prepared" primarily referred to Cassie, it had a strong similarity with "have high expectations" because both elements describe the standards and expectations held by Cassie for both herself and the students.

The reason for the high similarity between "use online resources" and "have high expectations" is not readily apparent and although the two elements share ratings on many constructs, this appears to be coincidental rather than reflective of a connection or relationship between the activities.

Construct dendogram. The construct dendogram has two main branches, one of which comprises "dependent on what is being taught/learned – constant regardless of what is being taught/learned" and "practical in nature – theoretical in nature." The second main branch comprises three sub-clusters. The first sub-cluster comprises "students play an active role – students play a passive role" and "focuses on content – focuses on application," the second sub-cluster comprises "teacher is the main character – students are the main character" and "individual work – requires student cooperation," and the third sub-cluster consists of the single construct "occurs outside the classroom – occurs inside the classroom."

Table 16 displays the similarity scores of constructs on Cassie's repertory grid that exceed a match of 80%.

Table 16
Matches of 80% and Higher between Constructs in Cassie's Repertory Grid

Constructs	*Match %*
Pair 1	87.5
Focuses on content – Focuses on application	
Students play a passive role – Students play an active role	
Pair 2	82.5
Students are the main character – Teacher is the main character	
Requires student cooperation – Individual work	
Pair 3	82.5
Students are the main character – Teacher is the main character	
Focuses on content – Focuses on application	
Pair 4	80.0
Focuses on content – Focuses on application	
Practical in nature – Theoretical in nature	
Pair 5	80.0
Occurs in the classroom – Occurs outside the classroom	
Students play a passive role – Students play an active role	

The ratings of all elements on "students play a passive role – students play an active role" and "focuses on content – focuses on application" are similar (87.5% match), with five elements receiving the same rating and five elements being rated one point different. This is likely because when the classroom activity focuses on content (as opposed to application), students play a more passive role in the associated activity or instructional practice.

The matches between constructs indicate that when Cassie thinks of activities that focus on content, she plays a primary role in the activity, students are more passive, and the activity requires work more individual in nature. When Cassie thinks of activities that focus on application, the students are the main characters, they play an active role, and often cooperate to complete the activity. The similarity scores of several constructs bear out this

argument. For example, the ratings of many elements on "students are the main character – teacher is the main character" and "requires student cooperation – individual work" are similar (82.5% match) and this is likely because when Cassie is the main character, the work is individual whereas when the students are the main character, they are often working in cooperation. Similarly, the ratings of many elements on "students are the main character – teacher is the main character" (whose polarity is reversed in the cluster analysis) and "focuses on content – focuses on application" are also similar (82.5% match). When students are the main character, they are usually actively participating in some activity that requires them to apply concepts, whereas when Cassie is the main character she is often presenting material and therefore the focus is usually on content.

Several of the constructs appear inter-related and difficult to disentangle. For example, activities the instructor identifies as having students as the main character receive similar ratings on "requires student cooperation – individual work" and "focuses on content – focuses on application." Indeed, "students are the main character – teacher is the main character" and "requires student cooperation – individual work" as well as "students are the main character – teacher is the main character" and "focuses on content – focuses on application" are matched at 82.5%.

The final construct similarity score discussed is that of "focuses on application – focuses on content" and "practical in nature – theoretical in nature," with a similarity match of 80% although their polarity is the reverse of one another. For example, elements that receive a high score on "practical in nature – theoretical in nature" receive a low score on "focuses on content – focuses on application" and elements that receive a high score on "focuses on content – focuses on application" receive a lower score on "practical in nature –

theoretical in nature." This inverse relationship is expected given the strong similarity in phrasing of the two constructs.

Summary. Cassie has a clear sense of the instructional practices she considers to be central to the preparation of future elementary education mathematics teachers. She quickly grasped the focus and aim of the repertory grid elicitation process and responded evenly and assuredly to each part of the process: element elicitation, construct elicitation, and rating the elements. The elements she proposed represented a range of activities one would expect to see in a teacher preparation classroom (lecture, discussion, group work) as well as activities specific to mathematics education (use of manipulatives). Some of the constructs seem to overlap (such as "focuses on content – focuses on application" and "practical in nature – theoretical in nature") and sometimes seem to describe very elementary characteristics (such as the main character of the activity, whether the activity is an individual or cooperative task, whether it takes place inside or outside of the classroom).

Cassie's repertory grid represents an instructor who is focused, logical, direct, and holds strong beliefs about how to effectively prepare future elementary education mathematics teachers. At the same time, she is passionate about mathematics education and demonstrates this through the high expectations she holds for her students and herself and a clear sense of what it means to prepare and be an elementary education mathematics teacher.

Enactment

To explore how Cassie enacted her personal theories about the instructional strategies most critical for preparing undergraduate education students to teach mathematics at the K-8 level, I conducted the same three analyses of the observation data as for Cora and Jamie. These included the following: a) quantitative analyses of the frequency and duration of the episodes assigned to each category related to the TPACK framework, b) statistical

analyses of the data to determine whether there were significant differences in the pattern of categories across sessions in a course, and c) analyses of her repertory grid elements and constructs to explore connections between her personal theories and their enactment in the classroom.

Distribution of classroom discourse across TPACK categories. This section analyzes the frequency and duration of the episodes assigned to each of the TPACK categories in the four sessions taught by Cassie. Of the total class time included in Cassie's dataset, 10.94% was coded as Logistics (9.78%) and Social (1.17%). These codes are not displayed in Table 17 and are not included in the associated analyses, as they are not relevant to the research questions explored.

Table 17 displays the percentage of the episodes assigned to the seven TPACK categories in all sessions as well as each individual session. Categories for which I established the sub-codes of technology-analog and technology-digital display the percentage for each sub-code as well as the total for the category.

Table 17
Percentage of Classroom Discourse Assigned to Each Coding Category in Four Sessions Taught by Cassie

	C	*P*	*T*		*PC*	*TC*		*TP*		*TPC*	
			T-A	*T-D*		*T-A C*	*T-D C*	*T-A P*	*T-D P*	*T-A P C*	*T-D P C*
All sessions	10.08	0.74	0.16		34.51	11.54		0.05		31.99	
			0.05	0.11		11.46	0.07	0.05	0.00	31.30	0.69
Session 1	14.11	1.28	0.31		32.40	29.69		0.20		11.41	
			0.00	0.31		29.69	0.00	0.20	0.00	11.41	0.00
Session 2	6.23	0.25	0.06		54.17	7.98		0.00		24.86	
			0.00	0.06		7.73	0.25	0.00	0.00	24.86	0.00
Session 3	9.57	0.76	0.03		26.38	3.18		0.00		51.96	
			0.03	0.00		3.18	0.00	0.00	0.00	51.96	0.00
Session 4	11.22	0.74	0.29		22.59	7.37		0.00		37.71	
			0.17	0.12		7.31	0.06	0.00	0.00	34.68	3.03

To visually represent the relationship of the talk in Cassie's class, Figure 16 shows an area-proportional Euler diagram in which the size and overlap of the circles correspond to the relative emphasis and overlap of classroom talk in all sessions as it related to pedagogy, content, and technology.

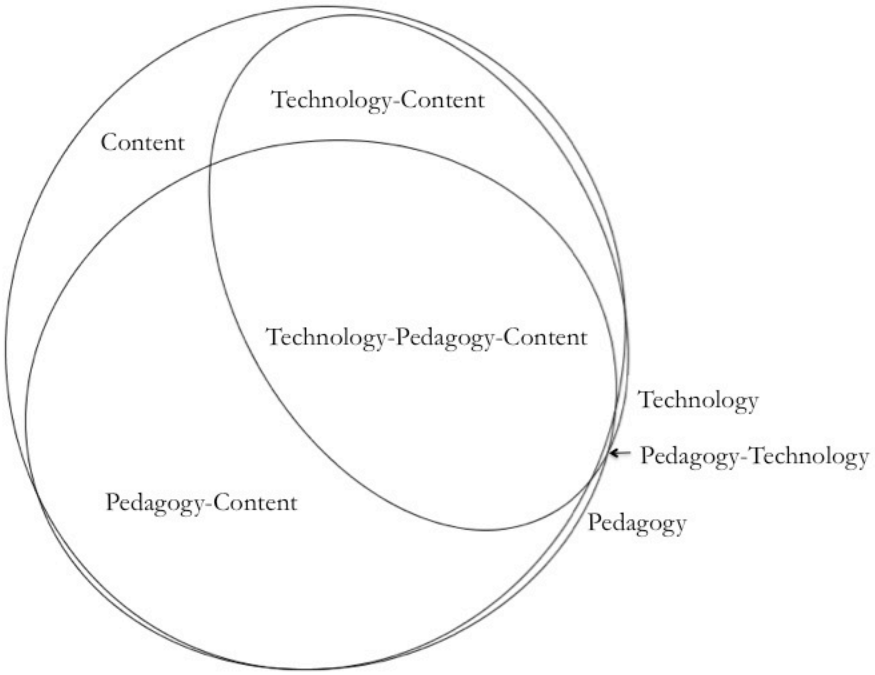


Figure 16. Area-proportional Euler diagram representing the discourse across four sessions of Cassie's class.

Examination of the tabular data and visual representation reveals trends about the distribution of talk in Cassie's classes. The course selected as the focus of this study addresses methods of teaching mathematics, not mathematics content or methods of teaching alone, so we expect to see the highest percentage of talk in the categories of pedagogy-content (34.51%) and technology-pedagogy-content (31.99%). The data support this expectation and particularly emphasize the large percentage of classroom discourse focused on these two topics, exceeding two-thirds of the classroom talk in the four observed sessions. In addition to providing students an opportunity to experience activities she

believes are critical to the teaching of mathematics to elementary education students, Cassie also modeled how to bridge the concrete activity to the abstract mathematical model and facilitated a discussion with the teacher candidates about the interweaving of the components and how varying one element affects another. In the excerpt below, Cassie discusses with students how they can assess student content knowledge by observing student use of the manipulatives to solve problems and then make the activity easier or harder for their students, so as to adjust for their level of mathematics knowledge and increase the challenge as student understanding and ability grow.

Cassie: Let's discuss some important things about this activity. If you have your first graders or your second graders play this game, and you walk around, and you observe the students the same way I was walking around and I was looking at what you were doing, what are some things that you can learn about your students' knowledge on base 10, operations, whatever, as you observe? What are some things that you can learn?

Susan: You can see how well and fast they can add. Like, do some of them have to put them all out or do they know that they can just pull out and switch out on their own.

Cassie: Okay, so, how well they can count their numbers, and then, make exchanges if they need to. [Another suggestion?]

Lisa: And how well they recognize the numbers on the dice.

Cassie: Right. Even before we go into the blocks, if they rolled a five and a one, do they know that, immediately, this is a six? Do they know that five plus one is six? Or do you have kids who actually count, "One, two, three, four, five, and one, six"? So, you evaluate their addition facts by playing a game that you are thinking about place value, but then, you get feedback on other things, too, okay? So, that's very good. Other things that I can get out of just observing my students?

George: That they can count by tens, or by twos to get to the number they need to.

Cassie: Okay, so, how they count – if they have a lot of blocks, do they count by twos? Do they count by tens? Okay, very good. Other things?

Vicky: Just their general manipulation, like trading out, moving things over, just all of those things – how they manipulate those.

Cassie: Right, how they manipulate numbers. And you say that in a very general term, but there are a lot of specific things that you can get out of your students, and I was walking around, and I saw different ways that even you adults use to make up your numbers, okay?

So, these are different strategies that you're going to see your students using. It doesn't tell you, "This person is smarter than the other person." It tells you, "I have students who understand place value a little bit differently and they're not at the same level yet." Some people need to see the ones in front of them, and other people can visualize, "Okay, nine plus one from here, that will make a ten, and I can make my exchange," okay?

Now, how can you make this game…a little bit easier if you have a group of kids who are still struggling with large numbers and you don't want to go to 100, how can you make this a little bit easier? Jean?

Jean: Use one dice.

Cassie: Okay, if you don't want them to be concerned about adding numbers, they can use only one die. That's going to take longer, of course, right? Other things that you can do? Elizabeth?

Elizabeth: Go to 50.

Cassie: Race to 50, right? If you don't want them to go all the way to 100, and sometimes time is important, make sure that you change your activity accordingly – Race to 30, Race to 50. What if you want to make this a little bit more challenging with older students?

Ashley: Race to 1,000 with six dice.

Cassie: Race to 1,000, wow. That's a big change. Ashley said, "you can say Race to 1,000" [holds up the 1000-block] and that's a good objective because that will force them to make exchanges with hundreds, but if you use only two dice, it's going to take forever, okay? She said, "Use more dice." That's a good idea, but then, maybe, the kids will have a hard time adding numbers with six. So, another thing that you can do here, Racing to 1,000, you can multiply – Elizabeth said – six and five. Multiply six times five, is 30, and they can put 30 on their organizer. Or, if you want to stay with the theme of place value, you can change the rules a little bit and say, "When you roll your dice, the high number is your tens; the smaller number is your ones." So, if I roll six and five, what number am I going to put on my organizer?

Ryan: Sixty-five

Cassie: Sixty-five, okay? So, I gave you one, simple activity, and you have this also in your…packet, but that doesn't mean that that's the only way you can play the game. You can change it depending on the group of kids that you have, how old they are, if you want to make it easier, if you want to make it more difficult. How can I make it even more challenging? If you want to play this game and really challenge the kids on place value, how can you make it even more difficult?…What can you do? Kathy?

Kathy: Go backwards.

Cassie: Go backwards, right? What if you start with 1,000 [holds up the 1000-block] and the goal is to go to zero? Okay? And you rolled a six and a five, and you need to take away 65 from this, the kids need to do what? Break this into ten hundreds; take 100, break it into 10 tens; take a ten, break it into 10 ones, and then, start taking away, okay? So, that's a great activity that's going to challenge your students and it will help them again with exchanging and understanding place value, and once they understand this – and the kids love these activities because they think of them as games – once they have a good understanding of this, then you can go into your four operations. You're going to see how quickly now I'm going to go over the four operations again because you understand these exchanges. But the main point here is that it's important for them to understand the 1:10 relationship, and you, the teacher, can collect very important information by observing your students. Oftentimes, you think that the only way to get feedback is to give the students a test or a quiz, and actually, the number one thing that you can do and get information is observing because if I walk around, and I see that Jennifer has a pile of ones, and she's collecting ones, and she's collecting ones, and she has a mountain of ones, do I need to give her a test to see if she understands place value? No, okay? So, if you see a student who behaves in a certain way, that information becomes your guidance for, "What am I going to do with this group of kids this week or this next week?"

The next-highest percentages of talk occurred in the categories of technology-content (11.54%) and content (10.08%), usually in the form of instructor or student talk about the mathematics knowledge and skills necessary to understand and then teach the mathematics to elementary education students. The percentage of talk coded as technology-content was similar to that coded as content alone, indicating that when Cassie and the students discussed mathematics content it was often with reference to an analog or digital

technology, such as manipulatives or visual models. This underscores the emphasis Cassie places on using models, usually in the form of physical manipulatives, to teach young children basic concepts first so they develop a strong understanding and then build on that strong conceptual understanding to teach advanced concepts and skills. In the excerpt below, Cassie works with the students and fraction circles to model teaching the fraction concepts of parts of a whole and equivalency.

Cassie: All right, so the first thing I'm going to ask you to do is go to your table and get these circular pie pieces out very quickly. And I want each person to create and have in front of you one pie, one whole pie, with the same color pieces. [Students work for 30 seconds.] Okay, Jan, how did you make your pie? How many pieces do you have there?

Jan: Three.

Cassie: Okay. So Jan says, "I used three out of three parts." Do you have one whole pie? [Writes "3/3 = 1" on board.]

Jan: Yes.

Cassie: Yes. So is it okay to say that three out of three parts is one?

Jan: Yes.

Cassie: Yes. Very good. Betty, how about yours?

Betty: Eight.

Cassie: Betty used eight out of eight parts. Do you have one? [Writes "8/8 = 1" on board.]

Betty: Yes.

Cassie: Yes, very good. Who has a different one? Traci?

Traci: Four.

Cassie: Traci used four out of four parts. Do you have one? [Writes "4/4 = 1" on board.]

Traci: Yes.

Cassie: Yes, okay. You get the idea? Who has another one that is not on the board? Kyle?

Kyle: I have one-half. I mean I have two.

Cassie: So you used two out of two, and do you have one whole pie? [Writes "2/2 = 1" on board.]

Kyle: Yes.

Cassie: Yes. And another one?

Justin: Um, six out of six.

Cassie: Justin used six out of six. And I have one. [Writes "6/6 = 1" on board.] So it took us 30 seconds, and here [fractions on the board] we have a very important concept. What is it?

Kristina: Parts of a whole.

Cassie: What is it? Give me this in English.

Kristina: Equivalency.

Cassie: Equivalency. Okay, what do you mean by that?

Kristina: That they're equal.

Cassie: They're all equal, yeah. They're all whole parts. Three over three is the same as eight over eight is the same as 100 over 100 is one. What else? How many pieces do I need to have to have one whole? All of the pieces, right? All of the pieces. If my pan of brownies is cut into 25 pieces, the whole pan is 25 of 25 pieces. If someone eats a piece, then I don't have one whole anymore…Take a look at the pieces that make up your pie. What can you tell me about those pieces?

Diane: They're equal.

Cassie: They're all the same size, very good. So with younger students, they may need to take them apart and stack them and put them on top of each other, but the discovery, again, is important. With a very simple activity like this, the kids can realize that when we're talking about fractional parts, we're talking about equivalent parts. So if I have a sandwich and I cut it into three parts, and I get one-third, and Jeff gets one-third, and Serena gets one-third, we all get the same amount, okay? So here's another important discovery. Fractional parts are equivalent parts.

The low percentage of talk focused on pedagogy (0.74%) and technology-pedagogy (0.05%) demonstrates that the instructor and students rarely focused on issues of pedagogy alone. Similarly, very little classroom talk focused on technology alone. As in Cora and Jamie's classes, the only pair of sub-codes in which talk about digital technology occurred more frequently than talk about analog technology was for the category of technology alone.

Examples of classroom discourse addressing the reciprocal relationship of analog-technology, pedagogy, and content abound in Cassie's class. Reassignment of all technology-content codes to content, technology-pedagogy to pedagogy, and technology-pedagogy-content codes to pedagogy-content highlight Cassie's extraordinarily high emphasis on the relationship of pedagogy and content. However, were it not for the sub-code established to capture instances of classroom discourse that relate to the way in which analog technology and content, as well as analog technology, content, and pedagogy, are reciprocally related, Cassie's ongoing emphasis on manipulatives and direct modeling as critical instructional and learning devices would be lost. Again, compare the distribution of codes shown in Table 17 and Figure 16 with that displayed in Table 18 and Figure 17, were we not to categorize and include analog technology use in the classroom.

Table 18
Percentage of Classroom Discourse in Cassie's Class if Codes were Re-Assigned

	C	*P*	*T*	*PC*	*TC*	*TP*	*TPC*
All sessions	10.41	0.43	0.03	45.87	0.08	0.00	0.69

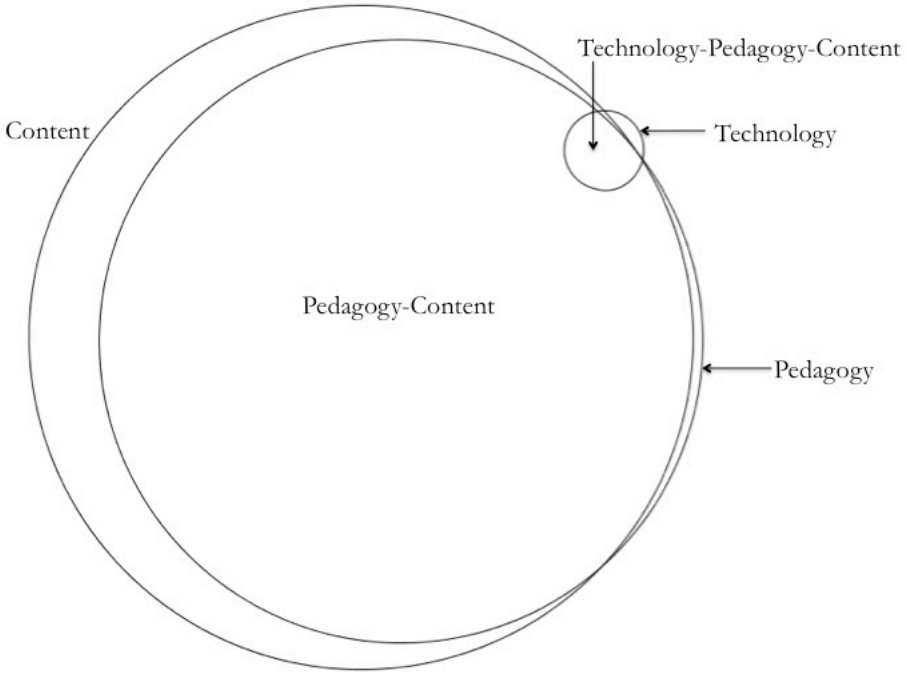


Figure 17. Area-proportional Euler diagram representing the discourse across four sessions of Cassie's class if the technology-analog sub-codes were re-assigned.

Test of independence. A chi-square test of independence was performed to examine whether there were significant differences in the pattern of categories across sessions taught by Cassie. The relation between these variables was significant, X^2 (18) = 85.98, $p < 0.05$, indicating heterogeneity in the distribution of the TPACK categories among observed class sessions. This suggests that the percentage of classroom discourse relating to each of the TPACK categories varied significantly from session to session or topic to topic.

Connections between personal theories and enactment in the classroom.

Cassie's repertory grid included ten elements. Using the interview transcript, I reviewed Cassie's description of each grid element to apply the appropriate code. However, depending on the function and context of each instructional strategy, most of the elements met the operational definition of more than one code and I found them difficult to assign. For example, when describing the element "lecture," Cassie stated that "In some cases you have

to talk to [the students] about certain things and they have to basically listen to the topic you are discussing." Depending on the lecture topic, this element might be coded as any one of the seven categories. The same challenge applied to other elements such as "work in groups" and "involve students in discussion." To address this, I returned to the interview transcript and verified that when we began the interview, Cassie clearly stated that "the first thing that came to my mind is I want my students to be knowledgeable about what they will be teaching, but I achieve that by doing a lot of other different things." In clarifying the focus of the interview and repertory grid, Cassie also articulated that each instructional strategy she listed would be couched within the context of preparing elementary education students to teach mathematics. For this reason, I decided to code many of the repertory grid elements as pedagogy-content.

Certain elements in Cassie's grid clearly referenced the use of technology as an essential component, indicating the relevance of applying the codes technology-content, technology-pedagogy, or technology-pedagogy-content. In particular, her definitions of "use manipulatives" and "use online resources" referenced the integration of technology, pedagogy, and/or content, as when Cassie described that she can:

> …talk to them about an activity using the base ten blocks and we have the base ten blocks in front of them and they do it – and then they can see the same virtual manipulative on the computer and they can see the base ten block and see that, oh, instead of actually breaking the ten into ten ones, I can click here and say undo and it breaks into ten ones.

For this reason, I did not restrict the coding of elements to P, C or PC as I did with Cora and Jamie's grids.

After coding each element, I calculated the frequency of each code assigned to the ten repertory grid elements. Content was assigned to zero of ten elements, pedagogy to two elements, pedagogy-content to six elements, and technology-pedagogy-content to two

elements. Next, I used linear regression to evaluate the strength of the relationship between Cassie's conception of the instructional strategies central to the preparation of future elementary education mathematics teachers and their enactment in the observed sessions. The distribution of codes assigned to the elements elicited in Cassie's structured interview were not a statistically significant predictor of the distribution of codes assigned to the classroom discourse in the observed sessions ($\beta = .63$, $p = 0.37$), likely because of the small dataset, yet Cassie's personal theories still accounted for 40% of the variance in their observed enactment in the classroom.

The discussion of elements during the rating phase of Cassie's interview provided insight regarding the evaluative judgment she uses in selecting the classroom activity and technology best suited to the identified learning outcomes for a particular lesson.

> Cassie: In other words…in some cases it's a good idea to use online resources, in other cases it's not. In some cases I have the students present activities and in other cases I don't have them, so it's not something I would have in the classroom all the time….
>
> Meredith: So [high expectations] you described as a constant, and then….
>
> Cassie: Yeah it's there all the time regardless of what you're teaching: decimals or fractions or the kids or the students are in discussion group or working with manipulatives or they do it online or not. For me [high expectations] should be there all the time. But [online resources and student presentations], I mean, I don't have – I don't use online resources on a daily basis….
>
> Meredith: From what you're saying these are based on –
>
> Cassie: Maybe…depending on what the concept is – depending on what the objective for that day is – [online resources and student presentations] may be there or may not be there….Yeah, I mean this is how I would describe it – that [online resources and student presentations] really depend on that specific day, that specific objective that you have, that specific concept that you're trying to teach that may be there or is not there. But the high expectations I feel should be there all the time regardless of what you're teaching.

This excerpt also demonstrates why the codes assigned to each episode in the classroom observations depend so strongly on the context of the classroom discourse and the function of the learning activity. In Cassie's class, the instructional strategy "involve students in discussion" might serve the function of sharing solution strategies to clarify mathematics content, of debating whether a mathematics activity would be more appropriate for a kindergartner or third grader, or of evaluating the merit of a technology for teaching some specific content. Since I assigned the repertory grid element a code of pedagogy-content to represent Cassie's overarching emphasis on the teaching of mathematics but corresponding classroom episodes were coded as content, pedagogy-content, or technology-pedagogy-content, this likely decreased the measured strength of the relationship between Cassie's personal theories as represented by the repertory grid and their enactment in the classroom observations.

Retrospection

I conducted a brief retrospective protocol with Cassie after each observed class to discuss a selected incident from the observed class with respect to elicited repertory grid elements and constructs. As with the structured interview that guided the repertory grid elicitation process, Cassie replied to my probing questions during the retrospective protocol evenly and assuredly. My interviews with her were the briefest of the three participants, but Cassie's responses describing her thinking about or in response to certain classroom events provided insight regarding the instantiation of her planned activities. The themes that emerged from these discussions align with those from the repertory grid and content analyses indicating Cassie's focus on using a range of instructional activities and encouraging strategic, integrated use of manipulatives to help students develop their mathematics knowledge and skills.

Cassie talked frequently with her students in class about the way children learn and how they, as teachers, can help students develop a strong conceptual understanding and move from the concrete to abstract. During a lesson on number operations in which students learned about CGI by working with manipulatives to directly model different CGI problem types, Cassie talked to them about the way they solve Join Change Unknown problems (for example, 4 + __ = 6) in comparison to the way young children solve them. In particular, she warned the teacher candidates not to describe such problems to students as subtraction problems, even though that is how an adult would solve them, because doing so would confuse the students and contradict the way children solve the problem using direct modeling. When asked about this, Cassie explained:

> So I'm hoping that they were able to get out of this the following: this is a problem the kids will approach in the correct way because it is an addition problem, and understand that you are approaching it in a different manner because you are not a child. So when it comes to applying the research, that's what I'm trying to help them understand, that [they] know this is what happens in the kids' heads and [they're] going to allow them to explore and explain and…[they] don't need to intervene and make them more confused.

Since a large part of the exploration she wants the teacher candidates to encourage in their students relies on the use of manipulatives to directly model problems, Cassie began coaching her students to use blocks in the second class session so they would become familiar with them and make them a part of their problem-solving process throughout the semester. Using manipulatives was something she had to prompt, though, as this was a new experience for most of the students.

> Another thing that I've noticed with this particular lesson…is when I give them the first problem none of them use the blocks at the beginning. They all go to paper and pencil….So then I kind of had to say, "okay, how are you going to use the blocks. Show me your strategy," and show [one student] the things and then after she used the blocks a lot of students started using the blocks.

In this way, Cassie explained how necessary it was each time she started a new semester that she encourage use of manipulatives in order for her students to experience their value and learn the importance of making manipulatives an integral part of their own classrooms.

As the teacher candidates gain experience and grow accustomed to using physical and visual models to solve problems, Cassie wants to make sure they can help their students move from the concrete model to the abstract concept. Just as they will need to do with their future students, Cassie provides as many examples as needed and then bridges from the practical approach to the theoretical. In the particular incident we discussed, she was working with the students on how to teach addition and subtraction of fractions with different denominators, represented by tiles of different shapes and colors.

> I try to help them understand how they can teach that concept to the kids using the manipulatives and trying to understand from the practical approach, okay, I'm going to do it and I have to understand why the 1/4 is now 3/12 or whatever and also help them understand how that relates to the way they learned it, which is the theory.
>
> That was the main the thing that I wanted to get out of them – to understand why we find a common denominator and the fact that before we used the same colors and we added and subtracted with the same colors and they realize, oh, it's easy if its' the same color and I can do it, but when it is two different colors, when it is two different denominators, I have to do something about it.

Circling the room during the individual or group work time and talking to the students allowed Cassie to identify how students were solving the problem, then call on them to share their strategies with the whole class. Cassie often called on students to share different ways of approaching the same problem, but also asked some students to describe their answer and approach knowing that it would provide an example of a common error or misconception.

> When I go over problem solving, one point where I always – it's not unexpected, I always expect it – is when I give them the fractions problem and they come up with 10,080 books and in the past I used to ignore it and say "that doesn't make sense" and continue. But I've seen it so many times now that I make it a point to stop and say, "okay, is there an answer? I know I'm putting you on the spot, but why is it that

> way? And why is it important that we teach the kids to stop and say, 'Is this a reasonable answer?'"

The detailed explanation Cassie provided in each of our post-observation interviews about the choices she made and how she taught her classes reiterated the way she designed each of her classes to model instructional strategies for her students, engage them in the same type of mathematic activities they should have their students complete, then facilitate a focused discussion about those strategies and the materials used and how to alter them for different student needs, content, or grade levels.

All Participants

To examine whether there were significant differences in the overall distribution of classroom talk across TPACK categories between the three instructors, I performed a chi-square test of independence. The relation between these variables was significant, X^2 (12) = 87.41, $p < 0.05$, indicating heterogeneity in the distribution of the TPACK categories among observed class sessions. This suggests that the percentage of classroom discourse relating to each of the TPACK categories varied significantly from instructor to instructor. This variation confirms the expectation that the percentage of talk focusing on each category would vary due to the way each instructor enacts their individual conception of the instructional strategies central to preparing future elementary education mathematics teachers.

I also confirmed that the distribution of talk differed when the class sessions taught by the instructors focused on the same topics, such as early number sense or fraction concepts. This analysis allowed me to explore whether the percentage distribution of TPACK categories related to the topic being presented or the instructional style of the

participant. All tests conducted indicated heterogeneity in the distribution of the TPACK categories among participants when the class sessions focused on the same topic.

Table 19
Results of Chi-Square Tests of Independence for Class Sessions Focusing on the Same Topic

Topic	*Participants*	*Results*
Problem solving	Jamie, Cassie	X^2 (6) = 42.55, $p < 0.05$
Early number sense	Cora, Jamie, Cassie	X^2 (10) = 207.30, $p < 0.05$
Numeration systems, number theory, and place value	Cora, Jamie, Cassie	X^2 (10) = 76.22, $p < 0.05$
Fractions	Cora, Jamie, Cassie	X^2 (12) = 161.39, $p < 0.05$

Summary

The results presented in this chapter describe the way three mathematics educators conceived of and enacted the instructional strategies they perceived to be most central to the preparation of future elementary education mathematics teachers. Table 20 summarizes the results for each teacher educator participant with respect to the four research questions.

Table 20
Summary of Results

	Conceptions	*Enactment*	*Retrospection*
Research question	What are teacher educators' personal theories about the instructional strategies most critical for preparing future mathematics teachers?	How do teacher educators enact their personal theories in the classroom? What does classroom discourse reveal about the relative emphasis and relationship between pedagogy, content, and technology as enacted by teacher educators?	What is the function of technology in teacher educators' personal theories about the teaching of mathematics and their practices as enacted in the classroom?
Cora	Focused on two primary themes of pedagogy and the students as future teachers.	Emphasized instructional material or task development: mathematics as activity. Technology-content and pedagogy-content received the most emphasis (54.13% total). Personal theories not a statistically significant predictor, but accounted for 94% of the	Technology enhances instructional activity.

		variance in the observed enactment.	
Jamie	Emphasized conceptual understanding of what it means to do mathematics and teach mathematics, more so than specific instructional techniques.	Emphasized mathematics content development: mathematics as content. Content and pedagogy-content received the most emphasis (55.13% total). Personal theories not a statistically significant predictor, but accounted for 31% of the variance in the observed enactment.	Technology develops core content.
Cassie	Evinced strong beliefs about specific instructional strategies she considers central to preparing future elementary education mathematics teachers.	Emphasized student cognitive development: mathematics as a way of thinking. Technology-pedagogy-content and pedagogy-content received the most emphasis (66.50% total). Personal theories not a statistically significant predictor, but accounted for 40% of the variance in the observed enactment.	Technology structures and reveals students' thinking.

As might be expected, there existed both similarities and differences in the conceptions the instructors held and the way they enacted those conceptions in their classrooms. Similarities included an expected emphasis on developing student understanding of what it means to do mathematics and teach mathematics so that the teacher candidates can in turn convey their mathematics knowledge and skills to future students. In teaching their students, however, each instructor emphasized different components of the TPACK framework used to analyze the data, with one instructor most emphasizing content, another technology-content, and a third technology-pedagogy-content. The significance of the results from the repertory grids, content analyses, and retrospective protocols is discussed in detail in the following chapter.

CHAPTER 5

DISCUSSION

To explore instructor conceptions about the teaching of mathematics and the enactment of those beliefs in the context of a pedagogy course on mathematics education, this study aimed to answer the following questions:

1. What are teacher educators' personal theories about the instructional strategies most critical for preparing future mathematics teachers?
2. How do teacher educators enact their personal theories in the classroom?
3. What does classroom discourse reveal about the relative emphasis and relationship between pedagogy, content, and technology as enacted by teacher educators?
4. What is the function of technology in teacher educators' personal theories about the teaching of mathematics and their practices as enacted in the classroom?

The results presented in Chapter 4 reveal three key findings related to these research questions. First, the conceptions that the three teacher educators hold about mathematics – mathematics as activity, mathematics as content, and mathematics as a way of thinking – govern the way in which they conceptualize and utilize technology. Triangulation of results from the repertory grids, content analysis, and retrospective protocols demonstrate three fundamentally different conceptions held by the three teacher educators about the nature of their course on methods of teaching mathematics in elementary education.

Second, instructors made good use of manipulatives and other non-digital technologies in their classes, but the emphasis on these tools of the trade was much less than one might expect and at times seemed almost ancillary to the primary emphasis on content or pedagogy-content. Even when defining technology to include both analog and digital

cases, technology use ranged from only 10% to 45% of the overall class time across the technology, technology-content, technology-pedagogy and technology-pedagogy-content categories for the three participants. Major constraints to the utilization of digital technologies exist in elementary education classrooms and either the higher education methods classes reflect common practice, the common practice reflects the mathematics methods courses, or the two settings are self-reinforcing.

Third, instructor and student discourse rarely referenced technology in isolation but instead almost always related it to pedagogy, content, or pedagogy and content. Identifying the functionality of a technology with respect to their own conception of the teaching of mathematics allowed the instructors to use the tool in a meaningful way in their classroom. Regardless of the participant, the technology provided a function useful and relevant to each individual's conception – mathematics as activity, content, or way of thinking – and therefore each instructor utilized the technology in a manner authentic to their practice.

The discussion below explores each of these key findings in detail.

Personal Theories about Instructional Strategies

To address Question 1, the results presented in Chapter 4 reveal that in responding to the guiding question of "what instructional practices do you consider most critical for preparing undergraduate elementary education students to teach mathematics" the three teacher educator participants primarily proposed general pedagogical strategies not specific to a particular content or technology. Elements such as "student (re)grouping," "make real-world connections," "build on prior knowledge," and "go to the classroom prepared" could be used in a variety of classroom settings to teach any number of content areas. However, when proposing constructs that distinguish the elements, all three participants referenced concrete examples of how they facilitate those instructional strategies in their mathematics

methods classes. For example, Cora described how her students draw connections to what they have observed or experienced as mathematics learners or in the mathematics classes where they are student teaching, and Cassie talked about arriving in advance of her students to configure the classroom space and prepare the mathematics manipulatives used in that day's activities. The general instructional strategies offer certain pedagogical benefits, but it is significant that in describing those practices the instructors articulated how they use them within the context of mathematical learning – to construct students' mathematical knowledge and develop their skills in teaching mathematics.

Grid elements such as "hands-on activities," "modeling," and "use manipulatives" made more direct allusions to the participants' use of physical artifacts as an essential component of certain strategies or activities. The description of activities as "hands-on" indicates student engagement with objects of some kind and "use manipulatives" emphasizes manipulatives as a resource central to mediating mathematics learning. Similar to their talk about general pedagogical strategies applied to the teaching of mathematics, in describing these elements the participants discussed how they use such objects to construct mathematics knowledge. For example, Cora described the number dice and dot dice her students worked with during hands-on activities about "counting on" as a mathematics addition strategy.

Although I asked a follow-up prompt of all three participants that was intended to activate the consideration of digital technology, there was little mention of such tools or resources during the element or construct elicitation phases. Cora and Jamie proposed the technology-related strategies of "providing links/URLs to fill in the gaps and tools to help them grow" and "using the document camera to show the ways models are represented." In doing so, they appeared to construe the function of digital technology as a tool or resource

for presenting or distributing content. Cassie qualified her suggestion of "use online resources" by describing how she draws connections between physical and virtual manipulatives to teach mathematical concepts and procedures. She provided the example of learning place value through base-10 blocks, whether physical or virtual, and alluded to the way the virtual manipulatives emulate the physical blocks, as well as the way students can use virtual manipulatives to construct simple mathematics addition problems. In these and other examples referenced during the structured interviews, the participants demonstrate that it is the way they enact their conceptions of instructional strategies – through integration of technology, pedagogy, and/or content – that provides meaningful learning experiences for their students.

Enactment of Personal Theories in the Classroom

To address Questions 2 and 3, the results presented in Chapter 4 illustrate models of the relative emphasis and relationship between pedagogy, content, and technology as enacted by each teacher educator participant and coded using the *a priori* scheme based on the TPACK framework. As expected, the models represent three unique individuals' enactment of their personal theories in the classroom, but also reveal three very different approaches to the preparation of their students to teach elementary mathematics. Whether conducting a similar activity or covering similar content, or addressing different instructional strategies or topics, how an instructor presents material, structures the lesson, facilitates activities, and questions students yield varying distributions of classroom talk across the TPACK components.

The frequency analysis and corresponding area-proportional Euler diagrams show that in Cora's class the categories of technology-content and pedagogy-content received the most emphasis (54.13% across the two categories), in Jamie's class the talk focused most on

content and pedagogy-content (55.13% total), and discourse in Cassie's class centered on technology-pedagogy-content and pedagogy-content (66.50% total). Using "development" as a theme for discussing these results, the models developed indicate that Cora emphasized instructional material or task development, Jamie emphasized mathematics content development, and Cassie emphasized student cognitive development (see Figure 18).

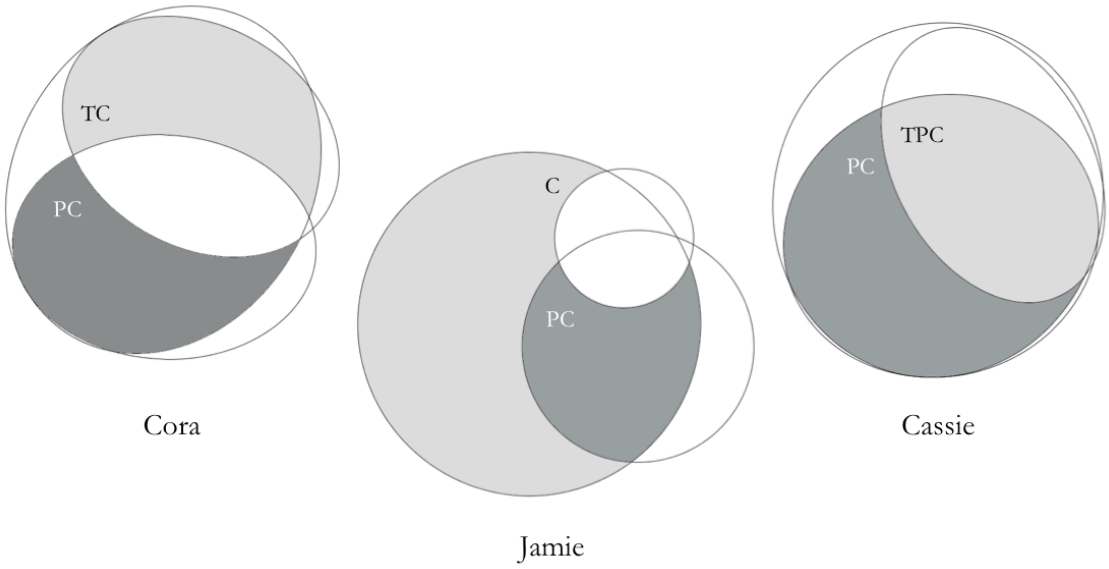


Figure 18. Models indicating the primary and secondary categories of focus for each participant.

In focusing on what you do as an elementary education mathematics teacher, Cora provided her students a veritable repository of activities and tasks that focus on mathematical reasoning and associated knowledge and skills. She engaged the future teachers in working through and discussing tasks so they might experience and learn from them prior to using them in their own classrooms. However, the classroom discussion focused more often on the nature of the task rather than the nature of students' learning and how the task and the learning work together in a coordinated, longitudinal structure. Cora's students worked with a large variety of hands-on manipulatives, such as beans, discs, popcorn and

marshmallows, and practiced activities that required them to develop models and visual representations of their mathematical thinking. Although Cora and the students discussed the way these objects and models, or technologies, could be used to help students develop their mathematical thinking, they did not usually address why those technologies were most appropriate for the particular mathematical task at hand and how the teacher candidates could most effectively use or modify them to meet certain pedagogical content needs.

Jamie's passion for mathematics and the enjoyment she has in sharing her mathematics knowledge and skills with students lead her to focus on the mathematical development of students, whether aware of this focus or not. Jamie often provided mathematics problems for students to solve at intermittent points during the class period, which allowed them to review mathematics content related to that class session's topic (such as problem solving or fractions). She asked students to share their solutions, but the ensuing discussion almost always emphasized the mathematical structure of the problem rather than the various strategies students used to solve the problem, what a teacher might learn about their students' understanding, and how to address common errors or misconceptions. Activities that might have focused on pedagogical content, such as a lesson on CGI problem types, served instead to underscore procedural elements, for example by having students write word problems of each CGI notational type but not discuss how to identify and help students who approach the problems in atypical ways.

Each lesson in Cassie's class followed a similar outline that allowed her to model instructional strategies while the students experienced an activity using manipulatives or visual models, shared solutions and the strategies used to derive them, then discussed the activity, the grade level at which it could be introduced, how to make it easier or more challenging for introductory or advanced concepts, the significance of the activity for

students' mathematical learning, how to modify the activity based on the resources available, why certain objects and models would be appropriate for the students to use, and more. Rather than approaching foundational mathematics knowledge and skills as facts or procedures, Cassie emphasized the conceptual understanding that provides the knowledge base necessary for students to develop their proficiency in mathematics. By creating situations in which her students could not draw on their adult mathematics knowledge, for example by having them solve addition problems based on an alphabetic rather than number system and therefore employ manipulatives or models rather than mathematics facts, Cassie forced them to experience mathematics in a way that emulates the mathematical thinking of their young students, then discuss the activity, the related mathematical concepts, and the young students' cognitive development. While the use of models and manipulatives served an extremely important function for Cassie in allowing students to externalize their mathematical thinking, the underlying value was the development of children's coherent and consistent thinking about mathematical relationships. Cassie's instruction emphasized that is not sufficient for an elementary education mathematics teacher to facilitate an activity with their students; they must also draw connections between what students are doing in the classroom and their cognitive development – from concrete to abstract and from individual to conventional forms of representation.

The linear regression analysis measuring the strength of the relationship between each instructor's conception of the instructional strategies central to the preparation of future elementary education mathematics teachers and their enactment in the observed sessions indicated that the instructors' personal theories accounted for 94%, 31% and 40% of the variance in their enactment in the classroom. While the results were not statistically significant because of the small dataset and the qualitative nature of this study, the practical

significance of these results warrants recognition. Classrooms are a complex space; the student body changes from class to class, the topics covered vary each week, and different resources and equipment are available. Yet, despite the small dataset that would lead one to expect much more variability, the relationship between conception and enactment remained very strong in all three participants. Indeed, the variables of pedagogy, content and pedagogy-content account for one-third to almost all of the overall variation in each instructor's observed class sessions. The data sources gathered for each participant demonstrate sufficient consistency to suggest the models developed have a verisimilitude to each person in their context and provide evidence that this line of investigation is worthwhile.

Function of Technology

To address Question 4, the discussion below weaves together the analyses and results from the structured interviews, classroom observations, and retrospective protocols to explore the function of technology in the teacher educator participants' beliefs about the teaching of mathematics and their practices as enacted in the classroom.

Together, in Isolation

Analyses of data from the classroom observations reveal that technology, pedagogy, and content often co-exist in the enactment of instructional strategies to prepare future mathematics teachers, but not in such a way that demonstrates deliberate interweaving of the three components. When coding data from all three sources, it would have been easy to identify instances of TPACK had I focused on presence alone, but it is the "thoughtful interweaving" (Mishra & Koehler, 2006, p. 1029) of these three elements that distinguishes co-existence from integration. In other words, the use of technology (T) for delivery or replication of some instructional practice (P) about a particular content (C) does not equate

TPACK. Showing a video of someone explaining or demonstrating an instructional strategy, using PowerPoint to display activity directions, or completing a handout or worksheet that does not require in the activity itself some modeling or visual representation – these are all examples of instructional activities that use technology as a tool to distribute information or facilitate an activity, but not to mediate cognitive change.

In determining whether to code an episode as TPC as opposed to PC, TC or TP, it was necessary to evaluate the function of the episode, both as I believed it to be intended by the instructor and also as construed by me as an observer (and possibly the students as well). At times, an instructor made explicit the purpose of a classroom activity as a model for students and preceded or followed the activity with focused discussion on affordances of a technology given the context, alternate technologies to use or ways to conduct the activity, and impact on student mathematical learning. In other cases, the components were used "together but in isolation" (Koehler, 2012), such as students experiencing a classroom activity (P) that used some artifact (T) related to a mathematical concept or skill (C) but the specific methods of teaching the mathematics using the technology were not made clear and discussion of the activity focused on the model or representation of mathematics rather than the rationale for using the technology to teach the content to students at a particular grade level, or how a teacher could use the technology-content activity to assess student understanding. For example, students might complete an activity that provided experience working with a visual or physical representation to teach a mathematics concept or skill, but the related discussion served no function in terms of students' understanding of the pedagogy relevant to teaching the mathematics using the technology. Such episodes were coded as T-A C rather than T-A PC because only two of the domains were addressed, as opposed to the integration of all three.

The excerpts below demonstrate examples of the type of learning objectives and questioning posed by two instructors before, during and after a similar activity on place value completed by students in two different courses. Segments of discourse in both classes were coded T-A PC, however the comments and questions made by Cora focused on procedural steps to complete the activity and other technologies that might be used as variants of the activity, yielding a higher frequency of segments coded PC or T-A C.

Excerpts from discourse during "Double Digits" activity in Cora's class:

> Each person takes a turn rolling the die. The number may be written in either the tens column or the ones column. So class, when a number is entered in the tens column, zero is written in the ones next to it. Thus four written in the tens column is really forty. Right? After each player has rolled the die seven times, the players add up the numbers. The players who are left in the game compare their totals. The person who is closest to 100 without going over is the winner.
>
> Class, Gary just did it wrong. What should I have done to make sure he understood the rules?
>
> I can see where I messed up. I didn't show you how to do the scoreboard. Class, I didn't explain the scoreboard. Some of you are doing it different ways.
>
> I noticed some of you used an algorithm to…I noticed some of you used arithmetic to record it. How did you guys record your scores, class? How did you record the scores?
>
> Here's another version….It's actually a dollar bill…that you would put the penny strips over, and some students seemed to like that one better…because they're actually creating a dollar bill….I also have another version where you use Unifix cubes. You're actually building your Unifix cubes and you would put them on the board. So there's different – You choose the one you want to use, and then if you get students that have problems putting it on there, then you would use probably the bigger board, etc. So there's three different versions of this game.

In comparison, Cassie's comments, questions and resulting student discussion addressed the mathematical learning objective of the activity and how to modify it to align with the K-12 students' level of mathematical knowledge. She also modeled the steps for completing the activity with a student and discussed the mathematical strategies for

evaluating placement of the numbers and demonstrating an understanding of place value. Her design of the pedagogical activity teaching mathematical content using technology and the 15-minute post-activity discussion focused on elements demonstrating integration of pedagogy, technology and content resulted in a higher frequency of T-A PC codes.

> Excerpts from discourse during "Place value with cards" activity in Cassie's class:
>
> Now, let's play a game. Kids love this game in 4th grade and it helps them with comparing numbers, and actually, putting numbers in order. Putting the sequence of numbers in order from the smallest to the larger, larger or largest to the smallest, is actually an item in the kids' AIMS test, okay?
>
> So, this is the objective: each person in the group will take turns getting a number out of this pile of cards, okay? …She needs to decide where to place this number [on the game card] so that when we're both done with all of our three numbers, she has the highest number.
>
> All right, how can you make this a little bit easier if you want to play this game with younger students?
>
> How can you make this more challenging if you want to give this to older students?

This "together, in isolation" phenomenon exists for the pedagogy-content, technology-content and technology-pedagogy domains as well. An instructor may have intended the function of an activity, such as writing examples of each CGI problem type, to be pedagogy-content, but without explicit discussion about the techniques or methods necessary to use those problems in a lesson in a meaningful way, the "intentional interweaving" and connection of pedagogy and content was not made and such episodes were coded as content only.

In other cases, the presence of two more components appeared coincidental, such as when an instructor called for volunteers to share solutions without knowing in advance whether a student would demonstrate a particular model, strategy or technology use. In situations where a student shared his or her thinking process and described a visual model

used to solve the problem but there was no follow-up discussion about how the teacher might use the shard solution to assess student understanding or help teach other students, the episode was coded as T-A C. In the excerpt below, Cora asks the classroom writ-large to share solution strategies without identifying specific students in advance. Additionally, from the student explanation it appears the team did not use the visual model to solve the problem but drew a model they knew represented the solution based on their existing mathematics knowledge (kids holding one dollar and a quarter rather than sharing the quarters equally among kids, resulting in five quarters each), therefore the segment was coded as content only.

Cora: Anybody else do it different and want to share their work with us? Come on, you've got to show this. Tell us what you did. [Cora holds up the student paper to show the other students but the illustration is not easily visible.]

Daryl: Well, we thought about money so there were four kids. So we gave each kid a dollar and then we were left with a dollar and had to break it up evenly. And so we gave each student a quarter.

Cora: Okay, so they first had a dollar. I don't see the – oh, they have a quarter in one hand and a dollar in the other.

In comparison, in the excerpt below, Cassie circled the room during the group work time and identified students using particular strategies she wanted them to share, then called on them and engaged the class in a discussion of what they could teach or learn about their elementary students' cognitive understanding of a mathematics concept from the use of such a strategy and technology. Many episodes during this lesson were coded as T-A PC.

Cassie: I'm going to stop for a couple of minutes and give some people the opportunity to share with everybody some of your strategies and see what works for some people, what works for other people and then we're going to continue with solving the problem. Okay…

So Ryan and friends, would you like to share with us and show us how you approached the problem?

[Sharing by students.]

So that was a fantastic strategy. It worked for them. I've seen other students who started with the S, C and T and maybe, it didn't work for them, and they started doing something else. So let's take a look at what Sarah and Ashley and friends are doing with their blocks. So one of you can start and then go back and forth. Sarah, do you want to start and tell us what you decided to do?

Sarah: Well, since the C, S, T thing didn't work for us, we just started building the sandwiches. And then we kind of got these ones done and then you started explaining, so we just built them along with Ryan's description.

Cassie: Can you bring me those so I can show everybody? So they actually started with the letters and then somewhere in the middle they got lost and they said, "Okay this is not going to work for us." So they used the blocks…

Sarah: Vicky started with the blocks from the beginning.

Cassie: So, let's take a look at what they did….

[Sharing by students.]

And then, I have another table over there that used, or at least they started using…let's see where I left my marker…right here, thank you…this strategy. What is this?

Jackie: A Venn diagram.

Cassie: A Venn diagram. Remember this from middle school, high school, MTE 180, 181 okay? This is a Venn diagram….

Alright. So we used drawings, letters and making the sandwiches with squares. We used Venn diagrams. We used the blocks. Which strategy is better?

Ryan: None. Neither. Whatever works for you.

Cassie: Whatever works for you. All of them are great strategies, but you use the one that makes sense for you, to you. So here I am going back to what I said before…the difference between going in and telling the students how to solve the problem. If I were to come in and tell you, "guys I want you to solve this problem and I want all of you to use Venn diagrams." Maybe some of you would have felt comfortable using Venn diagrams and others, maybe not. Or me coming in and telling you "don't use the blocks at all. I only want you to use paper and pencil." Some of you maybe would have felt uncomfortable because you want the blocks. You

> want to touch them. You want to put the sandwiches together as a…you know…as a young student. So it is important that you give the students time. You need to give them time and you need to give them the resources: blocks, sticks, paper, pencils, to solve the problem.

"You know this stuff," one instructor told her students in talking about number operations with single digits. Her purpose in saying this was to remind the teacher candidates that while the mathematics they are teaching their students may seem easy to them, it is necessary to break the content down and help students develop the underlying knowledge and skills they need to progress to more difficult mathematics concepts. Similarly, many activities presented in the classes of the three teacher educator participants provided an opportunity to develop the teacher candidates' ability to integrate technology, pedagogy, and content through modeling or experience with the classroom activities, but it often appeared that an instructor, as an expert in their own field, assumed students would identify and internalize the cognitive, social and developmental significance of a particular activity without needing the instructor to make explicit the nature of the mathematics knowledge, the way to organize and connect ideas, or the selection or use of a technology appropriate to the content and pedagogy. Without this focused discussion or elucidation, many opportunities to integrate knowledge, pedagogy and technology in the classroom fell short of their potential.

Digital Technology Need Not Apply

The results of this study reveal that although *digital* technology may not appear to be an essential component of an instructor's toolkit, *technology* can still play an integral role in teaching. The accuracy of this argument hinges on how we define technology. If we adopt a definition that equates technology with tools, machinery, and modern day digital devices, we mask technology use that comprises an individual's ability to evaluate the constraints and

affordances of an artifact, model, or representation and determine how to use it to teach some content in a given context. As evidenced by the models presented in Chapter 4, definitions that require us to categorize instances of "technology-content" as "content" and "technology-pedagogy-content" as "pedagogy-content" eliminate the visibility of such technology use and narrowly restrict the technology integration we perceive in classrooms. Refocusing the lens on technology as a system of knowledge, skills, and organization allows me to posit that an instructor who uses a non-digital technology in a strategic way to effect student learning could make the transition, if required, to using digital technology in the same way. In other words, it is the way that a person teaches and how she uses artifacts to effect student learning that are significant, not the tools themselves.

It is easy to make assumptions about the technology that will be available in classrooms, the value of using technology in the classroom, and judge instructors for their ability to use the technology in their teaching. However, access has been recognized as a barrier to technology integration in K-12 classrooms (M. Russell, Bebell, O'Dwyer, & O'Connor, 2003). The absence of what we might qualify as basic classroom technology, in this case an instructor computer and projector, greatly shaped the direction of this study but was likely less of an obstacle for the instructor herself. This highlights the cultural geography of the classroom – the "relationship among space, place, environment, and culture [that] provides a perspective for understanding how people function spatially" (Jordan-Bychkov, Domosh, Neumann, & Price, 2006, p. 2-3) – that we often forget about or take for granted but that has a profound impact on instructional opportunities. The space and equipment available allow or constrain the activities in which an instructor and students can engage, sometimes requiring an instructor to radically alter the design of an activity in order to work within established constraints. Distribution and collection of physical artifacts, relocation of

students, tables and chairs, creation or modification of analog resources if digital ones are not readily accessible – all these actions require extra time during or outside of class time and in turn affect the number of examples an instructor can complete with students, the number of problems students can work through independently or in groups, and the number of solutions students can share and discuss. Alternatively, a classroom or school may make technology available, but pose challenges to getting the technology into the hands of students in a setting where they can have the desired interactions.

This problem of access is not limited to digital technology, however, but applies equally well to non-digital artifacts such as mathematics manipulatives. An instructor may own only one demonstration set she can use at the front of the classroom, or enough sets for two to three groups of students, but not for every pair or individual student in the class. Cora and Cassie both discussed frequently the options for students in districts or schools that do not have the mathematics manipulatives specifically designed for instructional use, such as Unifix cubes, fraction tiles, and base-10 blocks. As Cassie told her students, "So, just because you don't have the fancy[1] manipulatives, that doesn't mean that you're not going to do the same activities that we're doing in here. There are other ways to achieve your objective." In addressing this directly, Cora and Cassie acknowledged the likely cultural geography of the classrooms their students will work in.

Indeed, it is possible that the use of non-digital manipulatives in the faculty members' own higher education classrooms is a deliberate choice designed to provide

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[1] In making reference to "fancy" manipulatives, Cassie alludes to the Unifix cubes the students used in class that day, not digital resources or virtual manipulatives. For many students, the Unifix cubes *are* fancy compared to the manipulatives Cassie encourages them to gather themselves if necessary, such as beans, pennies, kids…anything that can be counted, grouped, and used for direct modeling.

students with an experience as similar as possible to the classrooms where they will be teaching. Were the mathematics methods classes held in a highly technology-mediated classroom, it is easy to imagine that the instruction would not have varied considerably from what was observed, especially given the less frequent assignation of technology-pedagogy-content codes to discourse episodes from the observations conducted in the technology-mediated classrooms that had an instructor station, projector, document camera, and SMART Board. Due to the way technology was defined in this study, having such equipment available to two instructors did not unfairly bias the incidence of technology-pedagogy-content codes against the third instructor who did not (see Figure 19). In comparison, the class in which a variety of manipulatives were available each session, provided for each group of students, and selected to align with the day's activities demonstrated the highest use of modeling and visual representation by students as well as the largest amount of classroom talk focused on the integration of technology, pedagogy, and content. By observing approximately one-fourth (12 out of 45 hours) of the in-class time required for each of the three-credit classes, I decreased the likelihood that the sessions I observed happened to be ones in which technology was not used, or that the four sessions were anomalous in the way the instructors integrated, or did not integrate, technology with pedagogy and content.

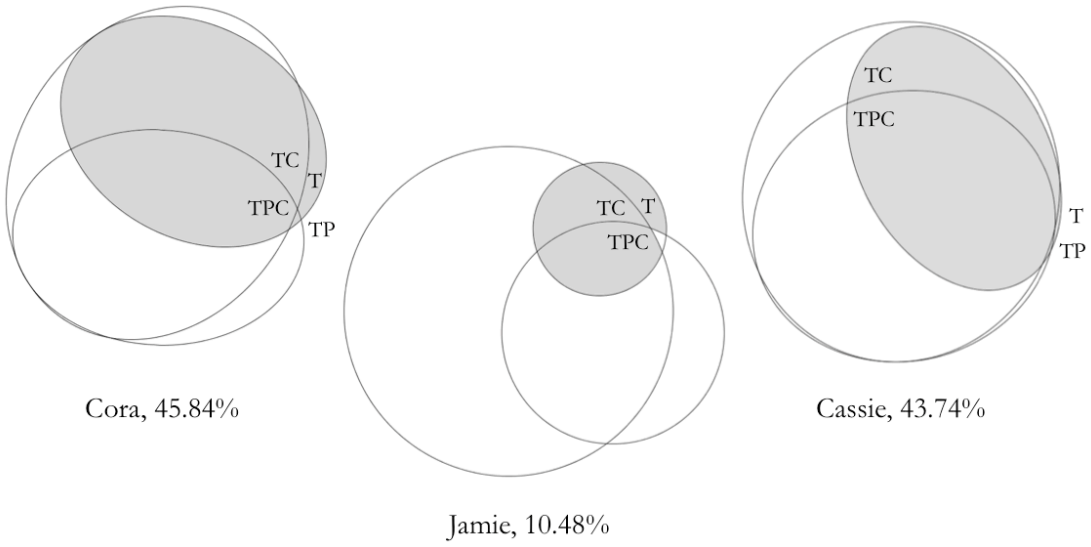


Figure 19. Models illustrating the percentage of technology use across all technology-related categories for each participant.

While "digital technology" codes of any kind were very low across all three participants, those assigned to episodes in Cora and Jamie's classes focused almost exclusively on problems with technology in the classroom, such as how to write on the SMART Board, how to turn on the document camera, why the Internet connection was not working, and why the video link from PowerPoint did not open correctly. In comparison, "digital technology" codes for Cassie's observed classes applied to an assignment she gave students to select from the National Library of Virtual Manipulatives (Utah State University, 2014) a manipulative similar to one they used in class and evaluate the type of activities it would be appropriate for, what it provided that was advantageous or disadvantageous compared to a physical manipulative, what grade level it would be appropriate for, and more.

Technically, Cora and Jamie did demonstrate usage of the digital technology in their classroom instruction, but functionally the classroom equipment allowed for little more than replication of what was already possible in analog format, such as using PowerPoint instead of overhead transparencies, slides or notecards and digital video in lieu of VHS or film.

Additionally, the function of the activities conducted with the technology served primarily to deliver content to the students by providing an outline and major points for the day's class or delivering a lecture or content that the instructor might otherwise have given. Not having calculators, student computer stations or a mandatory laptop requirement, and relevant mathematical software may have limited what the instructors could do in terms of integrating digital technology in the mathematics methods classroom. On the other hand, while it is nice to believe that those for whom technology – whether digital or non-digital – is truly important will find a way to make it available to their students, it's possible that others will decide it is not worth the extra effort.

Repurposing as Technology

Koehler et al (2011, p. 150-151) discuss the repurposing *of* technology – the need to repurpose new technologies for instructional use since few are designed with an educational purpose as their primary function.

> Most technologies teachers use have typically not been designed for educational purposes.…[A]s such, teachers must repurpose them for use in educational contexts. This is a process of *melioration*, or the "competence to borrow a concept from a field of knowledge supposedly far removed from his or her domain, and adopt it to a pressing challenge in an area of personal knowledge or interest" (Passig, 2007). Melioration acknowledges the importance and necessity of the cognitive skill of drawing on knowledge from varying domains and combining them in unique and effective ways. Such repurposing is at the heart of melioration and is possible only when the teacher knows the rules of the game, and is *fluent* enough to know which rules to bend, which to break, and which to leave alone.

This study puts forward the idea of repurposing *as* technology – the ability to repurpose items as models, tools, and visual representations and integrate them into the curriculum. If we believe that an object's purpose is not pre-defined by certain properties but that it is the interactions of the object, environment, and human psyche that determine its potential and significance, then an instructor who can evaluate an item and envision how to use it to

mediate cognitive, motivational, or social change may be able to do so with any number of artifacts. Considering this with respect to the results presented in Chapter 4, I posit that an instructor who demonstrates an ability to repurpose *analog* objects as technologies and integrate them with pedagogy and content is also likely to be able to repurpose *digital* technologies to transform cognition and learning.

Using a broad definition of technology allows for an exploration of the way instructors repurpose analog and/or digital technologies because it is their evaluative ability and creative use of objects that is significant, not their familiarity with any particular technology. During this study, I observed Cora, Jamie and Cassie model and discuss with their students alternate representations for traditional mathematics manipulatives such as dice, fraction tiles, two-color counters, base-10 blocks and Unifix cubes. Certainly, some of the manufactured manipulatives offer particular affordances, such as Unifix cubes being brightly colored in sets of ten, of a uniform size, and interlocking, but Cassie and Cora described numerous teacher-made alternatives such as beans in a cup that also allow for counting, grouping, exchanging, measuring and patterns (if the beans are distinguished by type or color), or alternatives for Unifix cubes:

> What if you don't have the blocks? You are at a school and you cannot find the manipulatives. Go to Fry's and buy a bag of beans, $0.35, right, and use those beans. Use the kids that you have in your class and say, I have 5 students around this table [gestures to one table] and 5 students around this table [gestures to second table] and 5 students around this table [gestures to third table]. How many kids do I have? 5, 10, 15 [gestures to 1, 2, 3 tables]. 3 times 5 is 15.

In describing low-cost and easily accessible options, Cora and Cassie underscore for their students the importance of manipulatives to allow students to model problems or their thinking process and reiterate that it is the use that is critical, not the object itself.

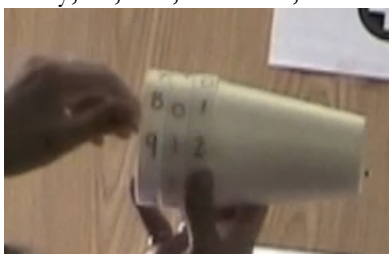
Cassie's commitment to providing the teacher candidates with opportunities to do problem solving using manipulatives, which many of the students have never done, in every single class serves to deepen the students' own understanding of how these models introduce and then reinforce concepts and relationships. This in turn helps them recognize examples and opportunities in the classrooms where they are doing their student teaching, which they describe during class, such as the excerpt below in which a student describes a model for representing standard notation using Styrofoam cups.

Cassie: Now, talking about creating your own, Andrea, do you want to share what you showed me during the break?

Andrea: Another teacher showed me this and she did her expanded notation – she was teaching place value. So, here's the ones, here's the tens, and here's the hundreds.



Okay, so, like, 912 – so, there's expanded, and then, here's standard.



Cassie: And you can see how easy and how inexpensive that is. Everybody can have their own, and they can move the cups and create the expanded notation. That's very good.

Types of objects that I observed being used or discussed as technology to teach mathematics content or methods of teaching mathematics included but were not limited to: popsicle sticks (problem solving, patterning), marshmallows and toothpicks (geometry), Unifix cubes (number operations, grouping, exchanging, patterning, sequencing, problem

solving), paper (fractions), pen/paper and marker/whiteboard (visual models), popcorn (geometry), fraction tiles (fractions), magnets (problem solving), base-10 blocks (place value, number operations), straws cut and bundled in groups of 10 (place value), counters or beans and different sized Dixie cups (number sense, number operations, exchanging, place value), clocks (multiplication, fractions), fingers (number sense, number operations), hula hoops (problem solving), penny strips (number operations), ten frames (number operations, place value), games (place value, exchanging, number sense, number operations, fractions), number line (counting, number operations, fractions), arrays (number operations), pennies (patterns, problem solving), pattern blocks or pattern cut-outs (geometry, fractions), egg cartons (fractions), music (fractions, patterns), and pipe cleaners (geometry). Through a variety of practice activities and discussion of manufactured and teacher-made manipulatives, students gained awareness of these cognitive or pedagogical technologies. It was not within the scope of this study to evaluate the extent to which the students learned to analyze the features of an object or technology, assess what it enabled students to think about and do, select a technology appropriate to the identified content and learning goals, and then integrate it in their teaching, however this line of inquiry would be worthwhile.

Focusing the study on the repurposing of objects as technology makes a unique contribution to the existing TPACK literature. Most published TPACK studies explore the themes of change in instructor technological pedagogical content knowledge through professional development, or evaluation of instructor or student technological pedagogical content knowledge through analysis of some curriculum that uses digital technology. However, this study demonstrates that when we explore TPACK in the context of a mathematics education course in which technology integration does not receive emphasis, we still find examples of the integration of technology, pedagogy, and content. Even though

an instructor is not using digital technology, we still observe strategic and meaningful interaction of analog technology, pedagogy, and content in their classroom. From this, we can learn about the way instructors teach when *not* being asked to integrate digital technology and gain insight about the function non-digital technology serves in their instruction.

f(technology) = y

If repurposing is what makes an educational technology, then one way to analyze classroom activity and discourse is to examine the *way* in which an instructor repurposes objects, rather than *which* ones are adapted and used. References to technology in isolation from pedagogy, content or pedagogy and content were rare (see Figure 20), underscoring the emphasis on *how* it was used, not *what* was used.

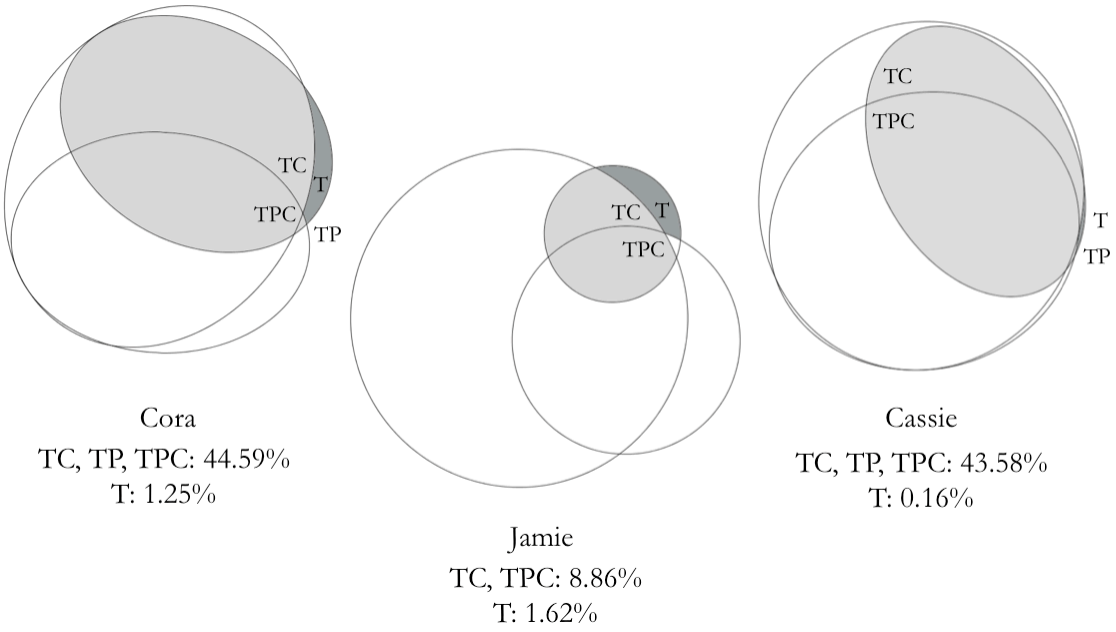


Figure 20. Models illustrating the predominant references to technology in relation to pedagogy, content, and pedagogy content rather than technology in isolation.

Indeed, technology served a variety of functions in the three participants' conceptions and enactment of the instructional strategies they believed to be most critical to preparing elementary education mathematics teachers,.

While technology can be used to affect cognitive, motivational and social elements of the learning environment (Pea, 1987), for the purposes of this discussion I define functional use as use that mediates some cognitive change situated in the context of teaching and learning mathematics and mathematics education. The function of technology to effect cognitive change does not originate with the creation of computers and digital media, nor do those in the field of educational technology claim as much. Pea's (1987) definition of cognitive technologies as "any medium that helps transcend the limitations of the mind (e.g., attention to goals, short-term memory span) in thinking, learning, and problem-solving activities" includes the systems of written language and mathematical notation and he acknowledges that every new technology, whether oral language, written language, pencil and paper, or computer, has transformed mathematics and mathematics education. This section explores the functions served by the technologies used by the teacher educator participants, thereby demonstrating the instructors' abilities to conceptualize and enact ways in which the items they repurpose as technology can effect cognitive change.

Technology as pivot. Vygotsky (1978) described the inability of young children to separate meaning from an object – without visually seeing the object they are unable to think about it; the object does not exist in the abstract. In finding a substitute object that can stand in for the original, however, the child begins to separate the meaning from the object itself. The substitute object serves as a "pivot," a fulcrum that allows the child to detach the meaning from the original and assign the meaning to another thing, indicating an ability to

think about the concept independently of the object itself. The child understands the substitute is not the original, but a direct representation of the original and its properties.

Just as meaning and object are fused, so too are problem situations and mathematical concepts. Initially, the object *is* the meaning; the problem *is* the mathematics. Manipulatives and other technologies serve as a pivot and allow students to detach the mathematics, or the "meaning," from a problem and transfer the meaning to a direct visual or physical representation. Whereas adults have developed the ability to represent problems symbolically, children must first directly model problems using substitute objects so as to identify and isolate the mathematical components or properties.

By discussing the ways different technologies can be used as visual or physical models for the mathematics in problem situations, Cora, Jamie and Cassie emphasized to the teacher candidates the need to help students pivot from the mathematics problem to a visual or physical representation and from there to a symbolic representation.

> So I want you to remember that kids need time to solve a problem and they also need something concrete…the blocks, their fingers, paper and pencil to draw a picture so that they discover certain things and they solve the problems. These strategies that you were using to solve these problems, the solution strategies that you discovered, the kids discover exactly the same strategies as they go through their math curriculum….
>
> You do exactly what the problem tells you to do and this first step in discovering solution strategies with very young students is called "direct modeling." I'm modeling directly. I'm doing what the problem is telling me to do. If the kids have clear directions, if the problem is presented to them in clear terms and they have time and manipulatives to model the problem, then the kids will be able to solve it, regardless of the operation that's involved, regardless of how large the number is, and regardless if it's a multi-step problem or it's a two-step problem, or it's only a one-step problem. If the kids can model it, they can act it out, then they will be able to solve it.

One of the most common and earliest pivots used by young children is their fingers, and both Cassie and Cora discussed this with their students:

Cora: What did we find out about the teacher that doesn't want the fingers?

John: Oh, that's my teacher.

Cora: Did you sell her on it or not?

John: No, I told her, "Hey, this kid's fingers are manipulatives." And she said that she saw in a workshop that kids that use their fingers rely on their fingers as they get older and they get left behind. I told her that's not true. I use my fingers even now and I'm 25 and I use my fingers.

Cora: Tell her your professor still uses her fingers on stuff.

John: I passed AEPA using my fingers.

Cora: The other tools that they use – the number lines and actual manipulatives – they're not going to bust those out in the grocery store. They're going to maybe count on their fingers. Well, we'll work on her. We're going to keep working on her and get her convinced.

As students develop their mathematical understanding, they decrease their reliance on pivots such as Unifix cubes, base 10 blocks, discs and other manipulatives to model a problem's properties. With repeated opportunity to practice the move from the concrete problem to the abstract mathematics and an instructor helping them make the connections, the students will learn to operate within the mathematics itself without relying on the technology, model or manipulative as an intermediary.

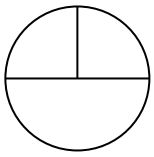
When solving problems or sharing solution strategies, the visual or physical models that the students shared with their instructor and peers also served another function, which was to externalize their mathematical thinking. This allowed students to compare problem-solving strategies as well as highlight misconceptions and use them as a learning opportunity, as in the example below:

Vicky: I know, can I come up?

Cora: Sure, come on up.

Vicky: Well, see. So remember, Mark's going to have the third, the third which is the smaller. It's like…we just had up there, where there was four

pieces, there was three triangles on the bottom, and one piece on top. [Draws illustration on board.]



Sarah: It's not a fraction.

Ryan: But it's not.

Vicky: Huh?

Sarah: It's not thirds, though.

Ryan: Yeah, it's not thirds.

Vicky: One, two, three?

Sarah: But they're not equal.

Vicky: Does the fraction have to be equal parts?

Sarah: Yeah.

Visually or physically modeling the problem, then talking through the solution strategies allows students to organize their mathematical thinking and express mathematical ideas to others. These activities help students develop not only mathematics content knowledge, but also ways to acquire and apply that knowledge.

Technology as model. In describing the function of technology as a model, I refer to models as didactical tools that provide "representations of problem situations" (Van den Heuvel-Panhuizen, 2003, p. 13) from Realistic Mathematics Education, or RME. In the Dutch approach to mathematics, a model can be generated in a number of ways, including sketches, physical models, diagrams, symbols and other visual renderings (Van den Heuvel-Panhuizen, 2003). In the classrooms observed, models often served as a bridge from the

concrete representation of a problem situation to the abstract, symbolic representation in the formal system of mathematics.

The instructors observed did not implement RME as their curriculum for teaching and learning mathematics, however characteristics of RME formed a part of their instructional design, in particular the use of models to elicit progress of "mathematization," or the human activity of arranging and rearranging reality mathematically (Freudenthal, 1968). Treffers (1987) developed a distinction between "horizontal" – leading from the "world of *life* to the world of *symbols*" (Freudenthal, 1991, p. 41) – and "vertical" – shaping, reshaping and manipulating within the world of symbols – mathematization.

Freudenthal (1991) expressed concern about whether mathematicians and mathematics educators would narrow their practice to vertical and horizontal mathematization, given their respective interests in working within the symbolic world and moving from the realistic to abstract. His assertion that both trajectories should be given equal status calls to mind the challenge the teacher educator participants in this study demonstrated in connecting technology-content, which I liken to horizontal mathematization, with pedagogy-content, a form of vertical mathematization, to thoughtfully interweave the three as technological pedagogical content knowledge.

In horizontal mathematization, students use mathematical tools to organize and solve problems situated in a realistic context. Mathematics educators facilitate this process by choosing an object, or technology, that embodies certain aspects of the relevant mathematics system and working with students to develop visual or physical models using the technology. For example, in working to solve a counting problem or numerical operation, students might work with beans, marshmallows or Unifix cubes as a representation of quantity and then apply structure to the set of objects. A number of classroom activities that Cora's students

experienced, such as drawing diagrams of word problems, creating three-dimensional shapes from toothpicks and marshmallows, and geometrically representing the location of a Hurkle on a grid, demonstrated this horizontal mathematization through the use of a tool or model to connect the realistic problem with the formal mathematics system. As already articulated in previous sections, these activities also demonstrated the careful integration of technology and content, using the technology as a form of representation of the mathematic subject matter, and made up a large percentage of the talk in Cora's classroom.

In comparison, Jamie's students more often experienced vertical mathematization and reorganizing within the mathematical system itself. In one such example, her interest in mathematical structure and passion for drawing connections between concepts and across grade levels led her to share with students a way to multiply binomials using vertical multiplication rather than the standard horizontal model. Her application of the vertical multiplication model demonstrated vertical mathematization not by virtue of the problem's physical layout, but by virtue of the connection drawn within the mathematical system by using the standard algorithm for multiplication as an alternative algorithm for multiplying binomials.

Jamie: What if you were in pre-algebra and you were looking at binomials and you had $(X + 3)(X + 1)$? How could you solve that problem? What would you have to do to solve that problem? You'd have to use the distributive property. What we end up with is…this horizontal technique. This is the way you typically do it when you get into algebra…

$(X + 3)(X + 1) =$
$(X \times X) + (X \times 1) + (3 \times X) + (3 \times 1) =$

Then we simplify right?
$X^2 + X + 3X + 3$

And then we simplify again by combining like terms…
$X^2 + 4X + 3$

I'll tell you a secret. Look at this. This is how you typically do it in algebra right? I'm not sure how often you see this anymore but what if you set it up vertically instead?

$$\begin{array}{l} (X \times 3) \\ \underline{\times\ (X \times 1)} \end{array}$$

Add it all together, what do you get?

$$\begin{array}{r} (X \times 3) \\ \underline{\times\ (X \times 1)} \\ 3 \\ X \\ 3X \\ \underline{X^2 \qquad\quad} \\ X^2+4X+3 \end{array}$$

Daryl: An algebraic equation.

Julie: Oh, my gosh! That would be so much easier than doing…

Daryl: This is way better.

Julie: Way easier.

Daryl: It's multiplication.

Jamie: It is. That's exactly what it is. It's multiplication.

The high percentage of discourse in Jamie's class sessions coded as content and pedagogy-content, in particular with respect to drawing connections across grade levels and discussing at what level particular content could be introduced, also suggests a general emphasis on this vertical mathematizing.

The ability, then, to move back and forth between horizontal and vertical mathematizing as appropriate given the problem situation, educational context, and mathematical knowledge of the individuals involved would demonstrate a fluency with the use of models to elicit progress of mathematization. Such an individual should also be able to facilitate the transition from a "model of" to a "model for" with the goal of bridging from the concrete but informal model of the original problem situation to the abstract and formal

model for reasoning other problems of a similar mathematic nature (Streefland, 1985, as cited in Van den Heuvel-Panhuizen, 2003). Cassie's class session on place value demonstrates this fluency as she moves from the physical representation of the base 10 blocks to the symbolic representation of expanded notation using horizontal mathematizing, then from expanded to standard notation using vertical mathematizing. Once students were comfortable with this model and the transition from horizontal to vertical, she repeated the process to focus on each of the four operations one at a time. She described this progress of mathematization to students:

> Again, look at my writing up here…Everything is done on the organizer [with the base 10 blocks], and then we record the information [using expanded notation]. This process where the students are doing it, and you are writing it, do it and write it, do it and write it, step-by-step, is called "bridging." I'm creating a bridge between the concrete manipulatives and the writing part of the algorithm. And once the kids understand this very well, then I'm going to move on to [the standard algorithm] and I'm going to use the same example [as with the base 10 blocks and expanded notation].

This pivoting from horizontal to vertical and back again as needed reflects an evaluative ability similar to that encapsulated in the concept of technological pedagogical content knowledge: the ability to render a representation of a concept or problem using an appropriate model or technology, explain why the model or technology is appropriate given the mathematical and learning contexts, and modify or extend the model as necessary to apply it to other mathematical problems or explain it to other individuals.

Technology as amplifier. An amplifying technology allows individuals to perform the same cognitive tasks they are already capable of, but easier, faster and more efficiently than might otherwise be possible (Pea, 1985; Pea, 1987). An activity from Cassie's class introducing problem solving as an NCTM standard for the process of learning mathematics (National Council of Teachers of Mathematics, 2000) provides an illustrative example. To

begin, Cassie asked the students to take 12 Popsicle sticks each from the bag on their table and make an arrangement like the one drawn on the board:



She then engaged them in a discussion about the attributes of the arrangement. Although Cassie could have directed students to replicate the drawing on a piece of paper, use of the popsicle sticks allowed her to discuss the arrangement knowing that as long as they used the sticks provided and arranged them as shown on the board, the student arrangements would all be the same. Each student had a common physical model and could discuss its attributes as a class, whereas had they each drawn their own diagram this might not have been the case. Furthermore, using Popsicle sticks with the same length and size ensured the students could apply mathematic knowledge to respond to a question about how they knew that the shapes were squares.

Cassie: What else can you tell me about the arrangement that you have right in front of you, Megan?

Megan: There's 5 squares.

Cassie: There are 5 squares…Okay I have a question for you. Are you certain that those are squares or are you assuming that they are squares because they look like squares?

Karen: Assuming.

Cassie: You're assuming. Nobody is certain that these are squares? Well, Gary, yes?

Gary: Well, I think I could say that they are squares because I could say that each of these [sticks] is the same length so each side of my square is the same length, which is the criteria for a square.

Cassie: Very good. So you're not assuming you know for sure that these are squares because the side of each square is exactly the same. The sticks

have the same sides so we have congruent sides, right? And how about the corners? Did they matter?

Kevin: Yes.

Cassie: The angles. Why, how?

Kevin: They have to be 90 degrees.

Cassie: Ok, are they 90 degrees?

Kevin: Yes, yes.

Cassie: Yes, yes, okay, they look like they're 90 degrees okay. Ok, so we're going to call those squares, all right?

After the initial discussion about its attributes, Cassie assigned the students a problem to solve using the arrangement in front of them and reiterated several times the need for them to physically move the sticks while trying to solve the problem.

> First task for today: I would like you to move three sticks, okay? You're going to place them somewhere else in your arrangement, so you're going to end up using all your 12 sticks again. But instead of having four squares in front of you I want you to end up having three squares. Okay. So let me repeat the directions. You're going to need to move three sticks…you're going to have to pick up three sticks, place them down again somewhere in your arrangement so that you're using all your sticks, the 12 sticks again, but instead of having four squares in front of you, you're going to end up having three squares in front of you.
>
> [Quiet in classroom as students look at the sticks and consider the problem.]
>
> And this is where you really have to touch the sticks. They're not going to move by looking at them [laughter among students]. Okay, so you need to touch and move the sticks around.

Using the sticks made it easier and more efficient for the students to guess and check via physical manipulation and in this way afforded more of a problem-solving experience. In comparison, pencil and paper drawings would have forced the students to think analytically rather than allowing them to "play around" until they generated the solution. However, had they opted to draw multiple copies of the arrangement as a way of representing their guess and check attempts, the students would have established a record of their work for review

and reference. Use of the popsicle allowed students to try different strategies to solve the problem, then think analytically about the pattern or algorithm that would allow them to solve a similar problem, whereas pencil and paper might have required them to be more analytical up front but also might have prevented them from solving the problem as quickly or easily.

The base 10 blocks shared with students by two of the instructors provided another example of an amplifying technology. During the classroom observations, I saw or heard reference to multiple representations for place value, including dots, lines and squares to represent ones, tens and hundreds, beans in a cup, bundles of straws, and base 10 blocks. While all the models allowed for the same kind of cognitive work by the students and some allowed students to physically manipulate them to represent numbers and complete problems using the four operations, the proportional representation of numbers afforded by the base 10 blocks offered a distinct advantage that provided a faster and more efficient way to teach place value concepts to students.

Teacher as Technology

After reviewing the results and discussion presented thus far, it becomes clear that a central question to ask is whether the class sessions observed would have been different if conducted in technology-mediated classrooms. Would the diagrams representing the relationship of technology, pedagogy, and content have looked different for any of the instructors? Of course, the type of technology available and the definition of "technology" used should affect the answer. From reviewing the course syllabi, I know that Cassie dedicated one lesson during the semester to "calculators/technology" but that neither Cora nor Jamie indicated by way of the class schedule that they would address a similar topic in a specific class session. If one of the instructors had wanted to, holding class in a room where

each student had access to a computer did not appear possible at the district-based locations and although some students brought laptops with them, Cora was the only instructor who did not ask students to "please put your computers away…put your laptops away" at the beginning of class. However, the less frequent assignation of technology-pedagogy-content codes to discourse episodes from the observations conducted in the technology-mediated classrooms with an instructor station, projector, document camera, and SMART Board indicates that it was not the presence or absence of such equipment that dictated whether an instructor used technology, as defined in this study, in teaching the mathematics methods classes.

If, however, we construe the *teacher* as the technology, then it is the instructor who steps in between the learning materials and the teacher candidates and serves as a mediational tool to guide students in learning new meanings for "old" artifacts. It is the instructor who informs the novices and teaches them how to repurpose objects in mathematical ways. Casting the teacher as the technology means that a change of location to a technology-mediated classroom would not change significantly the relationship of technology, pedagogy, and content represented in the model developed for each instructor, because the setting would not alter an individual's evaluative ability and creative use of objects. As already observed, some instructors would demonstrate thoughtful use and integration of the technology available, while others would use it to replicate existing practices, as they did when using a computer to show a video in lieu of lecturing or using a SMART Board to display a PowerPoint that presents information and structures the class.

Although manipulatives may embody mathematical concepts, the objects themselves do not have an inherent ability to communicate the mathematical system they represent and so teacher educators play a central role in enculturating teacher candidates into the practices

of the mathematics education community and teaching them how to help students construct connections between problem situations, material objects and mathematical systems. For example, although base 10 blocks provide a proportional model, they have no inherent association with place value and a teacher must guide and support students in making a connection between a problem scenario, the concrete, physical model and the abstract mathematical concepts and standard mathematics notation (Gravemeijer, 2002). In this regard, when students work with the base 10 blocks they are not just engaging with a technology, but also with the mathematics education community's purpose for the blocks that provide them mathematical meaning (Wertsch, 1991). Furthermore, by introducing such tools at very young ages, children learn differently (Vygotsky, 1978) – as an example, children could learn to conceive of blocks as not just blocks, but part of a mathematical system and, depending on the context, a physical representation of patterns, number systems, quantity, geometric shapes, and more.

Previous sections explored the ways in which the teacher educator participants repurposed non-digital objects and materials to serve functions attributed to both digital and non-digital technologies, thereby demonstrating the instructors' abilities to conceptualize and enact ways in which the items they repurpose as technology can effect cognitive change. The proposition that the instructor, not the materials or conceptual objects, is the technology references a recurring theme in educational research about teacher quality as a predictor of student performance (Darling-Hammond, 2000; Hanushek, 1992; Rivkin, Hanushek, & Kain, 2005; Wenglinsky, 2000). More research is needed about how to design and carry out lessons that deliberately and thoughtfully integrate instructional tools and organizational structures to promote the learning of mathematical concepts (M. A. Simon, 1994; M. A. Simon, 2008).

Summary and Lines of Future Research

This research study serves as the initial step in a long-term, coherent research program intended to address the need for high-quality teacher preparation and professional development related to methods of teaching content such as science, mathematics, literacy and language arts, and history and social studies. Working with higher education faculty in the field of mathematics education, this study advances knowledge of teacher educators' personal theories about methods of teaching mathematics and how they enact those theories in their undergraduate teacher preparation classrooms. Results show that although *digital* technology may not appear to be an essential component of an instructor's toolkit, *technology* still plays an integral role in teaching.

As new standards for teacher preparation continue to emerge (International Society for Technology in Education, 2008; National Council for Accreditation of Teacher Education, 2008; National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010; NGSS Lead States, 2013), individuals and organizations will design professional development based on these guidelines. If we are to gather accurate and useful information about the use of technology in the instruction of mathematics and other content areas, it is imperative we articulate a very clear and functional definition or else risk overlooking or neglecting opportunities for meaningful technology integration.

What must we put in place to provide meaningful professional development and facilitate authentic technology integration rather than replicate existing practices? First, we must work in alignment with teachers' goals and provide tools that promote activity and require students to work individually or collectively to construct knowledge. Then, the tools must structure and reveal deeper, rigorous mathematical (or other) content. Finally, the tools must structure and reveal students' mathematical (or other) thinking as they grapple with the

deeper content. In this way, we bring together technology, pedagogy, and content in a way that has meaning for instructors. By emphasizing the functionality of technology – the way it can enhance an activity, develop core content, and/or reveal and structure students' thinking – we provide a way for instructors to map the way they think about their content, categorize what they do as teachers, and reflect on what they emphasize in their classroom and how they could better focus on or articulate the intersection of two or more domains of technology, pedagogy, and content.

Continued pursuit of research related to conceptions and enactment of technology would allow me to explore whether an instructor who repurposes non-digital technologies in a strategic way to effect student learning could make the transition, if required, to using digital technology in the same way. Additionally, after observing the three instructors who participated in this study and the creative ways in which they repurposed objects as technology for the purposes of teaching mathematics, I am eager to explore the strategic integration of digital technology, pedagogy and mathematics in the classroom of an instructor who exhibits that fluency. Finally, the research methods used in this study can transport across other major content areas in order to explore similarities and differences between individuals and subgroups, such as areas of concentration.

Knowledge generated from such studies can form the foundation for a trajectory of research that aims to improve technology integration in undergraduate teacher preparation programs through: development of mental models of "technology" as conceived of by instructors who do not strategically integrate technology in their teaching, those who strategically integrate non-digital technology, and those who strategically integrate digital technology; the exploration of linkages between the ways faculty members conceive of technology and how they integrate technology in their instruction; ways to emphasize and

explore the functionality of technology with respect to individuals' instructional goals; the design and testing of professional development for education faculty to develop their ability to evaluate, repurpose and integrate technology in their classrooms; the way interactive communication with peers during professional development episodes affect an individual's personal theories about teaching and technology; and how changes in an individual's practices affect his or her beliefs regarding technology and its integration with content and pedagogy.

It is my intention to pursue a research agenda that combines instructional design and classroom-based research (Cobb, 2001) through the direct involvement of practitioners to help clarify the problem, identify potential solutions, and iteratively test and refine a prototype learning environment (Cobb, Confrey, diSessa, Lehrer, & Schauble, 2003; van den Akker, 1999), as well as posit new design principles grounded in learning theory and contribute to the development of an informed theoretical perspective for instructional technology research (Barab & Squire, 2004).

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APPENDIX A

ASU HUMAN SUBJECTS INSTITUTIONAL REVIEW BOARD APPROVAL

ASU Knowledge Enterprise Development

Office of Research Integrity and Assurance

To:	James Middleton Payne
From:	Mark Roosa, Chair Soc Beh IRB
Date:	07/20/2011
Committee Action:	**Exemption Granted**
IRB Action Date:	07/20/2011
IRB Protocol #:	1107006654
Study Title:	Modeling Mathematics of Educators' Conceptual Understanding and Enacted Interactions of Pedagogy, Content and Technology

The above-referenced protocol is considered exempt after review by the Institutional Review Board pursuant to Federal regulations, 45 CFR Part 46.101(b)(1) (2) .

This part of the federal regulations requires that the information be recorded by investigators in such a manner that subjects cannot be identified, directly or through identifiers linked to the subjects. It is necessary that the information obtained not be such that if disclosed outside the research, it could reasonably place the subjects at risk of criminal or civil liability, or be damaging to the subjects' financial standing, employability, or reputation.

You should retain a copy of this letter for your records.

APPENDIX B

LETTER SOLICITING PARTICIPANT INVOLVEMENT

Dear [Participant Name]

I am a doctoral student in the Curriculum and Instruction, Mathematics Education concentration at Arizona State University conducting research under the supervision of Professor Jim Middleton. The purpose of my research study, entitled "Modeling Mathematics Educators' Conceptual and Enacted Interactions of Technology, Pedagogy, and Content," is to explore the instructional strategies that teacher educators believe are most critical to preparing elementary educators to teach mathematics at the K-8 level with the goal of understanding how instructors conceive of and implement the relationships between pedagogy, content, and technology.

For the purposes of this study I am focusing on the sections of EED 412: Mathematics in Elementary Schools offered by ASU at the iTeachAZ district sites and I am respectfully asking you to consider participating in my research project. The study will collect four primary types of data: 1) background information related to your educational preparation, teaching experience, and mathematics content background; 2) one structured interview approximately one hour in length, conducted at the convenience of the participants; 3) observations of 5-10 class sessions over the course of the semester; and 4) a brief discussion of approximately 10 minutes after each observed class session. I plan to gather all data during the spring 2012 semester.

I would like to audiotape the interviews and videotape and audiotape the observations and follow-up discussions. The interviews and observations will not be recorded without your permission. Please let me know if you do not want the interview or observations to be taped; you also can change your mind after the interview or observations start, just let me know.

Your participation in this study is voluntary. If you choose not to participate or to withdraw from the study at any time, there will be no penalty. There are no foreseeable risks or discomforts to your participation.

If you agree to participate, I will provide an official informed consent document that outlines the full terms of the research study and participation that you can review. If you have any questions concerning this research study, please contact me at (602) 405-1365 or my faculty advisor, Dr. Middleton, at (480) 965-9644. If you have any questions about your rights as a subject/participant in this research, or if you feel you have been placed at risk, you can contact the Chair of the Human Subjects Institutional Review Board, through the ASU Office of Research Integrity and Assurance, at (480) 965-6788.

I appreciate your time and consideration and look forward to hearing from you regarding whether you wish to be a part of the study.

Respectfully,

Meredith Toth

APPENDIX C

INTERVIEW SCRIPT FOR STRUCTURED INTERVIEWS

The interview script below is to be read by me prior to beginning the structured interview.

"Before we begin, I want to thank you for taking the time to meet with me today. This interview forms a part of my doctoral research study on how instructors give meaning to technology as it relates to pedagogy and content.

Today, we will be conducting a structured interview about the topic of instructional practices for preparing future mathematics teachers. The goal of this interview is for me to understand what you think about this topic, using your own concepts and terms. Since I'm trying to understand how and what you think about this topic, there are no "right answers." This particular interview technique allows us to be very precise about what you think about these instructional practices, but how much detail we go into is up to you. As part of this process, I'll be asking you to make a series of systematic comparisons about methods for teaching mathematics to future teachers and this interview process will probably take about one hour to complete.

Data from this interview will be used for the purposes of the study only. Any transcripts, reports, published articles or conference proceedings related to this study will refer to research participants by a pseudonym that is assigned and documented in a code sheet kept secure and available only to myself and my advisor.

Is everything I've mentioned acceptable? Do you have any questions before we start?"

The following phrases are intended to help the interviewee identify a set of elements to be discussed during the interview. The first question will not mention the use of

technology. The second question will introduce technology so as to activate its consideration by the interviewee while discussing additional elements.

- Describe for me a situation from your regular teaching practice that is important to your conception of what your students need to be developing as future elementary education mathematics teachers.
- Describe for me another situation from your regular teaching practice that achieves the same purpose, this time one that incorporates technology in some significant or meaningful way.

The following phrases are intended to help the interviewee identify additional elements for discussion and rating:

- Describe for me a teaching practice you consider critical for preparing undergraduate education students to teach mathematics.
- Describe for me an instructional practice you think would appeal to students interested in using technology in an educational setting.